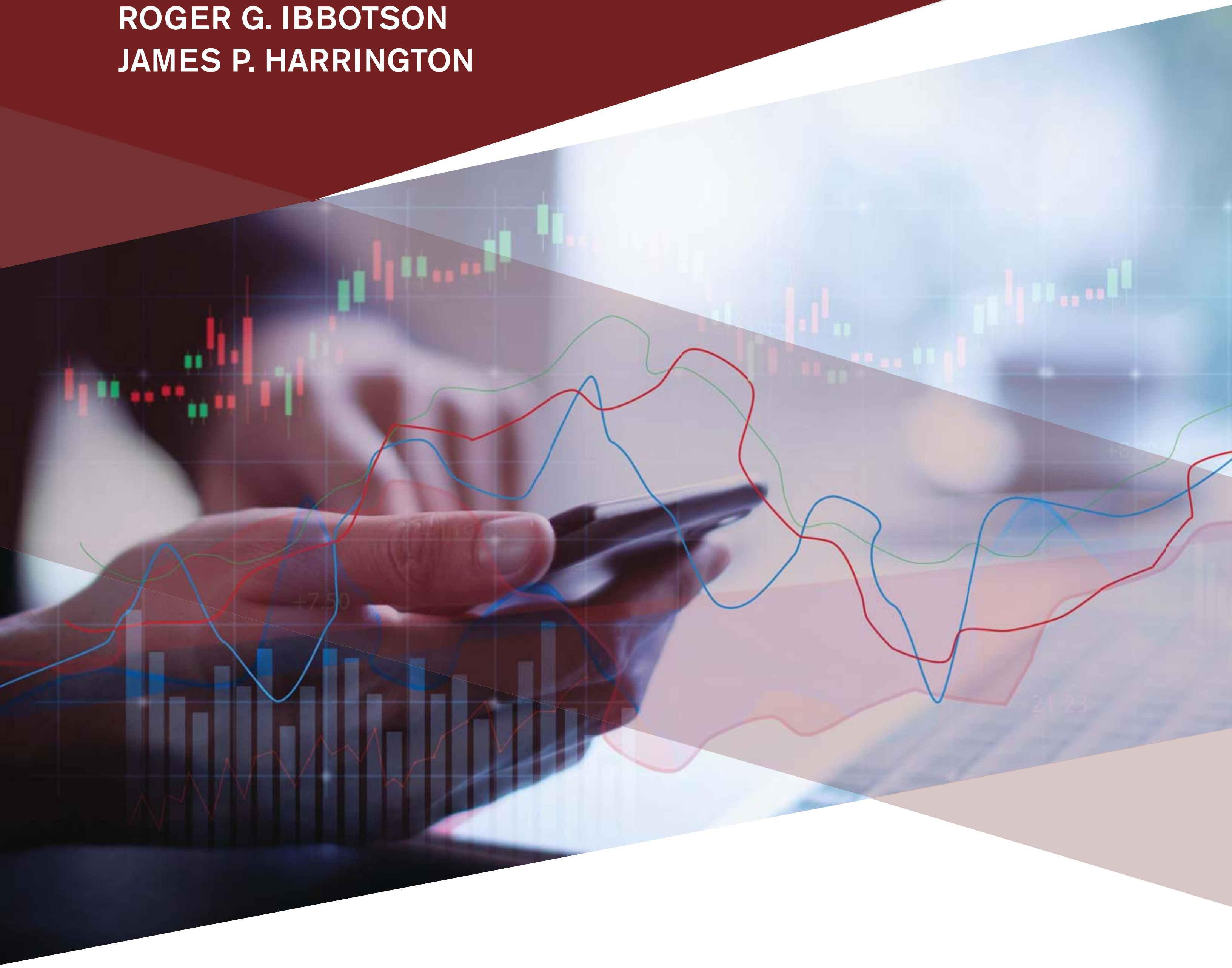
# STOCKS, BONDS, BILLS, AND INFLATION® (SBBI®)

## 2021 SUMMARY EDITION

ROGER G. IBBOTSON
JAMES P. HARRINGTON



DUFF & PHELPS
A KROLL BUSINESS

MORNINGSTAR

CFA Institute Research Foundation

DUFF & PHELPS
A KROLL BUSINESS

# Stocks, Bonds, Bills, and Inflation® (SBBI®) 2021 Summary Edition

## Interpretive Analysis and Insights

Through December 31, 2020

MORNINGSTAR®

CFA Institute Research Foundation

# About Duff & Phelps, A Kroll Business

For **nearly 100 years**, Duff & Phelps has helped clients make confident decisions in the areas of valuation, real estate, taxation and transfer pricing, disputes, M&A advisory and other corporate transactions.

Kroll is the world's premier provider of services and digital products related to **governance, risk and transparency**. We work with clients across diverse sectors in the areas of valuation, expert services, investigations, cyber security, corporate finance, restructuring, legal and business solutions, data analytics and regulatory compliance.

The firm's nearly **5,000 professionals** are located in **30 countries and territories** around the world.

For more information visit: www.duffandphelps.com.

The **Valuation Digital Solutions** group within Duff & Phelps, A Kroll Business ("D&P/Kroll"), strives to empower companies and finance professionals with high-quality valuation data that enables them to make sound business decisions. We share similar beliefs with CFA Institute in that a focus on education, research, and dissemination of data on financial markets benefits the overall investment profession. Personally, as a CFA charterholder, it is an honor to have D&P/Kroll collaborate with CFA Institute in this endeavor.

**Carla S. Nunes, CFA**
*Managing Director, D&P/Kroll Valuation Digital Solutions*

The *SBBI® 2021 Summary Edition* is a proud partnership with the following organizations:

# CFA Institute Research Foundation

Since 1965, CFA Institute Research Foundation has been providing independent, practitioner-focused research that helps investment management professionals effectively fulfill their duties with prudence, loyalty, and care. With the generous support of CFA Institute and thousands of donors from around the world, we are proud to offer this publication free to all. Please view additional Research Foundation content at our website: https://www.cfainstitute.org/en/research/foundation and follow us on LinkedIn: https://lnkd.in/e66zSKD and twitter: @CFAResearchFndn.

**Bud Haslett, CFA**
*Executive Director, CFA Institute Research Foundation*

# Morningstar

Morningstar, Inc. is a leading provider of independent investment research in North America, Europe, Australia, and Asia. The Company offers an extensive line of products and services for individual investors, financial advisors, asset managers, retirement plan providers and sponsors, and institutional investors in the debt and private capital markets. Morningstar provides data and research insights on a wide range of investment offerings, including managed investment products, publicly listed companies, private capital markets, debt securities, and real-time global market data. Morningstar also offers investment management services through its investment advisory subsidiaries, with approximately $227 billion in assets under advisement and management as of December 31, 2020. The Company has operations in 29 countries. For more information, visit www.morningstar.com/company. Follow Morningstar on Twitter: @MorningstarInc.

**Bryan Yelvington**
*Senior Vice President at Morningstar*

# About the D&P/Kroll, “Cost of Capital Navigator”

D&P/Kroll, has transitioned its U.S. and international (i) cost of capital data resources and (ii) industry-level statistics data resources to a new online platform, the “Cost of Capital Navigator.” The Cost of Capital Navigator is an interactive, web-based platform that guides finance and investment professionals through the process of estimating cost of capital, globally. The Cost of Capital Navigator includes four modules:

### ►U.S. Cost of Capital Module

Provides U.S. size premia, equity risk premia, risk-free rates, betas, industry risk premia, and other risk premia that can be used to develop U.S. cost of capital estimates. Studies included: CRSP Deciles Size Study, Risk Premium Report Study.[1] Excel Add-in Included.

### ►U.S. Industry Benchmarking Module

Provides industry-level cost of equity, debt, and WACC estimates, performance statistics, valuation multiples, levered and unlevered betas, capital structure, and additional statistics for approximately 170 U.S. industries. Industries are defined by GICS codes.

### ►International Cost of Capital Module

Provides measures of relative country risk for over 175 countries from the perspective of investors based in over 50 countries. Other data includes equity risk premia for 16 countries, risk-free rates for developed markets, industry betas for a global index as well as for developed markets, and long-term inflation expectations and corporate income tax rates for over 175 countries. Full country risk premia (CRPs) and relative volatility (RV) factor Tables by country.[2]

### ►International Industry Benchmarking Module

Provides industry-level cost of equity, debt, and WACC estimates, performance statistics, valuation multiples, levered and unlevered betas, capital structure, and additional statistics for approximately 90 industries in four global economic areas: (i) the World, (ii) the European Union, (iii) the Eurozone, and (iv) the United Kingdom. Each of the four global economic area’s industry analyses are presented in three currencies: (i) the euro (€ or EUR), (ii) the British pound (£ or GBP), and (iii) the U.S. dollar ($ or USD). Industries are defined by GICS codes.

To learn more, visit dpcostofcapital.com.

---

1 CRSP® is a registered trademark and service mark of Center for Research in Security Prices, LLC and has been licensed for use by D&P/Kroll. The D&P/Kroll publications and services are not sponsored, sold or promoted by CRSP®, its affiliates or its parent company. To learn more about CRSP, visit www.crsp.com.

2 Depending on subscription level.

# About the Authors

## Roger G. Ibbotson

Roger G. Ibbotson is Professor in the Practice Emeritus of Finance at Yale School of Management. He is also Chairman of Zebra Capital Management, LLC, a global equity investment and hedge fund manager. He is founder, and former chairman of Ibbotson Associates, now a Morningstar Company. He has written numerous books and articles including *Stocks, Bonds, Bills, and Inflation* with Rex Sinquefield (updated annually) which serves as a standard reference for information and capital market returns.

Professor Ibbotson conducts research on a broad range of financial topics, including popularity, liquidity, investment returns, mutual funds, international markets, portfolio management, and valuation. He has published *The Equity Risk Premium, Lifetime Financial Advice,* and most recently *Popularity: A Bridge between Classical and Behavioral Finance*. He has also co-authored two books with Gary Brinson, *Global Investing* and *Investment Markets*. He is a regular contributor and editorial board member to both trade and academic journals.

Professor Ibbotson serves on numerous boards including Dimensional Fund Advisors' funds. He frequently speaks at universities, conferences, and other forums. He received his bachelor's degree in mathematics from Purdue University, his MBA from Indiana University, and his PhD from the University of Chicago where he taught for more than ten years and served as Executive Director of the Center for Research in Security Prices (CRSP).

## James P. Harrington

James P. Harrington is a Director at D&P/Kroll (www.duffandphelps.com). James provides technical support on client engagements involving cost of capital and business valuation matters and is a leading contributor to D&P/Kroll' efforts in the development of studies, surveys, online content and tools, and firm-wide valuation models.

Previously, James was director of valuation research in Morningstar's Financial Communications Business where he led the group that produced the *Stocks, Bonds, Bills, and Inflation® (SBBI®) Valuation Yearbook*, *Stocks, Bonds, Bills, and Inflation® (SBBI®) Classic Yearbook*, *Cost of Capital Yearbook*, various international cost of capital reports, and also created a website dedicated to cost of capital issues.

James is co-author of the D&P/Kroll "Valuation Handbook" series with colleagues Carla Nunes and Roger Grabowski. The Valuation Handbooks were published as physical books starting in 2014; as of 2020 the information and data previously published in the Valuation Handbooks has been transitioned over to the D&P/Kroll Cost of Capital Navigator. The D&P/Kroll Cost of Capital Navigator (dpcostofcapital.com) is an interactive, web-based platform that guides analysts through the process of estimating cost of capital, globally.

# Table of Contents

# Foreword

Welcome to the *Stocks, Bonds, Bills and Inflation® (SBBI®) 2021 Summary Edition ("SBBI® 2021 Summary Edition")*. CFA Institute Research Foundation is delighted to offer this content free to everyone in the global investment community.

Although the Research Foundation is offering this publication free to all, it is the wonderful folks at D&P/Kroll and co-authors Roger Ibbotson and Jim Harrington, through a licensing agreement with Morningstar, who deserve the credit for developing the content for the *SBBI® 2021 Summary Edition.*[3] The Research Foundation is delighted to be entering a long-term partnership with D&P/Kroll for the annual publication of this content, and we hope that year after year, it becomes a valuable addition to your portfolio of investment knowledge.

