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**BEFORE THE WASHINGTON**

**UTILITIES & TRANSPORTATION COMMISSION**

WASHINGTON UTILIITES AND TRANSPORTATION COMMISSION,

Complainant,

v.

AVISTA CORPORATION d/b/a AVISTA UTILITIES,

Respondent.

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Myron Gordon et al.: *Capital Equipment Analysis*

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# CAPITAL EQUIPMENT ANALYSIS: THE REQUIRED RATE OF PROFIT

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The interest in capital equipment analysis that has been evident in the business literature of the past five years is the product of numerous social, economic, and business developments of the postwar period. No conclusive listing of these developments can be attempted here. However, four should be mentioned which are of particular importance in this search for a more systematic method for discovering, evaluating, and selecting investment opportunities. These are: (1) the high level of capital outlays (in absolute terms); (2) the growth in the size of business firms; (3) the delegation of responsibility for initiating recommendations from top management to the profit center, which has been part of the general movement toward decentralization; and (4) the growing use of "scientific management" in the operations of the business firm.

These developments have motivated the current attempt to develop objective criteria whereby the executive committee in a decentralized firm can arrive at a capital budget. Since each of its profit centers submits capital proposals, the executive committee must screen these and establish an allocation and a level of capital outlays that is consistent with top management's criteria for rationing the firm's funds. Capital budgeting affords the promise that this screening process can be made amenable to some established criteria that are understandable to all the component parts of the firm. Consequently, capital budgeting appeals to top management, for, in the first place, each plant manager can see his proposal in the light of all competing proposals for the funds of the enterprise. This may not completely eliminate irritation among the various parts of the firm, but a rational capital budgeting program can go a long way toward maintaining initiative on the part of a plant manager, even though the executive committee may veto one or all of his proposals. In the second place, the use of a capital budgeting program serves to satisfy top management that each accepted proposal meets adequate predetermined standards and that the budget as a whole is part of a sound, long-run plan for the firm.

What specifically does a capital budgeting program entail? The focal points of capital budgeting are: (1) ascertaining the profit abilities of the array of capital outlay alternatives, and (2) determining the least profitability required to make an investment, i.e., a cut-off point. Capital budgeting also involves administrative procedures and organization designed to discover investment opportunities, process information, and carry out the budget; however, these latter aspects of the subject have been discussed in detail by means of case studies that have appeared in publications of the American Management Association and the

National Industrial Conference Board and in periodicals such as the *N.A.C.A. Bulletin*.[1] Hence, we will not concern ourselves with them here.

There are at least four methods for establishing an order-preference array of the capital expenditure suggestions. They are: (1) the still popular "payoff period"; (2) the average investment formula; (3) the present value formula with the rate of interest given; and (4) the present value formula used to find the rate of profit. It is not our intention in this paper to discuss these various methods specifically, since critical analyses of these alternatives are to be found in papers by Dean, by Lorie and Savage, and by Gordon in a recent issue of the *Journal of Business*,[2] which is devoted exclusively to the subject of capital budgeting.

However, it is of interest to note that in each of these methods the future revenue streams generated by the proposed outlays must be amenable to measurement if the method is to be operational. However, improvements in quality, more pleasant working conditions, strategic advantages of integration, and other types of benefits from a capital outlay are still recognized only in qualitative terms, and there is a considerable hiatus in the literature of capital budgeting with respect to the solution of this problem. Hence, in the absence of satisfactory methods for quantifying these types of benefits, the evaluation of alternative proposals is still characterized by intuitive judgments on the part of management, and a general quantitative solution to the capital budgeting problem is not now feasible. It appears to us that this problem affords one of the most promising opportunities for the application of the methods of management science. In fact, we anticipate that techniques for the quantification of the more important factors now treated qualitatively will soon be found.

Given the rate of profit on each capital outlay proposal, the size of the budget and its allocation are automatically determined with the establishment of the rate of profit required for the inclusion of a proposal in the budget. In the balance of this paper, a method for determining this quantity is proposed and its use in capital budgeting is analyzed.

## II

We state that the objective of a firm is the maximization of the value of the stockholders' equity. While there may be legitimate differences of opinion as to whether this is the sole motivation of management, we certainly feel that there can be no quarrel with the statement that it is a dominant variable in manage-

[1] American Management Association, *Tested Approaches to Capital Equipment Replacement*, Special Report No. 1, 1954; American Management Association, *Capital Equipment Replacement*; *AMA Special Conference*, May 3–4, 1954 (New York, 1954, American Management Association, 105 pp.); J. H. Watson, III, National Industrial Conference Board, *Controlling Capital Expenditures*, Studies in Business Policy, No. 62, April, 1953; C. I. Fellers, "Problems of Capital Expenditure Budgeting", *N.A.C.A. Bulletin*, 26 (May, 1955), 918–24; E. N. Martin, "Equipment Replacement Policy and Application", *N.A.C.A. Bulletin*, 35 (February, 1954), 715–30.

