



Note on Value Drivers¹

Value-based management assumes that value creation should be a primary consideration in managerial decision making. It requires a thorough understanding of what creates value and why as well as the ability to measure value accurately. The goal of this note is to highlight the determinants of equity value and, in doing so, provide a framework for making financial, strategic, and investment decisions. In particular, the note describes three value drivers: profitability, advantage horizon, and reinvestment. Using both a theoretical model and a numerical example, it shows how each value driver affects equity value and explains why. It also presents empirical evidence to support the relation between the value drivers and value creation.

Theoretical Equity Valuation Model

Discounted cash flow (DCF) analysis translates future cash flows into current market values. For example, given a stream of equity cash flows (ECF) and a discount rate equal to the cost of equity (K_E), the market value of equity (E_{MV}) is the present value of future equity cash flows:

$$E_{MV} = ECF_1 / (1 + K_E) + ECF_2 / (1 + K_E)^2 + \dots \quad (1)$$

When the equity cash flows and discount rate are constant over time, this series is a stable perpetuity which can be written as:

$$E_{MV} = ECF / K_E \quad (2)$$

Assuming that the equity cash flows are equal to the accounting return on equity (ROE) times the book value of equity (E_B) at the beginning of the period, then equation 2 can be rewritten as:

$$E_{MV} = [(ROE) * (E_B)] / K_E \quad (3)$$

where $ROE = \text{Net Income} / E_B$

While the assumption that equity cash flows are equal to accounting earnings is convenient for expositional reasons, this assumption is clearly not valid except in very special circumstances. For example, non-cash items such as depreciation or deferred taxes, and cash-items that do not flow through the income statement such as changes in working capital and fixed assets both cause cash

¹ Much of the material in this note appears in Fruhan (1979), chapter 1.

Professor Benjamin C. Esty prepared this note as the basis for class discussion.

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flows to deviate from reported net income. Nevertheless, this assumption is not a bad approximation and, as will be shown in the next section, seems to generate reasonable empirical predictions.

After dividing each side of equation 3 by the book value of equity, the left side of the equality becomes the market-to-book ratio (the market value of equity divided by the book value of equity):

$$\text{Market/Book} = E_{MV} / E_{BV} = \text{ROE} / K_E \quad (4)$$

Equation 4 says that a firm's market-to-book ratio equals the ratio of its return on equity to its cost of equity. This simple valuation model, or variations of it, can be used to analyze the relation between profitability, growth, and value.

Profitability

The first value driver, profitability, is immediately clear from equation 4. For a given industry, more profitable firms—those able to generate higher returns per dollar of equity—should have higher market-to-book ratios. Conversely, firms which are unable to generate returns in excess of their cost of equity should sell for less than book value.

<u>Profitability</u>	<u>Value</u>
If $\text{ROE} > K_E$	then $\text{Market/Book} > 1$
If $\text{ROE} = K_E$	then $\text{Market/Book} = 1$
If $\text{ROE} < K_E$	then $\text{Market/Book} < 1$

One implication of this model is that firms can increase equity value by increasing their return on equity. The Du Pont formula decomposes ROE into three components and provides some guidance on how to increase it:

$$\begin{aligned} \text{ROE} &= (\text{Net Income/Equity}) \\ &= (\text{Net Income/Sales}) * (\text{Sales/Assets}) * (\text{Assets/Equity}) \\ &= (\text{Profit Margin}) * (\text{Asset Turnover}) * (\text{Financial Leverage}) \end{aligned}$$

For example, increasing the profit margin through higher prices or lower costs will increase the ROE. Similarly, increasing the asset turnover by increasing inventory turnover or reducing days receivables will increase the ROE. However, increasing financial leverage has dual, and possibly contradictory, effects. It increases not only the ROE through the Du Pont formula, but also the cost of equity.