## Why SBBI?

As a young finance student in the 1970's I recall my professor showing me *Stocks, Bonds, Bills, and Inflation: The Past (1926-1976) and the Future (1977-2000),* originally published by the Research Foundation in 1977[4], and being fascinated by all of the useful information it contained. Now, over four decades later, the "SBBI" is returning home to the RF with the publication of the *SBBI® 2021 Summary Edition.*

## Purpose of the SBBI Summary Edition

The primary purpose of the *SBBI® 2021 Summary Edition* is to accompany the raw U.S. historical SBBI® stock and bond data files that are available to all CFA Institute members[5] on the CFA Institute Research Foundation website:
**https://www.cfainstitute.org/en/research/foundation/sbbi**.

The online raw SBBI® dataset files include capital appreciation, income, and total returns of the major asset classes of the U.S. economy: large-cap stocks, small-cap stocks, corporate bonds, government bonds of various maturities, and inflation (January 1926 to present, monthly).

The *SBBI® 2021 Summary Edition* includes interpretive analysis and insights through December 31, 2020, and is meant to enable investors to understand how to calculate, interpret, and use the U.S. historical SBBI® stock and bond data that we have made available to our members. The *SBBI® 2021 Summary Edition* includes formulae and methodology for using the raw SBBI® dataset to properly calculate summary performance statistics, index values, and optimization inputs.

---

3 The *SBBI® 2021 Summary Edition* is an abridged version of the full-version *2021 SBBI® Yearbook*, available from D&P/Kroll here: dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook. The full-version *2021 Stocks, Bonds, Bills, and Inflation® (SBBI®) Yearbook* includes all the raw data in printed form with a complete set of graphs and additional interpretive analysis, plus precalculated tables of summary performance statistics, index values, inflation-adjusted returns, various "building block" premia (e.g., bond default premium, bond horizon premium, small stock premium, etc.), and optimization inputs.

4 A free download of *Stocks, Bonds, Bills, and Inflation: The Past (1926–1976) and the Future (1977–2000)* in PDF format is available at https://www.cfainstitute.org/en/research/foundation/1977/rf-v1977-sbbi-past-and-future.

5 In the mainland of China, CFA Institute accepts CFA charterholders only.

A secondary purpose of the *SBBI® 2021 Summary Edition* is to provide those individuals just starting out in the investment industry (but who are not yet CFA Institute members) an excellent way to gain a quick understanding of the major asset classes, to see how returns are calculated, and to get a sense of the long run perspective. Additionally, those investment professionals who are more established in their careers will likely be aware of the SBBI® data but still may not have direct access to the raw SBBI® data. This summary edition will help them to understand the data and its many uses, including the impact of size, value/growth, and liquidity on returns. For this reason, CFA Institute Research Foundation has made the *SBBI® 2021 Summary Edition* available to all investment professionals free of charge.[6,7]

## Special Thanks

As with any project such as this, there are many people responsible for its success. Thanks go out to Roger Ibbotson, for getting the ball rolling, and for Roger and Rex Sinquefield for creating the original SBBI® way back in the 1970s. Many thanks to Bryan Yelvington and Daniel Ortiz at Morningstar who were instrumental with getting the SBBI® dataset up and running on our website and approving of the publication for the *SBBI® 2021 Summary Edition*. In addition, Morningstar's Benjamin Cheaney, Joscelyn MacKay, and Stephen Schmitt provided valuable contributions.

Carla Nunes, CFA, and Jim Harrington from D&P/Kroll were both crucial to this publication, and it would not have been published without the efforts of these two. Kevin Madden, Anas Aboulamer, Zach Rodheim, Kevin Latz, and Aaron Russo (all of D&P/Kroll) were also instrumental in its publication.

At CFA Institute Research Foundation, many thanks to our Vice-Chair Ted Aronson, CFA, for his tireless work on behalf of CFA Institute and the Research Foundation and his generous, multi-year donation that helped fund this project. Thanks too to Chair Joanne Hill, and incoming Research Committee Chair Bill Fung and all-of-the Research Foundation board members for their comments and suggestions along the way. And a special thanks to Research Foundation Research Director Larry Siegel who contributed to the original SBBI in the 1970s and is a vital part of today's Research Foundation.

CFA Institute is absolutely critical to the operation of the Research Foundation and provides much of our funding and staffing needs. We thank Marg Franklin, CFA, Paul Andrews and Rhodri Preece, CFA for their continued support of the project, and Jessica Lawson for her work as Research Foundation Project Manager.

6 The *SBBI® 2021 Summary Edition* is available to all investment professionals free of charge; access to the raw monthly U.S. historical SBBI® stock and bond data on the CFA Institute Research Foundation's website is available to CFA Institute members only.

7 Some investment professionals may prefer to purchase the full-version *2021 SBBI Yearbook*, which includes *precalculated* statistics. The full-version *2021 SBBI Yearbook* (dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook) provides an excellent way for an analyst to *cross-check* their own calculations while learning how to properly use and analyze the raw monthly SBBI® data.

Finally, we would like to thank the thousands of donors whose contributions, from $10 to over $1 million (an especially generous donation from Gary Brinson, CFA), have gone directly to support this project and the publication of our content. We couldn't do it without their help.

Our goal for this project is to increase the global investment community's knowledge of quantitative investment strategies by providing the data, and tools to analyze the data, to CFA Institute members and others in the global investment community. An aspirational goal is to provide free/low-cost tools so that the next legendary investment mind, such as Benjamin Graham, William Sharpe, Peter Bernstein, Fischer Black, Martin Leibowitz and the others, can emerge and lead the investment field for the coming decades. By providing a forum for sharing this information, we hope to provide a platform that unlocks the potential of that next great investment mind or minds. We hope this is a good start in that direction.

**Bud Haslett, CFA**
*Executive Director, CFA Institute Research Foundation*

# Preface by Roger G. Ibbotson

This *Stocks, Bonds, Bills and Inflation® (SBBI®) 2021 Summary Edition* is meant to enable investors to understand how to calculate, interpret, and use the U.S. historical stock and bond data that the CFA Institute Research Foundation has made available to members.

The data includes the capital appreciation, income, and total returns of the major asset classes of the U.S. economy: large-cap stocks, small-cap stocks, corporate bonds, government bonds of various maturities, and inflation. Most of the data starts in 1926, and is presented in monthly, annual, decade, or longer period form. The raw data itself is mostly monthly but is not included in this summary volume. Rather, this summary volume includes illustrative data, methodology, formulae, and analysis that will help investors and analysts learn how to use market time series raw data, which is separately available.

The SBBI® dataset was originally created by Rex A. Sinquefield and myself back in 1976, and initially published in two academic journal issues before being updated in CFA Institute Research Foundation monographs in 1977, 1979, and 1982. Starting in 1983, Ibbotson Associates Inc. published the *Stocks, Bonds, Bills, and Inflation® (SBBI®) Yearbook* every year until 2006, when after acquiring Ibbotson Associates, Morningstar, Inc. continued the annual publication. Starting in 2016, D&P/Kroll (duffandphelps.com) has published the full-version *SBBI® Yearbook* under license agreement from Morningstar Inc. Now through an agreement that the CFA Institute Research Foundation made with Morningstar, Inc., CFA Institute members can access the raw SBBI® directly through the Research Foundation site.

This *SBBI® 2021 Summary Edition* is meant to accompany the raw data files. All the calculation formulae are incorporated herein, along with illustrative examples. Analysts and investors are shown how to link returns together to create asset class indices and returns of any frequency, e.g. monthly, quarterly, or annualized. Users are also shown how to create real (i.e., inflation adjusted) series, as well as derived series measuring various risk premiums (e.g., the equity risk premium, the small-cap premium, the bond horizon or interest rate premium, the bond default premium, or the real interest rate). Investors who just want to examine the results without actually accessing the electronic *SBBI®* raw data are encouraged to purchase the full-version *2021 Stocks, Bonds, Bills, and Inflation® (SBBI®) Yearbook* published each year by D&P/Kroll. The full-version *2021 SBBI® Yearbook* includes all the raw data in printed form with a complete set of graphs and additional interpretive analysis, plus *precalculated* tables of summary performance statistics, index values, and optimization inputs. The full-version *SBBI® Yearbook* is available at: dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

Even though many readers of the *SBBI® 2021 Summary Edition* will not have direct access to the raw SBBI® dataset, the summary edition is still likely to be of broad interest. CFA Institute is making this edition available for free, and it is an excellent way for those starting out in the investment industry to gain a quick understanding of the major asset classes, how returns are calculated, and to get a sense of the long run perspective. Many of those investment professionals who are more established in their careers will be aware of the SBBI® data, but still may not have direct access to the raw SBBI® data. This summary edition will help them to understand the data and its many uses, including the impact of size, value/growth, and liquidity on returns. Of course, CFA Institute members will also have free access to the raw SBBI® data itself, and they will be able to develop their own optimization and forecasting techniques, as well as be able to do all sorts of analysis with the SBBI® data by itself, or combined with other time series data from other external sources. It is my hope that this free *SBBI® 2021 Summary Edition* will be of use to investment professionals at all levels of their career, whether or not they have access to the raw SBBI® data, and whether or not they step up to acquire the full-version *2021 Stocks, Bonds, Bills, and Inflation® (SBBI®) Yearbook.*

Today we have such immediate access to stock and bond markets data, that we often take it for granted. We can access real time data as it happens at the individual security level and for a multitude of indices, both for U.S. data and international markets. We often let the multitude of data obscure the big picture. It is the long-term data that can best inform us on how to create vast wealth. Perhaps the strongest consistent relationship over time is that stocks outperform bonds. While not true every year, large-cap stocks have out- performed U.S. Treasury bills in 64 out of the last 95 calendar years. During the period 1926–2020, large cap stocks had a total return of over 10% compared to the U.S. Treasury bill return of just over 3%. This difference is a measure of the equity risk premium.

One might wonder why we created the SBBI® data and the subsequent *SBBI® Yearbooks*. To get a perspective on this we have to go back in time. During the 1950s, Harry Markowitz had developed the mathematics of risk and diversification. By the late 1950s and early 1960s, academics had discovered that stock returns behaved nearly as Random Walks. By the mid-1960s, William Sharpe, John Lintner and others had developed the Capital Asset Pricing Model (CAPM), formalizing a risk return relationship. Soon afterward, Eugene Fama had posited the Efficient Market Hypothesis. And especially important to empiricists, James Lorie and Lawrence Fisher had created the Center for Research in Security Prices (CRSP) data set at the University of Chicago, measuring the monthly returns of NYSE stocks starting in 1926.

By the 1970s when Rex and I entered the scene, risk and return were part of a continuing discussion. Index funds were starting to be developed, because investors were beginning to get interested in investing in the market portfolio. Holding the market portfolio was actually an outcome of the CAPM, and investing in a market index was consistent with random walks and efficient markets. Studies from Fisher and Lorie had demonstrated that stocks had high historical returns, but at the time of our studies the Fisher and Lorie data only went through 1968. Investors were hungry for updates.

In 1976, Rex and I published our original historical SBBI® results in the University of Chicago's now defunct *Journal of Business*.[8] We included a large-cap total return series based upon the Standard & Poor's 500 Index (S&P did not have a total return series until many years later). We also included corporate bonds, long-term U.S. Treasury Bonds and Bills, and the CPI inflation

[8] "Stocks, Bonds, Bills, and Inflation: Year-by-Year Historical Returns (1926-1974)", Roger G. Ibbotson and Rex A. Sinquefield, *The Journal of Business*, Vol. 49, No. 1, January 1976.

index. This enabled us to measure all sorts of risk premiums. The most important was the Equity Risk Premium, both relative to long and short horizon bonds.[9] But other risk premiums were important too. We measured a default premium, a horizon or interest rate premium, and a real (inflation-adjusted) interest rate. We were able to measure all of the series in both real and nominal terms.