[2] *Journal of Business*, Vol. XXVIII, No. 3 (October, 1955).

ment's decisions. It has been shown by Lutz and Lutz in their *Theory of the Investment of the Firm*[3] and by others[4] that this objective is realized in capital budgeting when the budget is set so as to equate the marginal return on investment with the rate of return at which the corporation's stock is selling in the market. The logic and operation of this criterion will be discussed later. Now, we only wish to note the role assigned in capital budgeting to the rate of profit that is required by the market.

At the present time, the dividend yield (the current dividend divided by the price) and the earnings yield (the current income per share divided by the price) are used to measure the rate of profit at which a share is selling. However, both these yields fail to recognize that a share's payments can be expected to grow, and the earnings yield fails to recognize that the corporation's earnings per share are not the payments made to the stockholder.

The practical significance of these failures is evidenced by the qualifications with which these two rate-of-profit measures are used by investment analysts. In the comparative analysis of common stocks for the purpose of arriving at buy or sell recommendations, the conclusions indicated by the dividend and/or the earnings yield are invariably qualified by the presence or absence of the prospect of growth. If it is necessary to qualify a share's yield as a measure of the rate of profit one might expect to earn by buying the share, then it must follow that current yield, whether income or dividend, is inadequate for the purposes of capital budgeting, which is also concerned with the future. In short, it appears to us that the prospective growth in a share's revenue stream should be reflected in a measure of the rate of profit at which the share is selling. Otherwise, its usefulness as the required rate of profit in capital budgeting is questionable.

In his *Theory of Investment Value*[5], a classic on the subject, J. B. Williams tackled this problem of growth. However, the models he developed were arbitrary and complicated so that the problem of growth remained among the phenomena dealt with qualitatively. It is our belief that the following proposal for a definition of the rate of profit that takes cognizance of prospective growth has merit.

The accepted definition of the rate of profit on an asset is the rate of discount that equates the asset's expected future payments with its price. Let $P_0$ = a share's price at $t = 0$, let $D_t$ = the dividend expected at time $t$, and let $k$ = the rate of profit. Then, the rate of profit on a share of stock is the value of $k$ that satisfies

$$(1) \qquad P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t}.$$

[3] Friedrich and Vera Lutz, *The Theory of Investment of the Firm* (Princeton, N. J., 1951, Princeton University Press, 253 pp.), 41–43.

[4] Joel Dean, *Capital Budgeting: Top Management Policy on Plant, Equipment, and Product Development* (New York, 1951, Columbia University Press, 174 pp.); Roland P. Soule, "Trends in the Cost of Capital", *Harvard Business Review*, 31 (March, April, 1953), 33–47.

[5] J. B. Williams, *The Theory of Investment Value*, (Cambridge, Massachusetts, 1938, Harvard University Press), 87–96.

It is mathematically convenient to assume that the dividend is paid and discounted continuously at the annual rates $D_t$ and $k$, in which case

$$P_0 = \int_0^\infty D_t e^{-kt}\, dt. \tag{2}$$

Since $P_0$ is known, estimating the rate of profit at which a share of stock is selling requires the determination of $D_t$, $t = 1, 2, \cdots, \infty$.

At the outset it should be made clear that our objective is not to find the rate of profit that *will actually be earned* by buying a share of stock. This requires knowledge of the dividends that will be paid in the future, the price at which the share will be sold, and when it will be sold. Unfortunately, such information is not available to us. The rate of profit of interest here is a relation between the present known price and the *expected future dividends*. The latter will vary among individuals with the information they have on a host of variables and with their personality. Therefore, by expected future dividends we mean an estimate that (1) is derivable from known data in an objective manner, (2) is derived by methods that appear reasonable, i.e., not in conflict with common sense knowledge of corporation financial behavior, and (3) can be used to arrive at a manageable measure of the rate of profit implicit in the expectation.