A firm's cost of equity, or equivalently investors' expected return on equity, can be estimated using the Capital Asset Pricing Model (CAPM). According to the model, the expected return on equity is a function of a firm's equity beta (β_E) which, in turn, is a function of both leverage and asset risk (β_A):

$$K_E = R_F + \beta_E (R_M - R_F) \quad (5)$$

where:

$$\begin{aligned} R_M &= \text{return on the market portfolio} \\ R_F &= \text{risk-free rate of return} \\ \beta_E &= [\beta_A - \beta_D (D/V)] (V/E) \end{aligned} \quad (6)$$

because:

$$\beta_A = \beta_D (D/V) + \beta_E (E/V) \quad (7)$$

and

$$\text{Firm Value (V)} = \text{Debt Value (D)} + \text{Equity Value (E)} \quad (8)$$

Assuming riskless debt, meaning the beta of debt is zero, then equation 6 can be written as:

$$\beta_E = \beta_A (V/E) \quad (9)$$

As financial leverage (D/V) increases, the ratio of firm value to equity value (V/E) increases, the equity beta increases, and, according to equation 5, the expected return on equity increases. The expected return increases because equity cash flows are riskier: leverage increases debtholders' fractional claim on the firm's cash flows. As a result, an increase in leverage can either increase or decrease the ratio in equation 4 depending on whether the return on equity (the numerator) or the cost of equity (the denominator) increases faster.

Advantage Horizon

Equation 4 presents a firm's market-to-book ratio as a stable perpetuity under the assumption that its profitability remains constant forever. An alternative, and more realistic assumption, is that firms generate positive abnormal returns—returns in excess of their cost of capital—for only a limited number of years. The period during which firms generate positive abnormal returns is known as the advantage horizon.

Using a variation of the simple valuation model in equation 4, Appendix 1 derives the market-to-book ratio as an annuity rather than a stable perpetuity. It assumes that a firm's equity returns can be divided into two parts: *normal* returns equal to the firm's cost of equity (K_E) and *abnormal* returns equal to the actual ROE less the cost of equity ($ROE - K_E$). Viewed in this fashion, one can think of abnormal returns and the advantage horizon in the same way Stewart (1991) defines economic value added (EVA) and the competitive advantage period (CAP). Equation A1.8 from the Appendix 1 is:²

$$\text{Market/Book} = 1 + (ROE - K_E) * [(1/K_E) - (1/(K_E(1+K_E)^n))] \quad (10)$$

where the advantage horizon is defined as n years. According to this formula, the greater the spread between a firm's return on equity and its cost of equity ($ROE - K_E$), the longer the advantage horizon (increasing n), and the sooner abnormal returns occur (positive abnormal returns in early years), the higher the market-to-book ratio. Firms that earn normal returns ($K_E = ROE$) in all periods should have market-to-book ratios equal to one; firms that generate negative abnormal returns during the advantage (disadvantage) period should have market-to-book ratios less than one.

Equation 10 is more realistic than equation 4 because most firms earn positive abnormal returns for only a limited number of years. The presence of positive abnormal returns encourages entry by new firms and increased competition by existing firms. Over time, competition reduces excess returns to the point where firms just earn the expected, or normal, rate of return. Although there is typically an inverse relation between the magnitude of positive abnormal profits and the length of the advantage horizon, this model implies that firms should seek to extend the advantage horizon as long as possible for a given level of profitability.

Ghemawat (1991) refers to this ability to preserve competitive advantage as sustainability and asserts it is a key determinant of value creation. Sustainability, he maintains, depends on a firm's ability to create scarcity value and for the firm's owners to capture or appropriate this value. Threats to scarcity value include imitation and substitution. A firm can defend against imitation by erecting barriers to entry or forestalling entry through aggressive positioning; a firm can defend against substitution by continually improving or augmenting its product. Threats to appropriability include

² This formula is a variation of the accounting-based valuation methods described in Bernard (1994); Palepu, Bernard, and Healy (1996), and Ohlson (1995).

slack and hold-up both of which result from misaligned incentives. Slack occurs when firms fail to create as much value as they are capable of creating; hold-up occurs when non-owners, instead of owners, capture value. Non-owners are often able to capture value when they provide complementary, and necessary, inputs.

Reinvestment

The third value driver, reinvestment, builds on the other two factors and incorporates the concept of growth. Firms that have attractive investment opportunities, meaning that investments are expected to generate positive abnormal earnings, can create equity value by reinvesting earnings or by investing additional equity. Appendix 2 derives a valuation model which allows for reinvestment of earnings at rate γ where γ equals the retention rate or the fraction of net income reinvested in the firm. The quantity γROE is a firm's sustainable growth rate, the rate at which it can grow its assets (or sales if they are proportional to assets) without changing its capital structure or raising external equity. With reinvestment, the valuation model becomes (equation A2.4):

$$\text{Market/Book} = [\text{ROE}(1 - \gamma)] / (K_E - \gamma\text{ROE}) \quad (11)$$

When a firm pays out all of its earnings as dividends, then the retention rate is zero ($\gamma = 0$) and equation 11 reduces to the simple valuation model in equation 4. Assuming a firm has attractive investment opportunities in which it can generate positive abnormal returns ($\text{ROE} > K_E$), then it can increase value by retaining a larger fraction of earnings and investing them in the business. Thus reinvestment and growth creates value only when a firm can generate positive abnormal returns on future investment opportunities. Those firms with the greatest number and the most profitable investment opportunities should have the highest market-to-book ratios provided they are able to fund the projects.