Once we recognized that we had many risk premiums across multiple asset classes, we also recognized that it would be the risk premiums that follow the random walks, rather than the nominal series themselves. This is because inflation is itself predictable from current bond yields. The Treasury yield curve provides a year-by-year term structure of forward interest rates, with each forward rate containing three components: the expected inflation rate, a horizon premium, and an expected one-year real interest rate. Since the yield curve is directly observable, it is only necessary to estimate the three component parts, and then add the historical risk premium distributions to create forecasts of all the asset classes.

The second SBBI® paper that Rex and I published in 1976 provided a year-by-year forecast of stocks, bonds, bills, and inflation from 1976 to 2000.[10] This forecast was not similar to conventional forecasts in several ways. First, it was based upon the idea that the bond market was efficient. Thus, the forward rates from the yield curve represented investors' unbiased predictions, after adjusting for the term structure of expected horizon premiums. It was also based upon the idea that in an efficient market, the various risk premiums would follow a random walk. Thus, the historical payoffs were extrapolated and predicted to continue. But most important, all premiums were drawn from historical distributions, so not only the expected returns were forecast, but also risk as well as the entire distribution of returns!

We were using 50 years of data to forecast the next 25 years. As it turned out, we were very accurate on the nominal stock market returns, although we under-forecast the bond returns, given the double-digit bond yields in the early 1980s. Nevertheless, the overall forecasts were reasonably within our probability distributions. I had separately forecast the Dow in 1974 (when it was about 800) to reach 10,000 by the year 2000, and when it did in 1999, I went on a television tour. Of course, the 10,000 in 2000 was not an exact forecast, but just near the center of the probability distribution.

Perhaps the most astonishing thing about analyzing long term data is the vast wealth that can be created by exponential growth. Investing in the SBBI® large cap index at the beginning of 1926 and reinvesting all the dividends until the end of 2020 provided a total return of over 10% per year. A single dollar invested over this 95-year period at this annualized rate (geometric mean) would have grown to nearly $11,000! Of course, this is a gross return. An actual investor would have had to pay transactions costs, fees, and taxes. But if these costs could be kept under control, the index investors would have still dramatically increased their wealth, even without any expertise in stock selection.

---

9 U.S. "historical" and "supply side" 1926–present equity risk premia, (as well as U.S. size premia) are available in the D&P/Kroll Cost of Capital Navigator's U.S. Cost of Capital Module at dpcostofcapital.com.

10 "Stocks, Bonds, Bills, and Inflation: Simulations of the Future (1976-2000)", Roger G. Ibbotson and Rex A. Sinquefield, *The Journal of Business*, Vol. 49, No.3, July 2016.

I hope you will find this *SBBI® 2021 Summary Edition* helpful. I also hope that many of you can experiment with the raw data from the CFA Institute Research Foundation website. And I remind you, that this summary version does not contain all the data, tables, graphs, and analysis that are available in the full-version *2021 Stocks, Bonds, Bills, and Inflation® (SBBI®) Yearbook* from D&P/Kroll, available at dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

**Roger G. Ibbotson**
*Professor Emeritus of Finance at the Yale School of Management, Chairman of Zebra Capital LLC, and former Chairman and founder of Ibbotson Associates, now part of Morningstar, Inc.*

# Chapter 1
# Results of U.S. Capital Markets in 2020 and in the Past Decade

This chapter presents newsworthy events in the market from 2020, as well as the returns for the seven basic SBBI® asset classes and describes the construction of these returns. More detail on the construction of some series can be found in the January 1976 *Journal of Business* article, referenced in the Introduction. Annual total returns and capital appreciation returns for each asset class are formed by compounding the monthly returns the CFA Institute Research Foundation has made available to members. Annual income returns are formed by summing the monthly income payments and dividing this sum by the beginning-of-year price. Returns are formed assuming no taxes or transaction costs, except for returns on small capitalization stocks that show the performance of an actual, tax-exempt investment fund including transaction and management costs, starting in 1982.

## An Extraordinary Year

The year 2020 was an extraordinary year in many respects. A once in a century pandemic (i.e., COVID-19) uprooted the global economy, created chaos all around the world, and precipitated a global economic slowdown.[11] The world economy shrunk by the most on record, and the U.S. economy's GDP had the worst performance since World War II [12,13] In an attempt to slow the spread of the virus, the U.S. and other countries adopted unprecedented stay-at-home policies leading to a quasi-halt of economic activities in most countries in the world. This created a tremendous burden on businesses and populations throughout the world.

Given the uncertainty around the impact of the pandemic, financial markets experienced a high level of volatility. Policymakers sought to provide help to the U.S. economy through monetary and fiscal policy. The major themes that dominated U.S. financial markets during 2020 include the following: the COVID-19 pandemic and how to fight it, equity markets' volatility and Economic recovery, monetary policy and fiscal policy, and the political uncertainty and social unrest engendered by the pandemic.

---

11 The World Health Organization declared COVID-19 a pandemic on March 11, 2020.

12 According to the latest update form the International Monetary Fund, the world economy is expected to shrink by 3.5%, the worst performance on record.

13 According to data from the U.S. Bureau of Economic Analysis (BEA), the U.S. GDP decreased by 9.03% from a year earlier in the second quarter of 2020 and 8.98% from the previous quarter. For more details, visit: www.bea.org.

## The COVID-19 Pandemic

On December 31, 2019, the World Health Organization (WHO) was informed of an outbreak of "pneumonia of unknown cause" detected in Wuhan, a large city in the Hubei Province, China.[14]

According to Johns Hopkins University, the virus was determined to be a novel type of coronavirus.[15] On January 10, 2020, gene sequencing further determined that the Wuhan coronavirus was related to the Severe Acute Respiratory Syndrome virus (SARS-CoV) which impacted primarily mainland China and Hong Kong, a special administrative region of China, during 2002 and 2003 and the Middle Eastern Respiratory Syndrome virus (MERS-CoV)that began in Saudi Arabia in 2012.[16] However, the rate of infection of COVID-19 appeared to be higher than that of SARS-CoV and MERS-CoV.[17]

On March 11, 2020, the WHO announced that it was changing its classification of COVID-19 to a "pandemic," which meant the disease was spreading rapidly to different parts of the world.[18] By March 13, 2020, Europe became the epicenter of the pandemic with more reported cases and deaths than in any other part of the world.[19] By April 11, 2020, the U.S. recorded the highest number of COVID-19 deaths in the world, surpassing Italy and other European countries according to John Hopkins University.[20] According to the same source, December was the deadliest month in 2020 for the U.S., with more than 77,500 of the country's 346,000 COVID-19 deaths (as of that time) occurring in that month.[21]

A public-private partnership led to the development of vaccines in record time. On May 15, 2020, the White House announced the launch of "Operation Warp Speed" (OWS) with the objective to fund the development, manufacture, and distribution of COVID-19 vaccines, therapeutics, and diagnostics.[22] One of the major objectives of OWS was to provide substantial quantities of vaccines to Americans by January 2021. The objective was achieved with two vaccines approved for emergency use: Pfizer-BioNTech vaccine on December 11, 2020 and Moderna COVID-19

---

14 World Health Organization, "Pneumonia of unknown cause – China", January 5, 2020, accessible here: https://www.who.int/csr/don/05-january-2020-pneumonia-of-unkown-cause-china/en/.

15 Gardner, Lauren, "Mapping 2019-nCoV", Center for Systems Science and Engineering, Johns Hopkins University, January 23, 2020, accessible here: https://systems.jhu.edu/research/public-health/ncov/.

16 Ibid.

17 Cohut, Maria, "Novel coronavirus: Your questions, answered", MedicalNewsToday, March 19, 2020, https://www.medicalnewstoday.com/articles/novel-coronavirus-your-questions-answered.

18 World Health Organization, "WHO Director-General's opening remarks at the media briefing on COVID-19", March 11, 2020, available here: https://www.who.int/dg/speeches/detail/who-director-general-s-opening-remarks-at-the-media-briefing-on-covid-19-- 11- march-2020.

19 World Health Organization, "WHO Director-General's opening remarks at the media briefing on COVID-19", March 13, 2020, available here: https://www.who.int/director-general/speeches/detail/who-director-general-s-opening-remarks-at-the-mission briefing-oncovid- 19---13-march-2020.

20 "U.S. COVID-19 deaths reach 20,200, surpassing Italy as highest in the world." April 11, 2020, https://www.cbc.ca/news/world/u-s-covid-19-deaths-highest-in-world-1.5529861.

21 Maxouris, Christina and Jason Hanna "US surpasses 20 million Covid-19 cases while experts foresee tough times in January." January 1, 2021, https://www.cnn.com/2021/01/01/health/us-coronavirus-friday/index.html.

22 For more details, please see press release here: https://www.hhs.gov/about/news/2020/05/15/trump-administration announcesframework- and-leadership-for-operation-warp-speed.html.

vaccine December 18, 2020.[23,24] As of December 30, 2021, 2.8 million Americans had received a vaccine.[25,26]

## The U.S. Equity Market

The U.S. equity markets were volatile in 2020, moving with news of the virus, potential therapeutics and vaccines, monetary and fiscal policy interventions, and the political uncertainty surrounding the 2020 presidential election. Despite these events, the U.S. equity markets ended the year at new record highs.

In the first weeks of 2020 U.S. equity markets rose, and by mid-February all three major U.S. indices had achieved new all-time highs (on February 12, 2020 the Dow Jones Industrial Average (DJIA) reached 29,551.42, and on February 19, 2020 the S&P 500 and NASDAQ Composite Index reached 3,386.15 and 9,817.18, respectively).

As COVID-19 spread and Europe became the new epicenter of infection, fears arose that the virus might have a much bigger impact on the economy than anticipated, especially after various governments decided to close their borders and enact stay-at-home policies.[27] In response, the U.S. Federal Reserve (Fed) (i) lowered the Fed funds target rate by 50 basis points (b.p.) on March 3, 2020 to a range of 1.00%–1.25% and then lowered by an additional 100 b.p. on March 16, bringing the target range to 0% 0.25%, and (ii) provided liquidity to financial institutions and in some cases to non-financial institutions to help them navigate this crisis.[28] Subsequently, Congress issued a series of aid packages to help individuals and businesses.[29]

By March 23, 2020, the DJIA, the S&P 500, and the NASDAQ had fallen by 37.1%, 33.9%, and 30.1%, respectively. Movements in equity markets were so sudden that "circuit breakers" were activated four times in March 2020.[30] Circuit breakers are designed to prevent market crashes and help markets digest information before continuing to trade.[31] Trading was halted for 15

---

23 For more details, please see press release here: https://www.fda.gov/news-events/press-announcements/fda-takes-key actionfight- against-covid-19-issuing-emergency-use-authorization-first-covid-19.

24 For more details, please see press release here: https://www.fda.gov/news-events/press-announcements/fda-takes additionalaction- fight-against-covid-19-issuing-emergency-use-authorization-second-covid.

25 Spalding, Rebecca and Carl O'Donnell, "U.S. vaccinations in 2020 fall far short of target of 20 million people." Reuters, December 31, 2020, https://www.reuters.com/article/us-health-coronavirus-usa-vaccinations-idUSKBN29512W.

26 By mid-March 2021, over 120 million vaccine doses had been administered in the U.S. See: https://covid.cdc.gov/covid-data-tracker/#vaccinations.

27 According to the Center for Disease Control and Prevention (CDC), between the period March 1, 2020 and May 31, 2020, 73% of the 3,233 U.S. counties issued mandatory stay-at-home policies. See: Moreland, Amanda, Christine Herlihy, Michael A. Tynan, et al. "Timing of State and Territorial COVID-19 Stay-at-Home Orders and Changes in Population Movement – United States, March 1–May 31, 2020." Morbidity Mortality Weekly Report 2020; 69(35):1198–1203 at: https://www.cdc.gov/mmwr/volumes/69/wr/ mm6935a2.htm. For more details on the stay-at-home orders in the U.S. please visit: https://www.finra.org/rules-guidance/key-topics/covid-19/shelter-in-place.