We arrive at $D_t$ by means of two assumptions. One, a corporation is expected to retain a fraction $b$ of its income after taxes; and two, a corporation is expected to earn a return of $r$ on the book value of its common equity. Let $Y_t$ equal a corporation's income per share of common after taxes at time $t$. Then the expected dividend at time $t$ is

$$D_t = (1 - b)Y_t \tag{3}$$

The income per share at time $t$ is the income at $(t - 1)$ plus $r$ percent of the income at $(t - 1)$ retained, or

$$Y_t = Y_{t-1} + rbY_{t-1} \tag{4}$$

Equation (4) is simply a compound interest expression so that, if $Y_t$ grows continuously at the rate $g = br$,

$$Y_t = Y_0 e^{gt}. \tag{5}$$

From Equations (3) and (5)

$$D_t = D_0 e^{gt}. \tag{6}$$

Substituting this expression for $D_t$ in Equation (2) and integrating, yields

$$\begin{aligned} P_0 &= \int_0^\infty D_0 e^{gt} e^{-kt}\, dt \\ &= D_0 \int_0^\infty e^{-t(k-g)}\, dt \qquad (7) \\ &= \frac{D_0}{k - g}. \end{aligned}$$

The condition for a solution is $k > g$, a condition that is easily satisfied, for otherwise, $P_0$ would be infinite or negative.

Solving Equation (7) for $k$ we find that

$$k = \frac{D_0}{P_0} + g. \tag{8}$$

Translated, this means that the rate of profit at which a share of common stock is selling is equal to the current dividend, divided by the current price (the dividend yield), plus the rate at which the dividend is expected to grow. Since there are other possible empirical definitions of the market rate of profit on a share of stock, we will refer to $k$ as the growth rate of profit.

## III

Let us now review and evaluate the rationale of the model we have just established. Estimating the rate of profit on a share of stock involves estimating the future dividend stream that it provides, and the fundamental difference between this model and the dividend yield is the assumption of growth. The latter, as can be seen, assumes that the dividend will remain constant. Since growth is generally recognized as a factor in the value of a share and since it is used to explain differences in dividend yield among shares, its explicit recognition appears desirable. Future dividends are uncertain, but the problem cannot be avoided by ignoring it. To assume a constant rate of growth and estimate it to be equal to the current rate appears to be a better alternative.

Under this model the dividend will grow at the rate $br$, which is the product of the fraction of income retained and the rate of return earned on net worth. It is mathematically true that the dividend will grow at this rate if the corporation retains $b$ and earns $r$. While we can be most certain that the dividend will not grow uniformly and continuously at some rate, unless we believe that an alternative method for estimating the future dividend stream is superior, the restriction of the model to the assumption that it will grow uniformly at some rate is no handicap. Furthermore, the future is discounted; hence, an error in the estimated dividend for a year in the distant future results in a considerably smaller error in $k$ than an error in estimating the dividend in a near year.

It should be noted that this measure of the rate of profit is suspect, when *both* income and dividend are zero, and it may also be questioned when either falls to very low (or negative) values. In such cases, the model yields a lower rate of profit than one might believe that the market requires on a corporation in such difficulties. It is evident that the dividend and the income yields are even more suspect under these conditions and, hence, are subject to the same limitations.

There are other approaches to the estimation of future dividends than the extrapolation of the current dividend on the basis of the growth rate implicit in $b$ and $r$. In particular, one can arrive at $g$ directly by taking some average of the past rate of growth in a corporation's dividend. Whether or not this or some other measure of the expected future dividends is superior to the one presented earlier will depend on their relative usefulness in such purposes as the analysis of variation in prices among shares and the preferences of those who want an objective measure of a share's rate of profit.

So far, we have compared the growth rate of profit with the income and dividend yields on theoretical grounds. Let us now consider how they differ in practice, using the same measurement rules for the variables in each case. The numerical difference between the growth rate of profit and the dividend yield is simply the growth rate. However, the income yield, which is the measure of the rate of profit commonly recommended for capital budgeting, differs from the growth rate of profit in a more complex manner, and to establish this difference we first note that

$$b = \frac{Y - D}{Y} \text{ and } r = \frac{Y}{B} \tag{9}$$

where $B$ = the net worth or book value per share. The growth rate of profit, therefore, may be written as

$$k = \frac{D}{P} + br = \frac{D}{P} + \frac{Y - D}{B}. \tag{10}$$

Next, the income yield can be decomposed as follows:

$$y = \frac{Y}{P} = \frac{D}{P} + \frac{Y - D}{P}. \tag{11}$$

We see then that $y$ and $k$ will be equal when book and market values are equal. It can be argued that the income yield overstates a share's payment stream by assuming that each payment is equal to the income per share and understates the payment stream by assuming that it will not grow. Hence, in this special case where book and market values are equal, the two errors exactly compensate each other.