In fact, it is often convenient to think of firm value as consisting of two parts: the present value of assets in place and the present value of future growth opportunities (Myers, 1977). The former require little in the way of additional investment, while the latter are investment opportunities which are expected to earn positive abnormal returns. These investment opportunities are called "real" options because they resemble financial options, particularly call options. They can be interpreted and managed using option pricing theory and valued using option pricing techniques (see Luehrman, 1995).

Numerical Example

Combining equations 10 and 11 produces a single valuation model that incorporates all three value drivers. Exhibit 1 shows this model as well as the relation between a hypothetical firm's market-to-book ratio and the value drivers. The exhibit presents three cases with differing levels of reinvestment ($\gamma = 0\%$, 33% , and 66%). For each case, there is a sensitivity table showing how the market-to-book ratio depends on the advantage horizon and level of profitability (ROE).

Case #1 (no reinvestment) shows that more profitable firms have higher market-to-book ratios—the ratio increases as one reads across the rows. As stated earlier, the impact of the advantage horizon depends on whether a firm generates positive or negative abnormal earnings. The longer a firm can generate positive abnormal earnings, the greater its market-to-book ratio. However, because of discounting, abnormal earnings in later years have a smaller impact on the market-to-book ratio than abnormal earnings in early years. Alternatively, firms that generate negative abnormal earnings have market-to-book ratios less than one. Moreover, their market-to-book ratio falls as the advantage

(disadvantage) horizon gets longer. Finally, the market-to-book ratio is equal to one and is independent of the advantage horizon for firms that generate normal earnings (the case where $ROE = K_E$).

Cases #2 and #3 (with reinvestment rates equal to 33% and 66%, respectively) illustrate the impact of reinvestment. Like the advantage horizon, reinvestment creates additional value only for firms that generate positive abnormal earnings. When firms are able to generate positive abnormal returns ($ROE = 25\%$), have a long advantage horizon (30 years), and reinvest a large fraction of earnings ($\gamma = 66\%$), they create significant value. The difference between the market-to-book ratio in the high return/long horizon with no reinvestment (case #1) and with reinvestment (case #3) is large: 1.66 vs. 4.27.

Empirical Evidence

This section presents empirical evidence on the relation between the value drivers and value creation. Despite the assumptions imbedded in the simple valuation models, they do, nonetheless, yield predictions which are consistent with what we observe in practice.

Profitability

The model predicts that there is a relation between a firm's market-to-book ratio and the ratio of its return on equity to its cost of equity. Given a set of firms in a single industry, the model implies that there should be a positive relation between ROE's and market-to-book ratios for these firms assuming their costs of capital are approximately equal. To a first approximation, it is reasonable to assume that firms in the same industry will have similar capital costs because they hold similar assets and, typically, have similar capital structures.

Exhibit 2 shows the relation between market-to-book ratios and firm profitability for two quite different industries: grocery stores and oil field service companies. Whereas the grocery industry is a retail business with high inventories and low margins, the oil-field services industry is a service business with industrial customers and higher margins. Yet in both cases, there is a very clear, positive relation between equity value and ROE's: higher ROE's are associated with higher market-to-book ratios. Fruhan (1996) presents similar evidence for a much wider range of industries including newspapers, telecommunications, and specialty chemicals.

There are at least two reasons why this relation does not hold perfectly. First, not all firms in the same industry have the same leverage or same asset risk. Thus, financial and operating differences cause the cost of equity to differ across firms. Second, accounting data is subject to manipulation by managers. On the one hand, managers provide valuable information through their choice of accounting disclosures and policies. On the other hand, they are biased which may lead them to distort reported numbers. Fortunately, however, most distortions occur through accruals which eventually get reversed. Because accounting data is subject to this kind of manipulation, it is critical to understand whether the reported numbers reflect economic reality. To the extent high ROE's reflect economic reality, and not unreasonable deferral of costs or a one-time aberrations, then the relation shown in exhibit 2 will be stronger. When accounting data does not reflect economic reality, one must undo the distortions before trying to make substantive conclusions about the business or its prospects.