28 For more details about the actions of the Fed, see: Chenh, Jeffery, Tyler Powell, Fae Skidmore and David Wessel, "What's the Fed doing in response to the COVID-19 crisis? What more could it do?" Brookings-The Hutchins Center Explains Series, January 25, 2021 https://www.brookings.edu/research/fed-response-to-covid19/.

29 For more details on the actions of Congress, see: https://pingree.house.gov/coronavirus/congress-response-on-coronavirus.htm.

30 Circuit-breaker points represent the thresholds at which trading is halted market-wide for single-day declines in the S&P 500 Index. Circuit breakers halt trading on the nation's stock markets during dramatic drops and are set at 7%, 13%, and 20% of the closing price for the previous day. The circuit breakers are calculated daily. Source: https://www.nyse.com/markets/nyse/trading-info.

31 These circuit breakers were created after the October 19, 1987 crash. To learn more, see: Funakosi, Minami and Trabis Hartman "March madness", Reuters, March 18, 2020, https://graphics.reuters.com/USA-MARKETS/0100B5L144C/index.html.

minutes on March 9, 2020 after the index slid 7% in the first three minutes after the opening bell; this was the first time in 20 years that market wide circuit breaker kicked in.[32] The second trading halt was on March 12, 2020, just six minutes after the start of the session. The third was on March 16, 2020 immediately after the session open, and the last time occurred on March 18, 2020 at 12:57 PM EST. All these halts were Level 1, meaning they lasted only 15 minutes.

The circuit breakers did not prevent equities from recording some of the worst daily performances in decades. In March 2020, S&P 500 recoded the third and the sixth worst performance in its history.[33] The S&P 500 dropped 260.74 points (-9.51%) and 324.89 points (-11.98%) on March 12 and March 16, respectively. These negative milestones were only beaten by the 1987 and 1929 market crashes performances. Remarkably, March 2020 also saw S&P record some of its best daily performances. The S&P 500 recorded two of the top ten daily performances in history by gaining 230.38 points (9.29%) and 209.93 points (9.38%) on March 13 and March 24, respectively.

These movements in U.S. equity markets pushed volatility to new highs. The VIX index, dubbed “the fear index,” which measures volatility in equity markets, reached a new high of 82.69 on March 16, 2020. This new record surpassed the previous high of 80.86 recorded on November 11, 2008 at the height of the Global Financial Crisis (GFC). Unlike when the VIX shattered its records multiple times during the GFC, during the pandemic the Fed intervention helped soothe markets, and VIX retraced by more than 50% less than a month after it reached its peak.[34] The VIX spiked moderately in the summer and just before the 2020 presidential election on November 3, but it stayed range bound and finished the year at 22.75.[35]

U.S. equity indices ultimately recovered and reached new highs in 2020. The NASDAQ was the first major U.S. equity index to recover and register a new high. Propelled by gains in technology stocks, the NASDAQ closed at 9,924.75 on June 8, 2020, surpassing the previous all-time high set on February 19, 2020 (9,817.18). By the end of the year, the NASDAQ had increased to 12,888.28, representing a 43.64% increase for the year and a 87.9% increase from the 2020 low set on March 23, 2020.

The S&P 500 was the second major U.S. equity index to recover and register a new high. The S&P 500 (price index) closed at 3,389.78 on August 18, 2020, surpassing the previous all-time high set on February 19 (3,386.15). By the end of the year, the S&P 500 had increased to 3,756.07, which represented a 16.3% increase for the year, and a 67.9% increase from the 2020 low set on March 23, 2020.

---

32 Pisani, Bob, “Circuit breakers, triggered for the first time in 20 years, pass a crucial test.” CNBC.com, March 9, 2020, https://www.cnbc.com/2020/03/09/circuit-breakers-triggered-for-the-first-time-in-20-years-pass-a-crucial-test.html.

33 “Sizzlers and Fizzlers” S&P Global, Accessed on February 20, https://www.spglobal.com/spdji/en/indexology/djia-and-sp-500/sizzlers-and-fizzlers/.

34 VIX crossed 41.38 on April 23, 2020 and did not break this level for the remainder of the year.

35 Source: S&P Capital IQ.

Finally, the DJIA recovered and registered a new all-time high of 29,950.44 on November 16, 2020, surpassing the previous all-time high set on February 12, 2020 (29,551.42). By the end of the year, the DJIA had increased to 30,606.48, which represented a 7.2% increase for the year, and a 64.6% increase from the 2020 low set on March 23, 2020.

The recovery was not even across sectors of the economy, and some sectors never recovered by the end of the year. Given the very unusual character of this recession, some sectors of the economy benefited more than others. Technology stocks were the winners from the stay-at-home policies as people worked from home and rarely ventured outside. Companies like Amazon, Netflix, Zoom, and others expanded their customer bases, whereas companies in hospitality and energy were hit the hardest. The S&P 500 information technology sector index was up 42.21% over the year, whereas the S&P 500 Energy Sector index was down 37.31% over the same period. The uncertainty around work in the office and mall shopping led to a decrease in the performance of real estate stocks. The S&P 500 Real Estate Sector Index did not recover after the pandemic even though it started the year with a good performance (6.43% compared to 4.81% for the S&P 500 from December 31, 2019 to February 23, 2020). By the end of the year, the sector lost 5.17%, the second worst sector sectorial performance after Energy.

## The U.S. Economy

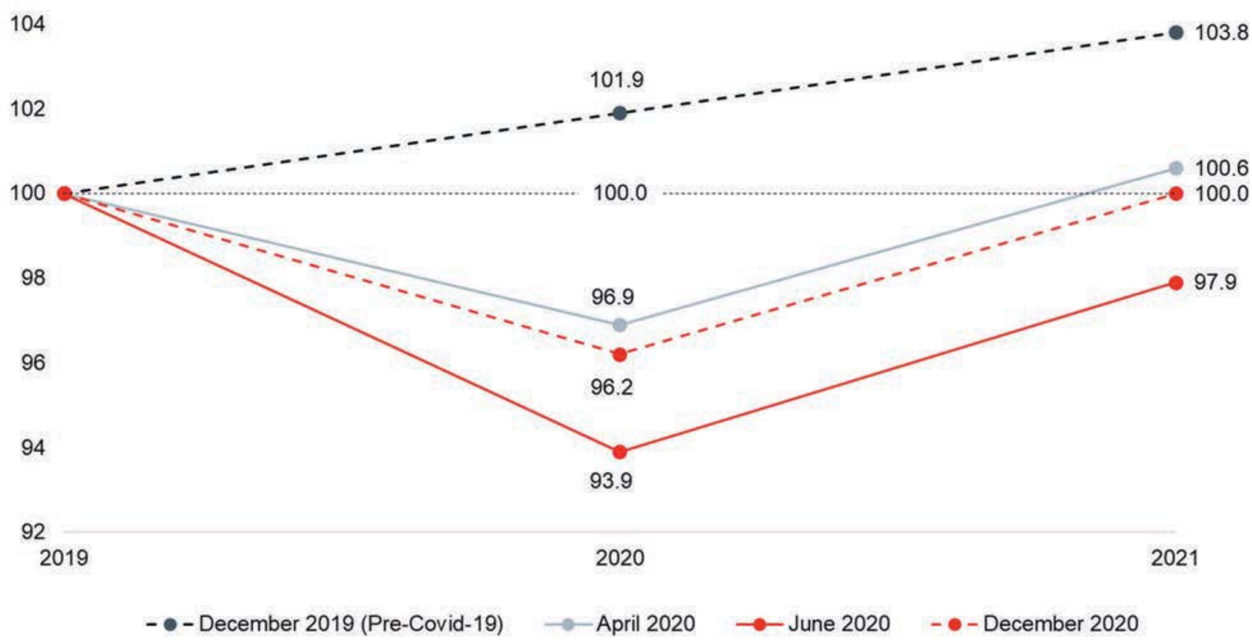
The stock market is not the economy. The economy did not recover by the end of the year as equity markets did. The uncertainty that followed the expansion of COVID-19 and the adoption of stay-at-home orders unsettled markets and made forecasting difficult. As more states started to announce policies to mitigate the expansion of the virus, economists started updating their U.S. real GDP forecasts to reflect the reality on the ground. However, given the unusual character of these events and the inability to understand their effects, forecasts were revised continually as we moved throughout the year.

Exhibit 1.1 compares U.S. GDP growth forecasts for the years 2020 and 2021 made at the end of 2019 (before the COVID-19 outbreak) to forecasts made over the course of 2020 (after the COVID-19 outbreak). The GDP growth estimates are reported as an index (year end 2019 = 100).

The gray dashed line at the top in Exhibit 1.1 represents the path of GDP growth that was expected at the end of 2019 before COVID-19 broke out in the first quarter of 2020. At the end of 2019, economists expected the economy to grow by 1.9% (101.9/100 – 1) by the end of 2020 and by 3.8% by the end of 2021 (compared to year-end 2019 levels).

By April 2020 (solid light gray line) the negative economic effects of COVID-19 began to come into focus, and GDP forecasts indicated a decrease of 3.1% (96.9/100 – 1) by the end of 2020, but it still held an expectation that the economy would recover and actually show a slight 0.6% increase by the end of 2021.

**Exhibit 1.1:** GDP Growth Estimates Before and After COVID-19




**Source of underlying data:** OECD, IMF, World Bank, Blue Chip Economic Indicators, Consensus Economics, Economist Intelligent Unit, Fitch Ratings, IHS Markit, Moody's Analytics, Oxford Economics, S&P Global Ratings.

By June 2020 (solid red line at bottom), as many governments imposed sweeping stay-at-home orders, forecasts turned bleak and the U.S. economy was not expected to recover by the end of 2021 as previously expected. As Exhibit 1.1 shows, economists believed that the U.S. economy would shrink by 6.1% (93.9/100 – 1) by the end 2020 and only recover approximately two thirds of that loss by the end of 2021 to a net growth of –2.1%.

At the end of 2020 the U.S. had two COVID-19 vaccines, the expectation of continued low interest rates through at least 2023, the resolution of the U.S. presidential election, and improved business confidence. As of December 2020 (dashed red line), economists' forecasts improved to show that the U.S. economy would shrink by only 3.8% by the end of 2020 and would recover completely by the end of 2021.

The degree to which the large changes in commerce, work life, schooling, travel, etc., that the COVID-19 pandemic forced upon societies in 2020 and 2021 will remain in place is unclear, and economic forecasts will undoubtedly be revised as time passes.

The performance of the U.S. economy was ultimately negative in the first and second quarters. The annualized quarter on quarter change in GDP in the first and second quarters were –5% and –31.4%, respectively. While the economy recovered in the third quarter, the fourth quarter showed that growth was losing steam. Since stay-at-home policies only started in March, the first quarter performance was negative but not as dismal as the second. The stimulus package offered by Congress and the expansionary monetary policy implemented by the Federal Reserve Bank helped the economy recover some of its loss. The annualized quarter on quarter performance was 33.4% in the third quarter.[36] This performance was not enough for the economy to recover the ground it lost. The last quarter of the year was relatively weak with a performance of 4.1%.[37] On a yearly basis, the U.S. real GDP decreased by 3.5% in 2020.[38]

## Unemployment

The labor market was devasted by the pandemic; the unemployment rate went from one of the best job markets in decades to the worst in the post-World War II period. As of February 2020, the U.S. unemployment rate was at 3.5%, the lowest level since December 1969. In two months, the unemployment rate reached 14.8%, the highest post-World war II level. As the stay-at-home order closed the economy, more and more people lost their jobs. The economy was shedding jobs at a higher rate than any other recession in the last 50 years.

The number of unemployment insurance claims filed during this period shows the level of devastation in the labor market. The cumulative number of claims filed as of the end of December 2020 surpassed any other recession on record. Since the National Bureau of Economic Research (NBER) declared the U.S. in a recession in February 2020, the monthly average initial claims filled was 6.77 million claims a month, around three times the highest monthly average number of initial claims filed in a recession since 1969.[39]

Exhibit 1.2 reports the cumulative number of initial claims each month since a recession was declared by NBER in the last 40 years. The cumulative numbers of claims reported each month during the 2020 recession overshadowed the numbers reported in any recession over the last 40 years. Eleven months into this recession in December 2020, the total number of claims in all recessions since 1981 to 2008 was 43.5 million, whereas the total number of claims filed in the 2020 recession alone was 74.5 million. In other words, the number of claims filed through December 2020 was 1.7 times (74.5 million ÷ 43.5 million) the sum of all previous recessions' cumulative claims combined over a similar period.