Commonly market and book values differ, and $y$ will be above $k$ when market is below book, and it will be below $k$ when market is above book. Hence, a share of IBM, for example, that is priced far above book had had an earnings yield of two to three percent in 1955. We know that the market requires a higher rate of profit on a common stock, even on IBM, and its growth rate of profit, $k$, is more in accord with the value suggested by common sense. Conversely, when U. S. Steel was selling at one-half of book value in 1950, the high income yield grossly overstated the rate of profit that the market was, in fact, requiring on the stock.

Furthermore, the growth rate of profit will fluctuate in a narrower range than the earnings yield. For instance, during the last few years, income, dividends, and book value have gone up more or less together, but market price has gone up at a considerably higher rate. Consequently, the growth rate of profit, dependent in part on book value, has fallen less than the earnings yield. Conversely, in a declining market $k$ would rise less rapidly than $y$.

There is a widespread feeling that many accounting figures, particularly book value per share, are insensitive to the realities of the world, and some may feel that the comparative stability of $k$ is merely a consequence of the limitations of accounting data. This is not true! The behavior of $k$ is not a consequence of the supposed lack of realism in accounting data. Rather, book value appears in the model because it, and not market value, is used to measure the rate of return the corporation earns on investment, which, we have seen, is the rate of return that enters into the determination of the rate at which the dividend will grow. The comparative stability of $k$ follows from the simple fact that, when a revenue stream is expected to grow, a change in the required rate of profit will give rise to a more than proportional change in the asset's price. Conversely, a change in the price reflects a less than proportional change in the rate of profit.

## IV

Given the rate of profit expected on each item in the schedule of available investment opportunities and given the rate of profit at which the corporation's stock is selling, what should the capital budget be? As stated earlier, the accepted theory is that the budget should be set so as to equate the marginal return on investment with the rate of profit at which the stock is selling. The reasoning is, if the market requires, let us say, a 10 percent return on investment in the corporation's stock, and if the corporation can earn 15 percent on additional investment, obtaining the funds and making the investment will increase the earnings per share. As the earnings and the dividend per share increase or as the market becomes persuaded that they will increase, the price of the stock will rise. The objective, it will be recalled, is the maximization of the value of the stockholder's equity.

The conclusion drawn implicitly assumes that the corporation can sell additional shares at or above the prevailing market, or if a new issue depresses the market, the fall will be slight, and the price will soon rise above the previous level. However, some other consideration may argue against a new stock issue; for example, the management may be concerned with dilution of control, or the costs of floating a new issue may be very high, or a new issue may be expected to depress the price severely and indefinitely for reasons not recognized in the theory. Hence, it does not automatically follow that a new issue should be floated when a firm's demand for funds exceeds, according to the above criterion, those that are internally available.

In determining whether the required rate of profit is above or below $r'$, the marginal return on investment, one can use $y$, the earnings yield, or $k$, the growth rate of profit as the required rate of profit. If $y$ and $k$ differ and if the reasoning in support of $k$ presented earlier is valid, using $y$ to estimate the direction in which a new issue will change the price of the stock may result in a wrong conclusion.

In arriving at the optimum size of a stock issue, the objective is to equate $r'$ and $y$ or $k$, depending on which is used. Internal data may be used to estimate the marginal efficiency of capital schedule. If the required rate of profit is considered a constant, its definition, $y = Y/P$ or $k = D/P + br$, provides its value. However, the required rate of profit may vary with the size of the stock issue or with the variables that may change as a consequence of the issue. In this event,