Advantage Horizon

Several researchers have studied the length of the advantage horizon. For example, Fruhan (1995) examined a sample of 87 "high-performing" firms defined as those firms with sales of greater than \$200 million and an average ROE of greater than 25% for five consecutive years between 1976-82. He calculated the median ROE for the firms from 1976-78 and from 1989-93, and then compared these medians against the average ROE for firms on the S&P 400 (see Exhibit 3). Whereas the median ROE for the high-performing subgroup was 21% above the average ROE for the S&P 400 in 1976-82, it was only 2% above in the later period. Thus the high-performing firms' abnormal earnings had largely dissipated over the fifteen year interval.

Palepu *et al* (1996, pp. 5.4-5.7) report similar findings: abnormally high or low ROE's tend to revert to normal levels, roughly between 10-14%, often within five years and usually within ten years.³ The reversion in ROE's is largely due to reversion in profit margins rather than reversion in asset turnover or leverage which remain relatively constant over time. The fact that advantage horizon lasts for five or ten years provides some justification for using five or ten-year projections in discounted cash flow analysis.

In another study, Ghemawat (1991) examined the returns on investment (ROI) for 692 business units from 1971-1980. After sorting the business units by their ROI in 1971, he divided the sample into two equal subgroups and calculated the average ROI for each subgroup over the next ten years. Initially, the top group had an average ROI of 39% compared to 3% for the bottom group. The 36% spread between the two groups decreased to less than 3% by the end of ten years: the average ROI for the top group had decreased to 21.5% while the average ROI for the bottom group increased to 18.0%.

While the evidence consistently shows that the advantage horizon is finite, firms like Coca-Cola, Wal-Mart, and Microsoft have been able to extend their advantage horizons for many years. These firms have been able to create tremendous value for shareholders by sustaining their ability to generate positive abnormal profits.

Reinvestment

The key insight from the model regarding investment is that reinvestment of earnings is value enhancing only when investment opportunities generate expected returns in excess of the cost of equity ($ROE > K_E$). Because investment opportunities vary across firms and vary over time for the same firm, it is impossible to make conclusive statements on the value of reinvestment. Nevertheless, there is some evidence that reinvestment creates value. Recent studies have shown that firms which announce major capital expenditure or research and development (R&D) programs experience positive abnormal equity returns.⁴ The market interprets these announcements as good news and their stock prices usually increase. While it may be the case that firms announce only their most positive NPV investments, Fruhan (1979, Table 1-6) provides evidence from a sample of almost 1500 firms that broadly supports the relation among high profitability, high reinvestment, and high equity valuations.

Acquisitions represent another form of investment for many firms. Jensen and Ruback (1983) review the many studies on acquirer returns surrounding merger announcements. They conclude that, on average, acquirer shareholders do not lose and target shareholders gain from merger

³ See also Freeman, Ohlson, and Penman (1982).

⁴ McConnell and Muscarella (1985) analyze capital expenditure announcements while Chan, Martin, and Kensinger (1990) analyze R&D expenditure announcements.

announcements. Thus, acquisitions create net gains for both firms combined even though they do not increase acquirer shareholder value.

Jensen (1986, 1993) presents an opposing view. He argues that managers often overinvest, i.e. invest in negative net present value projects, especially when their firms generate substantial free cash flow. Their incentive to overinvest results from their compensation being tied, indirectly, to firm size which, in turn, is a function of the amount investment. They are able to over invest because internal control systems such as board oversight are weak. In the absence of effective internal control systems, external forces such as the market for corporate control discipline investment activity. Jensen cites the oil industry in general and the Gulf Oil takeover in particular as examples where takeovers eliminated wasteful capital expenditures. Just as investing in positive NPV projects creates value, so, too, does eliminating negative NPV investments.

Warren Buffet, the prominent investor and chairman of Berkshire Hathaway, acknowledged the problem of overinvestment in his company's 1984 annual report:

Many corporations that show consistently good returns have, indeed, employed a large portion of their retained earnings on an economically unattractive, even disastrous, basis. Their marvelous core businesses camouflage repeated failures in capital allocation elsewhere (usually involving high-priced acquisitions). The managers at fault periodically report on the lessons they have learned from the latest disappointment. They then usually seek out future lessons. (Failure seems to go to their heads). . . In such cases, shareholders would be far better off if the earnings were retained to expand only the high return business, with the balance being paid in dividends or used to repurchase stock...