---

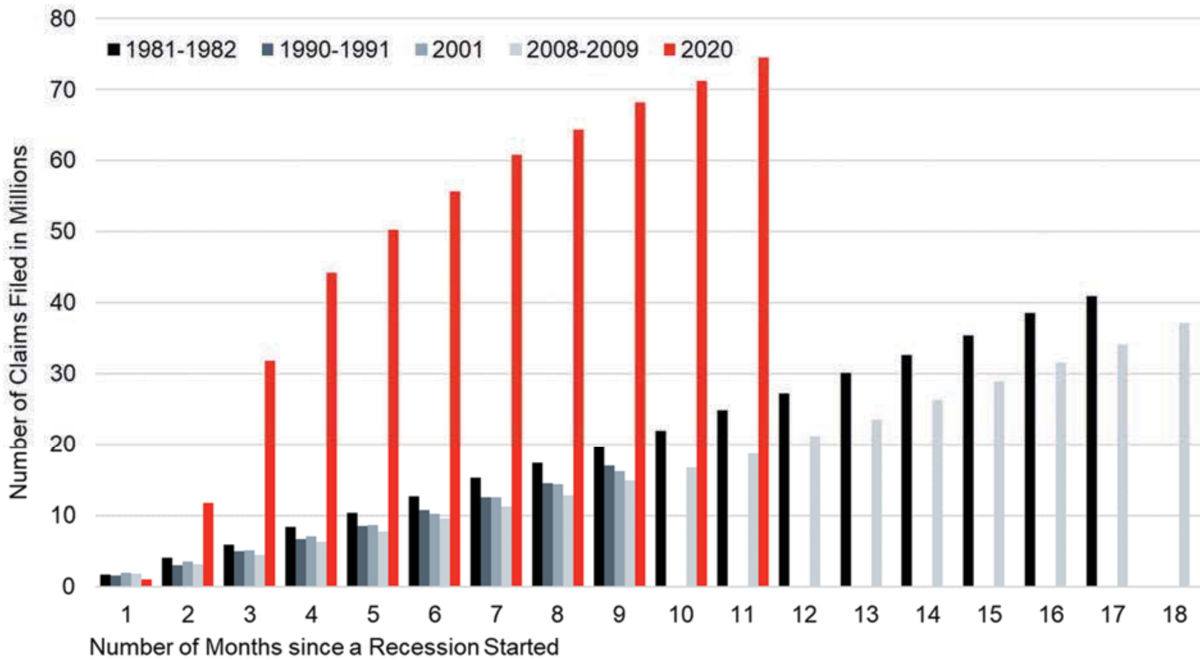
36 According to the Bureau of Economic Analysis, the year on year GDP growth in the first, second and third quarter was 0.3%, -9% and -2.8%, respectively. For more details visit: www.bea.gov.

37 The calculation of the fourth quarter U.S. GDP growth is based on the Bureau of Economic Analysis's second estimate. For more details visit: www.bea.gov.

38 See press release here: https://www.bea.gov/news/2021/gross-domestic-product-fourth-quarter-and-year-2020-second-estimate.

39 The highest average monthly number of unemployment insurance claims filled in a recession since 1969 was recorded in the 1981-1982 recession which was 2.41 million claims a month. The ratio is calculated as 6.77/2.41=2.8.

**Exhibit 1.2:** Cumulative Employment Insurance Claims for Each Recession Since 1981




**Source of underlying data:** Federal Reserve Banks of St Louis database, FRED

## Political Uncertainty

According to the Pew research, the U.S. became a politically polarized country over recent years.[40] During 2020, the impeachment of the U.S. President, the spread of the COVID-19 pandemic, the issue of racial justice, and the 2020 presidential election exacerbated disagreements between political rivals and led to heightened uncertainty.

A major political event of 2020 was the impeachment of President Donald Trump. On September 24, 2019, House Speaker Nancy Pelosi launched a formal House inquiry alleging that the president solicited foreign interference in the 2020 U.S. presidential to advance his chances of reelection.[41] The House Judiciary Committee voted to recommend two articles of impeachment: abuse of power and obstruction of Congress. On December 18, 2019, the Democrat controlled House voted to approve both articles, making President Trump the third president in history to be impeached. On January 16, 2020, the Senate trial began, and President Trump was acquitted of both charges three weeks later.[42]

---

40 Dimock, Michael and Richard Wike, "America is exceptional in the nature of its political divide.", PEW Research, November 13, 2020 https://www.pewresearch.org/fact-tank/2020/11/13/america-is-exceptional-in-the-nature-of-its-political-divide/.

41 Przybyla, Heidi and Adam Edelman "Nancy Pelosi announces formal impeachment inquiry of Trump.", NBC News, September 24, 2019, https://www.nbcnews.com/politics/trump-impeachment-inquiry/pelosi-announce-formal-impeachment-inquiry-trump-n1058251.

42 Kyle Cheney, Andrew Desiderio And John Bresnahan "Trump acquitted on impeachment charges, ending gravest threat to his presidency." Politico.com, February 5, 2020, https://www.politico.com/news/2020/02/05/trump-impeachment-vote-110805.

Research has shown that pandemics tend to increase the likelihood of social unrest.[43] The recent pandemic is no different. On May 25, 2020, George Floyd, an African American man, died while being apprehended by police in Minneapolis, Minnesota. The death of Mr. Floyd led to the eruption of protests in multiple cities across the U.S. in the name of racial justice and social equality. These protests morphed into an international movement. Protesters in countries as far away as Australia, Brazil, France, and Canada echoed the same slogans as in the U.S.[44] However, these protests meant people gathering in large crowds, which might hinder the efforts to contain the expansion of the virus and further the pain of the economy.[45] According to a Bank of America survey, investors are more worried about the pandemic than social unrest.[46] As a result, equity markets shrugged-off these events and continued their ascent.[47]

Mr. Trump lost the 2020 Presidential Election on November 3, 2020, but he contested the election in numerous states.[48] Mr. Trump and his legal team challenged the results in various courts and asked for recounts in states where it was possible, specifically in Georgia and Wisconsin.[49] As more states certified their results, the uncertainty around the results of elections decreased, and Mr. Joe Biden was sworn in as the 46th U.S. president. The resolution of the election uncertainty was seemingly welcomed by the equity markets, and the S&P 500 rose by 13.47% between November 2, 2020 and December 31, 2020.

Another important aspect of the November 2020 election was which party would control the House and the Senate. If the Democrats held the House and had a net gain of three seats in the Senate, the party would control of both houses of Congress.[50] Ultimately in the November 2020 election, Democrats held the House and achieved a net gain of one seat in the Senate. However, two Senate seats in Georgia required a special run-off election because none of the candidates reached the 50% mark.[51] The special run-off election took place on January 4, 2021 and sent two Democrats to the Senate, giving Democrats 50 seats and effective control of both the House and

---

43 Barrett, Philip, and Sophia Chen. "Social Repercussions of Pandemics." No. 2021/021. International Monetary Fund, 2021. Accessible here: https://www.imf.org/en/Publications/WP/Issues/2021/01/29/Social-Repercussions-of-Pandemics-50041.

44 Daragahi, Borzou "Why the George Floyd protests went global." June 10, 2020, https://www.atlanticcouncil.org/blogs/new-atlanticist/george-floyd-protests-world-racism/.

45 Reinicke, Carmen, "Here's how 4 financial experts think protests could negatively affect markets and the US economic recovery"June 1, 2020, https://markets.businessinsider.com/news/stocks/protests-negatively-affect-markets-us-economic-recovery-financial-expertsstocks- 2020-6-1029269622.

46 Garber, Jonathan, "Record stock rally faces risks from civil unrest, tech bubble" November 17, 2020 https://www.foxbusiness.com/markets/civil-unrest-stock-market-risk.

47 Wallace, Joe and Paul Vigna, "U.S. Stocks Close Higher Despite Social Unrest", Wall Street Journal, June 2, 2020 https://www.wsj.com/articles/global-stock-markets-dow-update-6-02-2020-11591072455.

48 The president's legal team contested the election in Wisconsin, Arizona, Nevada, Michigan, Minnesota, Georgia and Pennsylvania. See: Schartz, Matthew S., "Trump's Legal Losses Come Fast And Furious." NPR, December 5, 2020, https://www.npr.org/2020/12/05/943535299/trumps-legal-losses-come-fast-and-furious.

49 Breuninger, Kevin and Dan Mangan, "Trump campaign requests partial Wisconsin recount, deposits $3 million to challenge Biden victory" CNBC.com, November 18, 2020 https://www.cnbc.com/2020/11/18/trump-campaign-filing-for-partial-wisconsin recountchallenging- biden-victory.html; Jester, Julia and Dennis Romero "Trump campaign asks for another Georgia recount", NBC News, November 22, 2020 https://www.nbcnews.com/politics/2020-election/trump-campaign-asks-another-georgia-recount n1248538.

50 Before the election, the Republicans controlled the Senate by a majority of 53 to 47. A Democratic net gain of three seats would split the Senate 50-50, and Democratic Vice President Harris would cast the deciding vote in the case of ties. 1.41 Barrett, Philip, and Sophia Chen. "Social Repercussions of Pandemics." No. 2021/021. International Monetary Fund, 2021. Accessible here: https://www.imf.org/en/Publications/WP/Issues/2021/01/29/Social-Repercussions-of-Pandemics-50041.

51 "US Election 2020: Battle for US Senate to be decided in January." BBC, November 7, 2020, https://www.bbc.com/news/election-us-2020-54835724.

the Senate. Although their majorities are slim, democrat control could lead to the enactment of business-unfriendly legislation that lowers future after-tax corporate earnings.

## Monetary Policy

At the first FOMC meeting of the year 2020, the committee decided to leave the target range for the federal fund rate at 1.5%–1.75% which was considered by the FOMC an appropriate policy stance given the level of inflation and employment registered at that the time.[52] However, as more information about COVID-19 and its expansion emerged, the risk to the economy became more apparent. In an unscheduled meeting on March 3, 2020, the FOMC decided to lower the Fed funds target range by 50 b.p.[53]

In another unscheduled meeting on March 15, 2020, the FOMC lowered its rate even further and decided to use additional tools to cushion the effect of COVID-19 on the economy.[54] First, the FOMC relaunched its Quantitative Easing program where it announced the purchase of at least $700 billion of Treasury securities and agency mortgage-backed securities. Second, the committee decided to lower the banks' reserve requirement to zero and encouraged banks to use capital and liquidity buffers to provide loans to businesses and households affected by the expansion of the virus. Third, the committee announced a coordinated international action to provide U.S. dollar liquidity swap arrangement to the Bank of Canada, the Bank of England, the Bank of Japan, the European Central Bank, and the Swiss National Bank.[55]

Over the following week, the Fed issued multiple statements in which it announced additional measures to support the economy. On March 17, the Fed announced the creation of Primary Dealer Credit Facility (PDCF) that offers loans to large broker-dealer collateralized by a broad range of securities, including commercial papers and equity securities.[56] On March 18, the Fed announced the creation of Money Market Mutual Fund Liquidity Facility (MMLF) that offers collateralized loans to large banks who buy assets from money market mutual funds.[57]

On March 23, the Fed established new facilities and extended the reach of some of the previously announced ones. Three new emergency lending facilities were announced: Primary Market Corporate Credit Facility (PMCCF) to provide companies access to credit to maintain operations and capacity, Secondary Market Corporate Credit Facility (SMCCF) to support credit to large employers, and the Term Asset-Backed Securities Loan Facility (TALF) to support credit to consumers and businesses. These programs, which provide up to $300 billion in new financing options to firms, are backed by a $30 billion equity provided by the Treasury Department's

---

52 For more details, FOMC press release can be found here: https://www.federalreserve.gov/newsevents/pressreleases/monetary20200129a.htm.