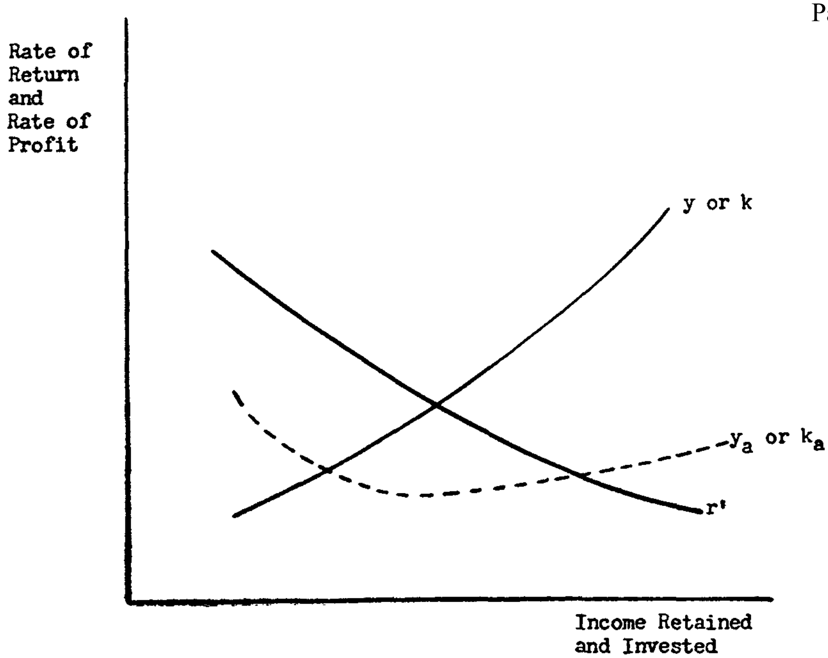


FIG. 1

finding the optimum size of a stock issue requires a model that predicts the variation in the required rate of profit with the relevant variables.

Borrowing is an alternative source of funds for investment. However, an analysis of this alternative requires the measurement of both (1) the variation in risk with debt, and (2) the difference between the rate of profit and the rate of interest needed to cover a given increase in risk. This has not been done as yet, which may explain the widespread practice of arbitrarily establishing a "satisfactory" financial structure and only borrowing to the extent allowed by it.

It has been stated by Dean[6] and Terborgh[7] that the long-term ceiling on a firm's capital outlays is the amount of its internally available funds. However, the share of its income a corporation retains is not beyond the control of its management; and, among the things we want from a capital budgeting model is guidance on whether the share of a corporation's income that is retained for investment should be raised or lowered.

Proceeding along traditional lines, the problem may be posed as follows. A firm estimates its earnings and depreciation allowances for the coming year and deducts the planned dividend to arrive at a preliminary figure for the capital budget. The marginal rate of return on investment in excess of this amount may be above or below the required rate of profit. We infer from theory that the two rates should be equated by (1) raising the budget and reducing the dividend

[6] Dean, *op. cit.*, 53–55.

[7] George Willard Terborgh, *Dynamic Equipment Policy* (New York, 1949, McGraw-Hill, 290 pp.), 228–29.

when the marginal return on investment is above the required rate of return, and (2) raising the dividend and reducing the budget when the reverse holds. The conditions under which this process yields an equilibrium are illustrated in Figure 1. The marginal return on investment, $r'$, should fall as the budget is increased, and the required rate of profit, $y$ or $k$, should increase or it should fall at a lower rate than $r'$. The latter case is illustrated by the line $y_a$ or $k_a$.

Changing the dividend so as to equate $r'$ and say $y$ should maximize the price of the stock. For instance, if $r'$ is above $y$, the company can earn a higher return on investment than stockholders require, and a dollar used this way is worth more to the stockholders than the dollar distributed in dividends. In other words, the price should go up by more than the income retained.

There are, of course, a number of problems connected with the use of this model for arriving at the optimum dividend rate. First, there is the question whether $y$ or $k$ should be used to measure the required rate of profit. Second, there is no question that the required rate of profit varies with the dividend rate. Hence, the current rate of profit given by the definition does not tell what profit rate will be required with a different dividend rate. This requires a model which predicts the variation in $y$ or $k$ with the dividend rate and other variables. Third, there is a very nasty problem of the short and the long run. It is widely believed, though the evidence has limitations, that the price of a share of stock varies with the dividend rate, in which case a corporation should distribute all of its income. However, it is quite possible that a change in the dividend gives rise to the expectation that earnings and future dividends are changing in the same direction. Further, in the short run, the market is not likely to be informed on a firm's marginal efficiency of capital schedule. For these and other reasons, it is likely that the dividend rate should not be made to vary with short-run changes in the marginal efficiency of capital, and more sophisticated methods than those now in use are needed to establish the variation in price or required rate of profit with the dividend rate.

## V

The major points developed in this paper may be summarized as follows. We presented a definition of the rate of profit required by the market on a share of common stock, and we noted some of its advantages. It is theoretically superior to the income and dividend yields because it recognizes that the revenue stream provided by a share can be expected to grow. Furthermore, its empirical characteristics are also superior to those of the income and dividend yields since its value is generally in closer agreement with common sense notions concerning the prevailing rate of profit on a share of stock and since its value fluctuates in a narrower range over time. We next examined some of the problems involved in using this definition of the rate of profit and the earnings yield in capital budgeting models. Finally, we saw that, before capital budgeting theory can be made a reliable guide to action, we must improve our techniques for estimating the future revenue on a capital outlay proposal, and we must learn a good deal more about how the rate of profit the market requires on a share of stock varies with the dividend, the growth rate, and other variables that may influence it.