Although stated in his characteristically droll way, Buffet's point is clear: reinvestment destroys value unless it generates an appropriate risk-adjusted rate of return.

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Exhibit 1: Numerical example of the relation between the value drivers and value creation

Combining equations 10 and 11 yields the following equation:

$$\text{Market/Book} = [(1+\gamma\text{ROE}) / (1+K_E)]^n + [\text{ROE}(1-\gamma) / (K_E - \gamma\text{ROE})] [1 - ((1+\gamma\text{ROE}) / (1+K_E))^n]$$

This Exhibit shows the hypothetical market-to-book ratios as a function of the three value drivers: profitability, advantage horizon, and re-investment; assuming the firm has a cost of equity equal to 15%. The three cases differ by the level of reinvestment which varies from 0% to 66%.

Case #1: Reinvestment rate (γ) = 0%

<u>Advantage Horizon</u>	Return on Equity (ROE)		
	5%	15%	25%
5 years	0.66	1.00	1.34
15 years	0.42	1.00	1.58
30 years	0.34	1.00	1.66

Case #2: Reinvestment rate (γ) = 33%

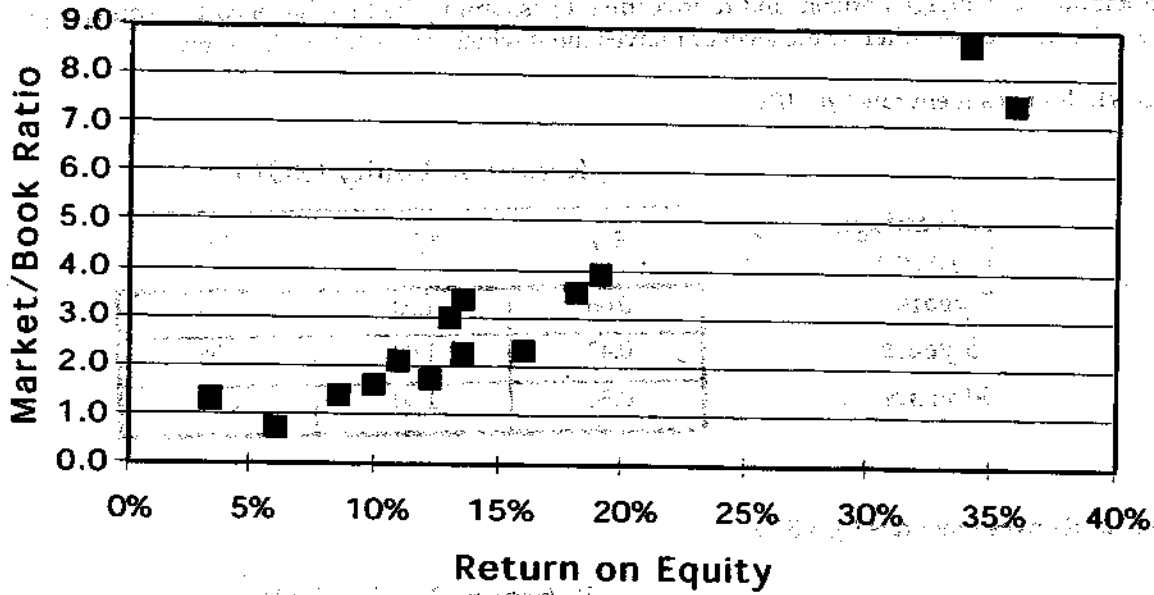
<u>Advantage Horizon</u>	Return on Equity (ROE)		
	5%	15%	25%
5 years	0.65	1.00	1.39
15 years	0.37	1.00	1.88
30 years	0.27	1.00	2.24

Case #3: Reinvestment rate (γ) = 66%

<u>Advantage Horizon</u>	Return on Equity (ROE)		
	5%	15%	25%
5 years	0.65	1.00	1.45
15 years	0.32	1.00	2.43
30 years	0.18	1.00	4.27

Exhibit 2: Relation between Return on Equity (ROE) and Market-to-Book Ratio

Grocery Stores (Value Line 11/15/96)



Oil Field Service Companies (Value Line 11/29/96)