53 For more details, please refer to the FOMC press release: https://www.federalreserve.gov/newsevents/pressreleases/monetary20200303a.htm.

54 For more details, please refer to the FOMC press release: https://www.federalreserve.gov/newsevents/pressreleases/monetary20200315a.htm.

55 For more details, please refer to the FOMC press release: https://www.federalreserve.gov/newsevents/pressreleases/monetary20200315c.htm.

56 For more details, please refer to the FOMC press release: https://www.federalreserve.gov/newsevents/pressreleases/monetary20200317b.htm.

57 For more details, please refer to the FOMC press release: https://www.federalreserve.gov/newsevents/pressreleases/monetary20200318a.htm.

Exchange Stabilization Fund (ESF).[58] On June 15, the Fed decided to expand the SMCCF and buy a portfolio index of U.S. corporate bonds rated investment grade as of March 22.[59]

As an effect of all these new lending facilities and interventions, the Fed's balance sheet increased from $4.2 trillion at the end of 2019 to $7.4 trillion at the end of 2020 (an approximate 76% increase). Although the dollar amount increase (approximately $3.2 trillion) was the largest on record, the expansion of the balance sheet during the GFC was larger in percentage terms (151%).[60]

## Fiscal Policy

As the pandemic intensified and the economy weakened, lawmakers enacted laws to help businesses and households through the crisis. Four major fiscal packages were enacted by Congress to fight the pandemic and its impact on the economy. The first fiscal package was the "Coronavirus Preparedness and Response Supplemental Appropriations Act" passed by the House and the Senate on March 4, and March 5, respectively, and signed into law by the U.S. President Donald Trump on March 6. This act was mainly directed to help with development of vaccines and therapeutics and the acquisition of medical supplies needed to fight the virus.

The second fiscal package enacted by Congress, the "Families First Coronavirus Response Act," was designed to help finance free COVID-19 tests, establish a 14-day paid leave for workers affected by the pandemic, and increase funding for food stamps. The act was passed by the House March 14, the Senate on March 18, and signed into law by President Trump on the same day.

The third (and possibly most important) fiscal package enacted in 2020 was called the "Coronavirus Aid, Relief, and Economic Security Act" or "CARES Act". It was passed by the Senate on March 25, by the House on March 27, and signed into a law on the same day by the president. This fiscal package was the biggest fiscal package ever voted in the U.S. history, with a total spending power of $2 trillion.[61] It included a direct cash payment to individuals and extra unemployment assistance payments, funding to small businesses, funding for sectors affected by the pandemic like Airlines, public health institutions and state and local governments, as well as relief for college students and graduates.[62]

The CARES Act boosted the Fed's actions by providing capital and legislation to offer more facilities to the economy. Three new facilities issued by the Fed were set up to help small

---

58 For more details, please refer to the FOMC press release: https://www.federalreserve.gov/newsevents/pressreleases/monetary20200323b.htm.

59 For more details, please refer to the New York Federal Reserve bank press release: https://www.newyorkfed.org/newsevents/news/markets/2020/20200615

60 According to data from the Federal Reserve Bank of St. Louis, the total assets of the Federal Reserve Bank were $7,363 billion and $4,165 billion as of December 30, 2020 and December 25, 2019, respectively; and $2,239 billion and $890 billion as of December 31, 2008 and December 26, 2007, respectively.

61 Pramuk, Jacob "Trump signs $2 trillion coronavirus relief bill as the US tries to prevent economic devastation", CNBC.com, March 27, 2020, https://www.cnbc.com/2020/03/27/house-passes-2-trillion-coronavirus-stimulus-bill-sends-it-to-trump.html.

62 Snell, Kelsey "What's Inside The Senate's $2 Trillion Coronavirus Aid Package", NPR, March 26, 2020 https://www.npr.org/2020/03/26/821457551/whats-inside-the-senate-s-2-trillion-coronavirus-aid-package.

businesses and local governments recover from the impact the pandemic. The first facility established was the Paycheck Protection Program Liquidity Facility (PPPFL) that would purchase Payment Protection Program (PPP) loans guaranteed by the Small Business Administration (SBA) from lenders.[63] The second facility was the Main Street New Loan Facility. With an equity $75 billion, the facility's objective is to purchase $600 billion of debt from companies employing up to 10,000 workers or with revenues of less than $2.5 billion, with any required payments on these loans deferred for a year.[64] The third facility is the Municipal Liquidity, designed to help local and state government with the loss of revenue from the disruption of economic activity. Its objective was to purchase $500 billion of debt from counties with a population of at least 500,000 and cities with a population of at least 250,000.[65]

As the third wave of the pandemic took hold of the nation, Congress agreed on providing a fourth relief package to further help households and businesses. The bill was signed into law in late December 23, 2020. The bill provided an additional direct cash payment to individuals, more funding for the Payment Protection Program, funding to expand unemployment insurance, funds for rental assistance, help to the transportation and healthcare sector. This last package had a price tag of $900 billion.[66]

## Commodities

Like other markets, the pandemic unsettled the commodities market as well, especially for energy. As more and more countries adopted stayed at home policies and air travel slowed, the price of oil started to decrease. The demand for oil collapsed and the price started to follow. This was compounded by a break in negotiation between the leading OPEC+ members, Saudi Arabia and Russia, that resulted in an undeclared oil price war and the flooding of the international oil market.[67]

In early 2020, COVID-19 ravaged the second largest economy in the world (China) as the country's crude imports slowed and refineries decrease their output.[68] In February 2020, the International Energy Agency forecasted that demand growth would fall to the lowest rate since 2011, with full-year growth falling by 325,000 bpd to 825,000 bpd[69] and a first quarter contraction in consumption by 435,000 bpd. Russia and OPEC's leading member, Saudi Arabia, started discussion on the level of cuts required to cope with this fall in demand, but they did not reach an agreement. In a retaliatory move, OPEC members decided to remove all limits on production. On

63 For more details, please refer to the FOMC press release: https://www.federalreserve.gov/newsevents/pressreleases/files/monetary20200409a6.pdf.

64 For more details, please refer to the FOMC press release: https://www.federalreserve.gov/newsevents/pressreleases/files/monetary20200409a7.pdf.

65 For more details, please refer to the FOMC press release: https://www.federalreserve.gov/monetarypolicy/muni.htm.

66 Rifis B., Jared, Kenneth A. Johnson and Zane S. Hatahet "Federal COVID Relief Bill passed by Congress - December 2020", The National Law Review, December 23, 2020, https://www.natlawreview.com/article/federal-covid-relief-bill-passed-congress-december-2020.

67 OPEC stands for Organization of the Petroleum Exporting Countries. List of member countries in OPEC+: Algeria, Angola, Azerbaijan, Bahrain, Brunei, Congo, Ecuador, Equatorial Guinea, Gabon, Iraq, Iran, Kazakhstan, Kuwait, Libya, Malaysia, Mexico, Nigeria, Oman, Russia, Saudi Arabia, South Sudan, Sudan, Venezuela, and UAE.

68 Aizhu, Chen "ChemChina becomes latest Chinese refiner to slash output due to coronavirus: sources", Reuters, February 13, 2002 https://www.reuters.com/article/us-china-health-chemchina-refinery/chemchina-becomes-latest-chinese-refiner-to-slash-output-due-to-coronavirus-sources-idUSKBN2070E8.

69 IEA (2020), Oil Market Report - February 2020, IEA, Paris https://www.iea.org/reports/oil-market-report-february-2020.

March 7, 2020, Saudi Arabia offered price discounts to customers in Europe, Asia, and the United States.[70]

The West Texas Intermediate (WTI), the U.S. oil benchmark fell by 24.59%, and Brent, the international oil benchmark, fell by 24.10%. The pressure on oil prices continued as demand continued to falter and the spat between Russia and Saudi Arabia intensified. Oil prices reached the lowest level since 2002.[71]

Following pressure from U.S. President Donald Trump, Russia and Saudi Arabia agreed to organize an emergency meeting.[72] During this meeting, both nations along with other members of OPEC+ decided to lower production by 9.7 million bpd until June 2020, and by 7.7 million bpd between July and the December 2020.[73] However, the damage to oil markets had already been done. The excess supply was still lingering in the market and filled storage facilities. The price of WTI May delivery contract expiring on April 21, 2020 turned negative on April 20, 2020 as traders were trying to get contracts off their hands. The price of a WTI contract ended the day at negative $37.62.[74] As the economy started to open up and the recovery started, oil prices started to recover as well. The WTI ended the year at $48.52, a decrease of 20.54%, and the Brent at $51.8, a decrease of 21.52% compared to the end of 2019.

Copper, an important industrial commodity, has seen a different path than Oil throughout 2020. The price of copper decreased as uncertainty unsettled investors. As such, Copper, which is known for its high correlation with the economic cycle, dropped by 23% to $2.15/lb on March 18, 2020 from $2.8/lb at the end of 2019.[75] Some experts were expecting a supply glut because of the pandemic and an important decrease of demand from China, the world largest copper importer.[76] However, this outlook did not materialize, and the demand for copper did not decrease as expected. To the contrary, supply could not keep up.[77] Copper ended the year at $3.52/lb, a performance of 25.54%.

Gold, the traditional safe haven and store value, reached record highs during 2020. As uncertainty around COVID-19 increased and extraordinary monetary measures were taken by central banks

70 "Saudi Arabia slashes April crude prices after OPEC's supply pact collapsed", Reuters, March 7, 2020, https://www.reuters.com/article/audi-oil-prices-idUSL8N2B00TK.

71 Stevens, Pippa, "Oil falls 24% in 3rd worst day on record, sinks to more than 18-year low" CNBC.com, March 18, 2020 https://www.cnbc.com/2020/03/18/oil-plummets-to-near-18-year-low-on-pace-for-worst-month-ever.html.

72 Gardner, Timothy Steve Holland, Dmitry Zhdannikov, Rania El Gamal "Trump told Saudis: Cut oil supply or lose U.S. military support – sources" , Reuters, April 30, 2020, https://www.reuters.com/article/global-oil-trump-saudi/special-report-trump-told-saudis cutoil- supply-or-lose-u-s-military-support-sources-idUSL1N2CH29V.

73 Jacobs, Trent "OPEC+ Moves To End Price War With 9.7 Million B/D Cut" ,Journal of Petroleum Technology, April 12, 2020, https://jpt.spe.org/opec-moves-end-price-war-10-million-bd-cut.

74 Lee, Nathaniel, "How negative oil prices revealed the dangers of the futures market", CNBC.com, June 16, 2020, https://www.cnbc.com/2020/06/16/how-negative-oil-prices-revealed-the-dangers-of-futures-trading.html#:~:text=A%20historic%20drop%20occurred%20on,around%20negative%20%2437%20per%20barrel.

75 Ashraf, Aoyon, "Copper Tapped as the Next Big Metals Trade of 2020" BNN-Bloomberg, December 16, 2019 https://www.bnnbloomberg.ca/copper-tapped-as-the-next-big-metals-trade-of-2020-1.1363467.

76 Ignacio, Reicelene Joy, "Global copper market in supply glut in 2020, 2021 – IWCC" , S&P Global Intelligence, May 27, 2020, https://www.spglobal.com/marketintelligence/en/news-insights/latest-news-headlines/global-copper-market-in-supply-glut-in-2020-2021-8211-iwcc-58811137.

77 Woodall, Toby, "Copper supply faces struggle to keep up with growing demand", S&P Global Intelligence, October 1, 2020, https://www.spglobal.com/marketintelligence/en/news-insights/latest-news-headlines/copper-supply-faces-struggle-to-keep-up-with-growing-demand-60471925.

around the world, investors turned to the yellow metal to protect themselves. Gold reached an all-time high of $2058.4/oz on August 8, 2020, but it retracted to end the year at $1895.1/oz, a performance of 24.75% compared to the end of 2019.

## Relative Performance of the SBBI® Series in 2020[78]

The relative performance of six U.S. asset classes plus inflation, as represented by the SBBI® series, is illustrated in Exhibit. 1.3. The relative performance of these series is reported for 2020 alone and as the average annual return over 1926–2020.