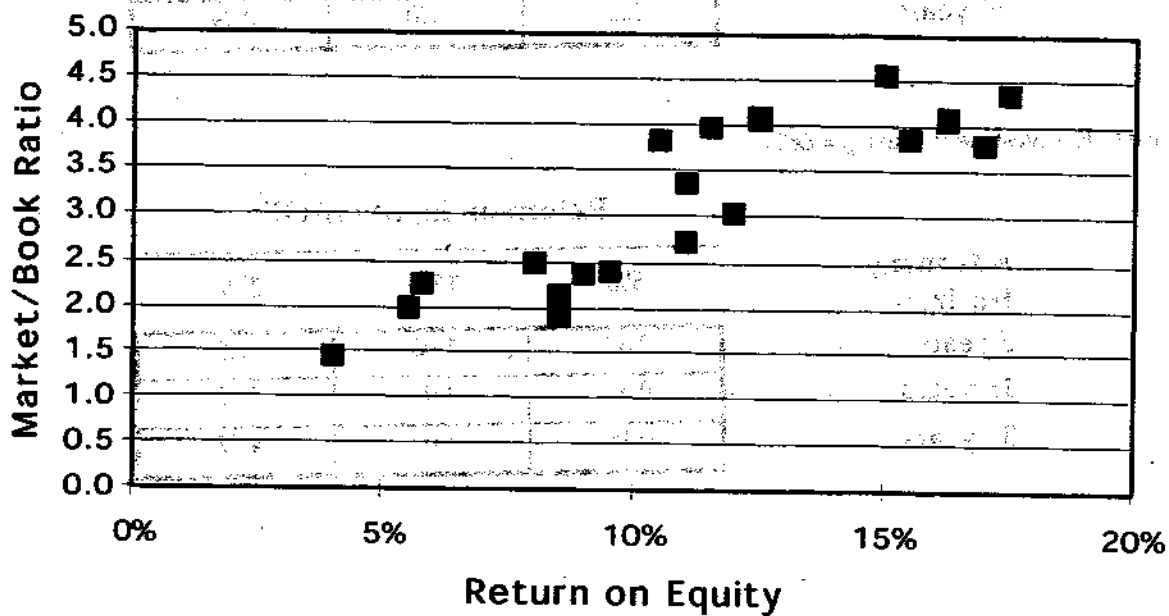


Exhibit 3: Advantage horizon

Fruhan (1995) analyzed the advantage horizon of a sample of 87 high-performing firms. To be included in the sample, firms had to have an average ROE of more than 25% for five consecutive years between 1976-82 and have sales greater than \$200 million. He found the following:

Top Performers:

1. Petrie Stores
2. H&R Block
3. Standard Microsystems
4. Airborne Freight
5. Wendy's International
6. Commerce Clearing House
7. Avon Products
8. Southwest Airlines
9. Charming Shoppes
10. Loctite Corp.

1976-78 Average ROE
2.03
1.45
1.43
0.77
0.74
0.67
0.67
0.63
0.56
0.56

For the period from 1976-78:

Median ROE for the top 87 firms = 37%
 S&P 400 Average ROE = 15%
 Spread = 21%

For the period from 1989-93:

Median ROE for the top 87 firms = 17%
 S&P 400 Average ROE = 15%
 Spread = 2%

Lesson: The advantage horizon is finite.

Appendix 1: Equity value and the advantage horizon

Equations 1 and 3 show that a firm's equity market value is a function of its return on equity (ROE) and cost of equity (K_E). Assuming no retention of earnings and constant returns, equity value is:

$$E_{MV} = ROE \cdot E_{BV} / (1+K_E) + ROE \cdot E_{BV} / (1+K_E)^2 + \dots \quad (A1.1)$$

dividing through by the book value of equity (E_{BV}) yields

$$\text{Market/Book} = E_{MV} / E_{BV} = ROE / (1+K_E) + ROE / (1+K_E)^2 + \dots \quad (A1.2)$$

The ROE can be divided into two parts: $ROE = (ROE - K_E) + K_E$. The first term ($ROE - K_E$) consists of "abnormal" earnings, returns to equity in excess of the cost of equity; the second term consists of "normal" earnings because that is the expected return on equity. Substituting back into equation A1.2 yields:

$$\text{Market/Book} = [(ROE - K_E) + K_E] / (1+K_E) + [(ROE - K_E) + K_E] / (1+K_E)^2 + \dots \quad (A1.3)$$

$$\begin{aligned} \text{Market/Book} &= (ROE - K_E) / (1+K_E) + [(ROE - K_E) / (1+K_E)^2 + \dots \\ &\quad + K_E / (1+K_E) + K_E / (1+K_E)^2 + \dots \end{aligned} \quad (A1.4)$$

Equation A1.4 is the sum of two geometric series, one of normal earnings and one of abnormal earnings. The present value of the normal earnings (using a perpetuity formula) is one:

$$1 = K_E / K_E = K_E / (1+K_E) + K_E / (1+K_E)^2 + \dots \quad (A1.5)$$

The present value of the abnormal earnings depends on how long the firm expects to earn abnormal earnings. It can be thought of as an annuity: The firm receives a stream of abnormal earnings for a period of n years. The present value of an annuity can be written as:

$$\text{present value} = (ROE - K_E) \cdot [(1/K_E) - (1/(K_E(1+K_E)^n))] \quad (A1.6)$$

Combining equations A1.5 and A1.6 yields:

$$\text{Market/Book} = 1 + [(ROE - K_E) \cdot [(1/K_E) - (1/(K_E(1+K_E)^n))] \quad (A1.7)$$

as n approaches infinity, equation A1.7 reduces to equation 4 in the note.

Appendix 2: Equity value and reinvestment

This appendix derives a model of equity valuation as a growing perpetuity. Given a firm with a constant return on equity (ROE), it can either retain its earnings or pay them out to equityholders as dividends. Assuming the firm retains a fraction of earnings (γ) and pays out the remainder, then the market value of equity can be determined as follows.

Time	Total Earnings	Amount Paid Out (ECF)	Amount Retained	Book Value of Equity
t=0				E_0
t=1	$ROE \cdot E_0$	$(1-\gamma) \cdot ROE \cdot E_0$	$(\gamma) \cdot ROE \cdot E_0$	$E_1 = E_0 + (\gamma) \cdot ROE \cdot E_0$ $E_1 = E_0 (1 + \gamma ROE)$
t=2	$ROE \cdot E_1$ $ROE \cdot [E_0(1 + \gamma ROE)]$	$(1-\gamma) \cdot ROE \cdot E_1$ $(1-\gamma) \cdot ROE \cdot E_0 (1 + \gamma ROE)$	$(\gamma) \cdot ROE \cdot E_1$ $(\gamma) \cdot ROE \cdot E_0 (1 + \gamma ROE)$	$E_2 = E_1 + (\gamma) \cdot ROE \cdot E_1$ $E_2 = E_1 (1 + \gamma ROE)$ $E_2 = E_0 (1 + \gamma ROE)^2$
t=3	$ROE \cdot E_2$ $ROE \cdot [E_0(1 + \gamma ROE)^2]$	$(1-\gamma) \cdot ROE \cdot E_2$ $(1-\gamma) \cdot ROE \cdot E_0 (1 + \gamma ROE)^2$	$(\gamma) \cdot ROE \cdot E_2$ $(\gamma) \cdot ROE \cdot E_0 (1 + \gamma ROE)^2$	$E_3 = E_2 + (\gamma) \cdot ROE \cdot E_2$ $E_3 = E_2 (1 + \gamma ROE)$ $E_3 = E_0 (1 + \gamma ROE)^3$
t=4	(etc.)			
Growth Rate	γROE	γROE	γROE	γROE

Value = discounted present value of payouts (equity cash flows)

$$= \frac{((1-\gamma) \cdot ROE \cdot E_0)}{(1+K_E)} + \frac{((1-\gamma) \cdot ROE \cdot E_0 (1 + \gamma ROE))}{(1+K_E)^2} + \dots \quad (A2.1)$$

$$= \frac{((1-\gamma) \cdot ROE \cdot E_0)}{(1+K_E)} \{ 1 + [(1+\gamma ROE)/(1+K_E)] + [(1+\gamma ROE)/(1+K_E)]^2 + \dots \} \quad (A2.2)$$

Equation A-2 is a growing perpetuity with growth rate equal to γROE . It can be rewritten as:

$$\text{Equity Value} = \frac{(1-\gamma) \cdot ROE \cdot E_0}{(K_E - \gamma ROE)} \quad (A2.3)$$

After multiplying through by the book value of equity (E_0), one gets the ratio of equity at market value to equity at book value ($E_{MV}/E_{BV} = V/E_0$):

$$\text{Market/Book} = \frac{(1-\gamma) \cdot ROE}{(K_E - \gamma ROE)} \quad (A2.4)$$