A few observations about the relationships in Exhibit 1.3:

- Large-Cap Stocks *outperformed* Small-Cap Stocks in 2020[79], counter to the average annual return over the 1926–2020 period where Large-Cap Stocks *underperformed* Small-Cap Stocks.

- Long-term (i.e., 20-year) U.S. corporate bonds and U.S. government bonds both significantly *outperformed* their 1926–2020 average annual returns in 2020, likely due to the Federal Reserve's shift to a more dovish monetary policy.[80]

- Long-term U.S. government bonds *outperformed* long-term U.S. corporate bonds in 2020. U.S. *corporate* bonds typically outperform equivalent-maturity U.S. government bonds due to investors' demand for greater compensation for investing in corporates due to default risk.[81] In 2020, a once in a century pandemic (i.e., COVID-19) uprooted the world economy and created chaos all around the world and precipitated a global economic slowdown. The outperformance in 2020 of U.S. government bonds compared to U.S. long-term corporate bonds in 2020 is likely due to a heightened demand for so-called "safe" securities (e.g., U.S. Treasuries) in times of crisis.

---

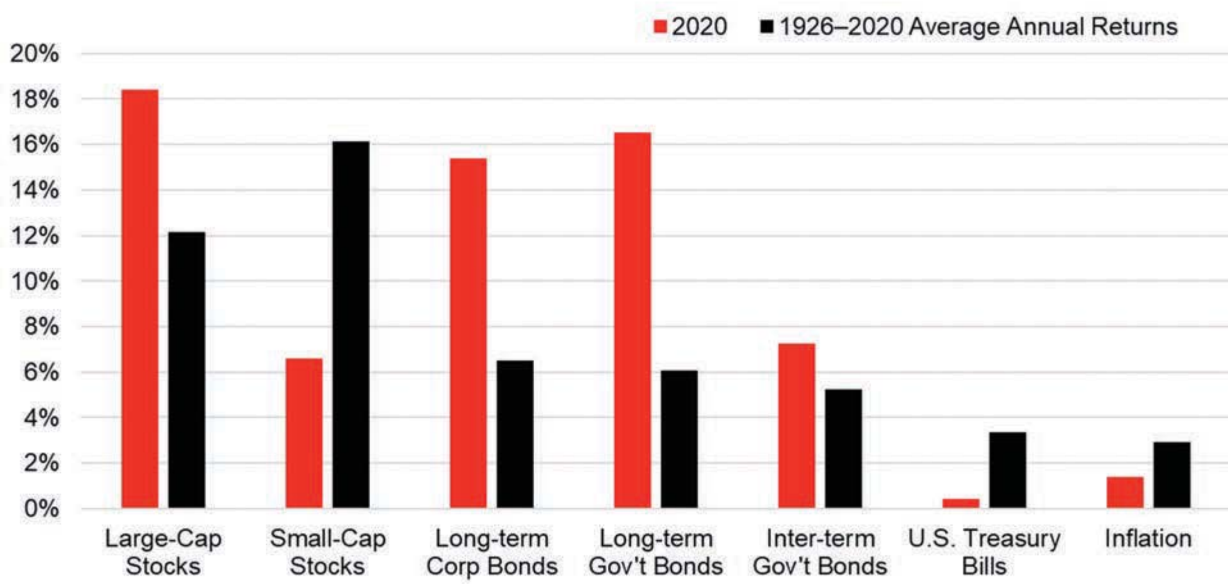
78 *Precalculated* summary statistics of annual returns (1926–2020) are presented in table format in the full-version *2021 SBBI® Yearbook* for the following Ibbotson Associates(IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: Large-Cap Stocks (total return, income return, and capital appreciation return), Small-Cap Stocks (total return), Long-term Corporate (i.e., 20-year) Bonds (total return), Long-term (i.e., 20-year) Government Bonds (total return, income return, and capital appreciation return), Intermediate-term (5-year) Government Bonds (total return, income return, and capital appreciation return), (30-day) U.S. Treasury Bills (total return), and Inflation. For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

79 One could argue that this is slightly atypical when measured over 12-month periods. For example, from January 1926–December 2020, there were 1,129 periods that were exactly 12 months in length. Small-Cap Stocks outperformed Large Cap Stocks 601 times, or approximately 53% of the time. When measured over longer periods, Small-Cap Stocks tend to outperform Large-Cap Stocks at an increasingly greater rate. For example, from January 1926–December 2020 there were 1,21 periods that were exactly 120 months (10 years) in length. Small-Cap Stocks outperformed Large Cap Stocks 686 times, or approximately 67% of the time. Over 20-year periods Small Cap Stocks did even better when compared to Large Cap Stocks, outperforming in 791 out of the 901 periods that were exactly 240 months (20 years) in length, or approximately 88% of the time.

80 Nick Timiraos, "Fed Signals Low Rates Likely to Last Several Years / Central bank also sets high hurdles for raising rates going forward", *Wall Street Journal*, September 16, 2020. See: https://www.wsj.com/articles/fed-signals-interest-rates-to-stay-near-zero-through-2023-11600279214.

81 Over the 1926–2020 time horizon the average annual return of U.S. long-term corporate bonds exceeded the returns of U.S. long-term government bonds (i.e., "Treasuries") (see Exhibit 1.3). On an annual basis, from 1926–2020 (95 years) U.S. long-term corporate bonds outperformed U.S. long-term government bonds 57 out of 95 years (60% of the time).

**Exhibit 1.3:** The Relative Performance of the Stocks, Bonds, Bills, and Inflations® (SBBI®) Series in 2020, and Over the 1926–2020 Time Horizon; Average Annual Returns




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## Relative Performance of the SBBI® Series by Decade

The great stock and bond market rise of the 1980s and 1990s was one of the most unusual in the history of the capital markets. In terms of the magnitude of the rise, these decades most closely resembled the 1920s and 1950s. These four decades accounted for a majority of the market's cumulative total return over the past 95 years. While the importance of a long-term view of investing is noted consistently in this book and elsewhere, the counterpart to this observation is this: to achieve high investment returns, one needs to participate only in the few periods of truly outstanding returns. The bull markets of 1922 to mid-1929, 1949–1961 (roughly speaking, the 1950s), mid-1982 to mid-1987, and 1991 1999 were such periods. More recently, in the 12-year period since the 2008 financial crisis and ending December 2020, an investor in large stocks would have realized an annual compound return of 15.0%, and an investor in small stocks would have realized an annual compound return of 13.1%.

It is interesting to place the decades of superior performance in historical context. The 1920s were preceded by mediocre returns and high inflation and were followed by the most devastating stock market crash and economic depression in U.S. history. This sequence of events mitigated the impact of the 1920s bull market on investor wealth. Nevertheless, the stock market became a liquid secondary market that decade, rendering it important for reasons other than return. In contrast, the 1950s were preceded and followed by decades with roughly average equity returns. The 1980s were preceded by a decade of “stagflation,” where modest stock price gains were seriously eroded by inflation and were followed by a period of stability in the 1990s.

The bond market performance of the 1980s and 1990s has no precedent. Bond yields, which had risen consistently since the 1940s, reached unprecedented levels in 1980–1981. (Other countries experiencing massive inflation have had correspondingly high interest rates.) Never before having had so far to fall, bond yields dropped further and faster than at any other time, producing what is indisputably the greatest bond bull market in history. Unfortunately, the boom came to an end in 1994. After falling to 21-year lows one year earlier, bond yields rose in 1994 to their highest level in over three years. Both long-term and intermediate-term government bond yields have generally fallen since 2000.[82]

The historical themes of the past decade, as they relate to the capital markets, can be summarized in three observations. First, the 17.5-year period starting in mid-1982 and ending in 1999 was a rare span of time in which investors quickly accumulated wealth.

Second, the postwar aberration of ever-higher inflation rates ended with a dramatic decrease in inflation in the early 1980s. In the 1990s, inflation was at a 2.9% compound annual rate, significantly lower than the 5.1% and 7.4% annual compound rate of the 1980s and 1970s, respectively, and lower than the longer-term compound annual rate at the end of that decade as measured over the 1926–1999 period (3.1%). The trend of relatively low inflation has continued through the 2000s and beyond. For example, the long-term compound annual rate of inflation over the 1926–2020 period (95 years) was 2.9%, but the compound annual rate over the 2000–2009 period, and the most recent 10-year period (2011–2020) were significantly lower at 2.5% and 1.7%, respectively.

Finally, participation in the returns of the capital markets since 1982 reached levels not approached in the 1920s, the 1950s, or even in the atypical boom period of 1967–1972. The growth since 1982 in the importance of pension funds and defined contribution pension plans, like the 401(k), as well as the rapidly increasing popularity of stock and bond mutual funds and exchange-traded products as basic savings vehicles, have enabled more individuals to experience the returns of the capital markets than ever before.

Exhibit 1.4 ranks the performance (as measured by compound annual rates of return) of the six basic U.S. asset classes plus inflation, as represented by the SBBI® series, for each decade from best performer (at top) to worst performer (at bottom). For example, in the 2010s the best performer was Large-Cap Stocks, and the worst performer was U.S. Treasury Bills.

82 The yield of SBBI® long-term (i.e., approximately 20 years maturity) U.S. government bonds at the end of 1999 was 6.82%; at the end of 2020 the yield of SBBI® long-term U.S. government bonds had fallen to 1.37%. The yield of SBBI® intermediate-term (i.e., approximately 5 years maturity) U.S. government bonds at the end of 1999 was 6.45%; at the end of 2019 the yield of SBBI® intermediate-term U.S. government bonds had fallen to 0.44%.

**Exhibit 1.4:** The Relative Performance of the Stocks, Bonds, Bills, and Inflations® (SBBI®) Series by Decade (Best Performer at Top, Worst Performer at bottom)

| 1920s* | 1930s | 1940s | 1950s |
|---|---|---|---|
| Large-Cap Stocks | Long-term Corp Bonds | Small-Cap Stocks | Large-Cap Stocks |
| Long-term Corp Bonds | Long-term Gov't Bonds | Large-Cap Stocks | Small-Cap Stocks |
| Long-term Gov't Bonds | Inter-term Gov't Bonds | Inflation | Inflation |
| Inter-term Gov't Bonds | Small-Cap Stocks | Long-term Gov't Bonds | U.S. Treasury Bills |
| U.S. Treasury Bills | U.S. Treasury Bills | Long-term Corp Bonds | Inter-term Gov't Bonds |
| Inflation | Large-Cap Stocks | Inter-term Gov't Bonds | Long-term Corp Bonds |
| Small-Cap Stocks | Inflation | U.S. Treasury Bills | Long-term Gov't Bonds |

| 1960s | 1970s | 1980s |
|---|---|---|
| Small-Cap Stocks | Small-Cap Stocks | Large-Cap Stocks |
| Large-Cap Stocks | Inflation | Small-Cap Stocks |
| U.S. Treasury Bills | Inter-term Gov't Bonds | Long-term Corp Bonds |
| Inter-term Gov't Bonds | U.S. Treasury Bills | Long-term Gov't Bonds |
| Inflation | Long-term Corp Bonds | Inter-term Gov't Bonds |
| Long-term Corp Bonds | Large-Cap Stocks | U.S. Treasury Bills |
| Long-term Gov't Bonds | Long-term Gov't Bonds | Inflation |
| **1990s** | **2000s** | **2010s** |
| Large-Cap Stocks | Long-term Gov't Bonds | Large-Cap Stocks |
| Small-Cap Stocks | Long-term Corp Bonds | Small-Cap Stocks |
| Long-term Gov't Bonds | Small-Cap Stocks | Long-term Corp Bonds |
| Long-term Corp Bonds | Inter-term Gov't Bonds | Long-term Gov't Bonds |
| Inter-term Gov't Bonds | U.S. Treasury Bills | Inter-term Gov't Bonds |
| U.S. Treasury Bills | Inflation | Inflation |
| Inflation | Large-Cap Stocks | U.S. Treasury Bills |

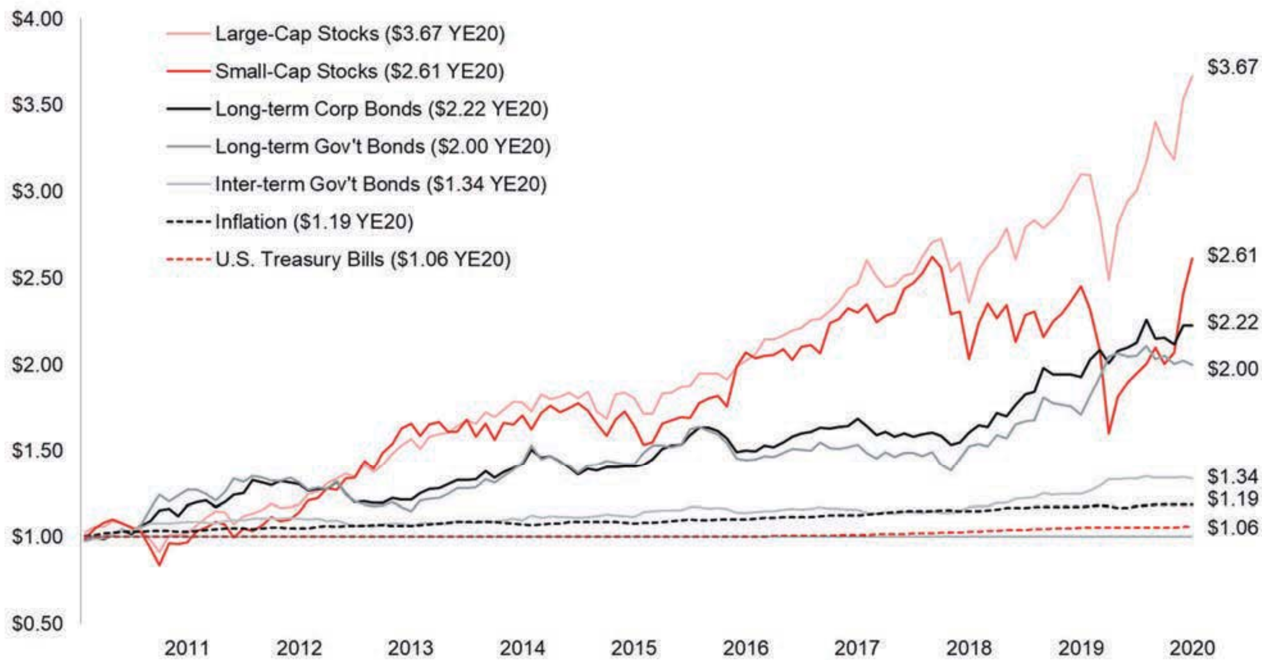
* Based on the period 1926–1929.

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. Performance measured by compound annual rates of return.

## Using Index Values to Measure Relative Performance

Relative performance can also be measured by using index values.[83] Exhibit 1.5 shows the market results for the most recent 10-year period (2011–2020) as illustrated by the growth of $1.00 invested on December 31, 2010 in each of the six basic U.S. asset classes plus inflation, as represented by the SBBI® series. A dollar invested at the end of 2010 in Large-Cap Stocks would have turned into $3.67 by the end of 2020, while a dollar invested at the end of 2010 in U.S. Treasury Bills would have turned into $1.06 by the end of 2020.

**Exhibit 1.5:** Wealth Indexes of $1.00 Investments in Each of the Six Basic U.S. Asset Classes Plus Inflation, as Represented by the Stocks, Bonds, Bills, and Inflation® SBBI® Series Over the Most Recent Decade (2011–2020) (Year-end 2010 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

83 To learn more about calculating index values, see Chapter 5, "Annual Returns and Indexes".

# Chapter 2
# The Long-Run Perspective

A long view of capital market history, illustrated by the 95-year period (1926–2020) examined here, uncovers the basic relationships between risk and return among the different asset classes and between nominal and real (inflation adjusted) returns. The goal of this study of asset returns is to provide a period long enough to include most or all of the major types of events that investors have experienced and may experience in the future. Such events include war and peace, growth and decline, bull and bear markets, inflation and deflation, and other less dramatic events that affect asset returns.

By studying the past, one can make inferences about the future. While the actual events that occurred during 1926–2020 will not be repeated, the event-types of that period can be expected to recur. It is sometimes said that only a few periods had unusual events, such as the stock market crash of 1929–1932 and World War II. This logic is suspicious because events that are deemed unusual happen with a certain regularity.[84] Some of the most unusual events of the century – the market crash of 1987, the equally remarkable inflation of the 1970s and early 1980s, the more recent events of September 11, 2001, the 2008–2009 financial crisis, and most recently, the market crash in the first quarter of 2020 that was precipitated by the spread of the COVID-19 virus – took place over the last three decades or so. To the degree that historical event-types tend to repeat themselves, the examination of past capital market returns is likely informative about what may be expected in the future.

## Using a Logarithmic Scale on Index Graphs

Previously in Exhibit 1.5 in Chapter 1, the market results over the most recent decade were illustrated by the growth of $1.00 invested on December 31, 2010 in each of the six basic U.S. asset classes plus inflation, as represented by the SBBI® series. A logarithmic scale was used on the vertical axis of Exhibit 1.5.[85] A logarithmic scale (see Exhibit 2.1a) allows for the direct comparison of the series' behavior at different points in time.

Specifically, the use of a logarithmic scale allows the following interpretation of the data: the same vertical distance, no matter where it is measured on the graph, represents the *same* percentage change in the series. For example, on a logarithmic scale, a 50% gain from $10 to $15 occupies

84 In 2010, Laurence B. Siegel, research director at the CFA Institute Research Foundationat the time, famously referred to these events as "black turkeys." The reference was to "black swans," the term author Nassim Nicholas Taleb gave to unfortunate events that aren't easily foreseeable. Siegel explained in a paper, "Black Swan or Black Turkey?" that market events like the global financial crisis are "everywhere in the data—(they) happen all the time" but investors are "willfully blind" to them. See: Laurence B. Siegel, "Black Swan or Black Turkey? The State of Economic Knowledge and the Crash of 2007–2009," *Financial Analysts Journal,* July/August 2010, Volume 66 Issue 4.

85 If creating a graph using Microsoft Excel, the vertical axis can be changed to logarithmic scale by right clicking the vertical axis and selecting "Format Axis…", and then selecting "Logarithmic scale" in the "Format Axis" dialog box.

the same vertical distance as a 50% gain from $1,000 to $1,500. On a linear scale, the same percentage gains look different (see Exhibit 2.1b). A logarithmic scale allows the viewer to compare investment performance across different periods; thus, the viewer can concentrate on rates of return without worrying about the number of dollars invested at any given time.

An additional (and practical) benefit of a logarithmic scale is the way the scale spreads the action out over time. It makes the graph easier to see and makes it easier to more carefully examine the fluctuations of the individual time series in different periods.

**Exhibit 2.1a:** Logarithmic Scale

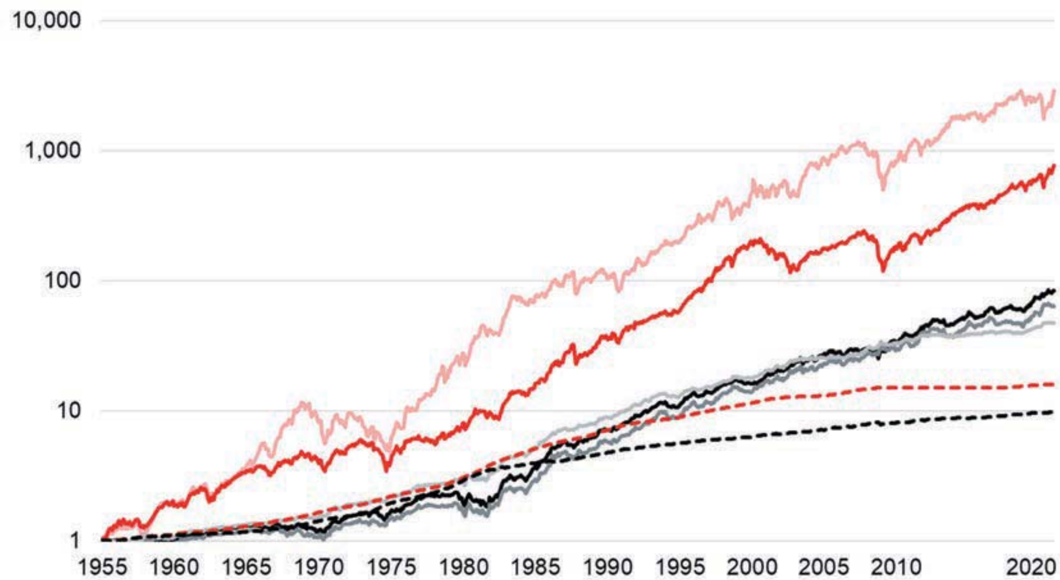
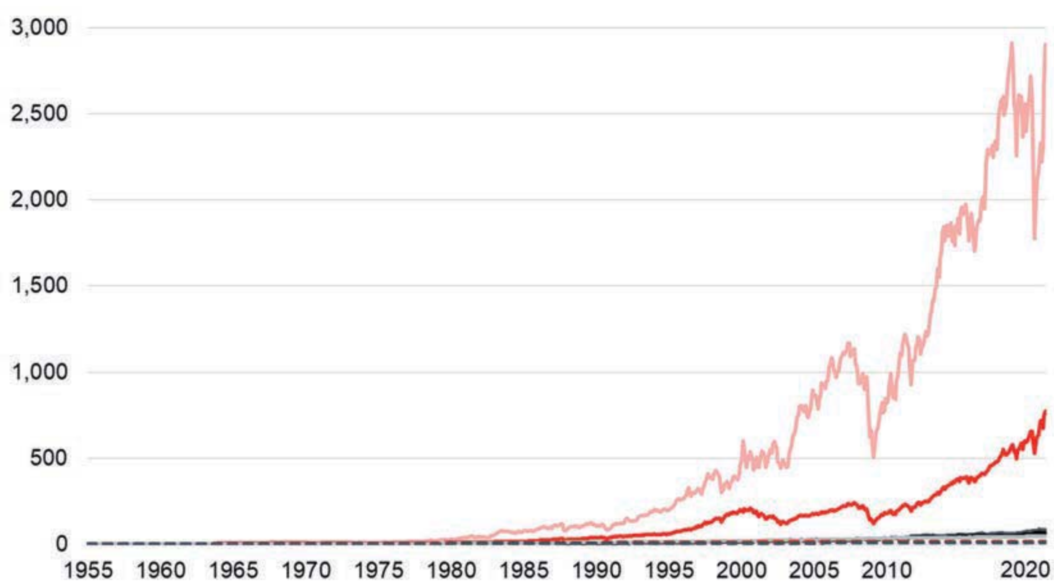



**Exhibit 2.1b:** Linear Scale



## Using Index Values to Measure the Relative Performance of the SBBI® Series

The relative performance of six U.S. asset classes plus inflation, as represented by the SBBI® series, was previously illustrated in Exhibit. 1.1 using average annual returns. In Exhibit 2.2 and Exhibit 2.3, the relative performance of the SBBI® series is illustrated using terminal index values. A "terminal index value" is defined here as the amount that an investment would have grown (or declined) to over a specific time period.[86]

Exhibit 2.2 illustrates the growth of $1 invested in the SBBI® large-cap stock series and $1 dollar invested in the SBBI small-cap stock series over the over the time horizon 1926 through 2020 (95 years total).[87]

A few observations about Exhibit 2.2:

- Small-cap stocks *outperformed* large-cap stocks over the 1926–2020 period: a hypothetical investment of $1 in small-cap stocks grew to nearly $42,000 by the end of 2020, and a hypothetical investment of $1 in large-cap stocks grew to nearly $11,000 by the end of 2020.[88]

- Both equity series in Exhibit 2.2 significantly *outperformed* the bond and bill series in Exhibit 2.3.

Over the long-term, smaller stocks tend to outperform larger stocks. This is known as the "size effect" and is discussed in greater detail in Chapter 7, "Company Size and Return".[89]

---

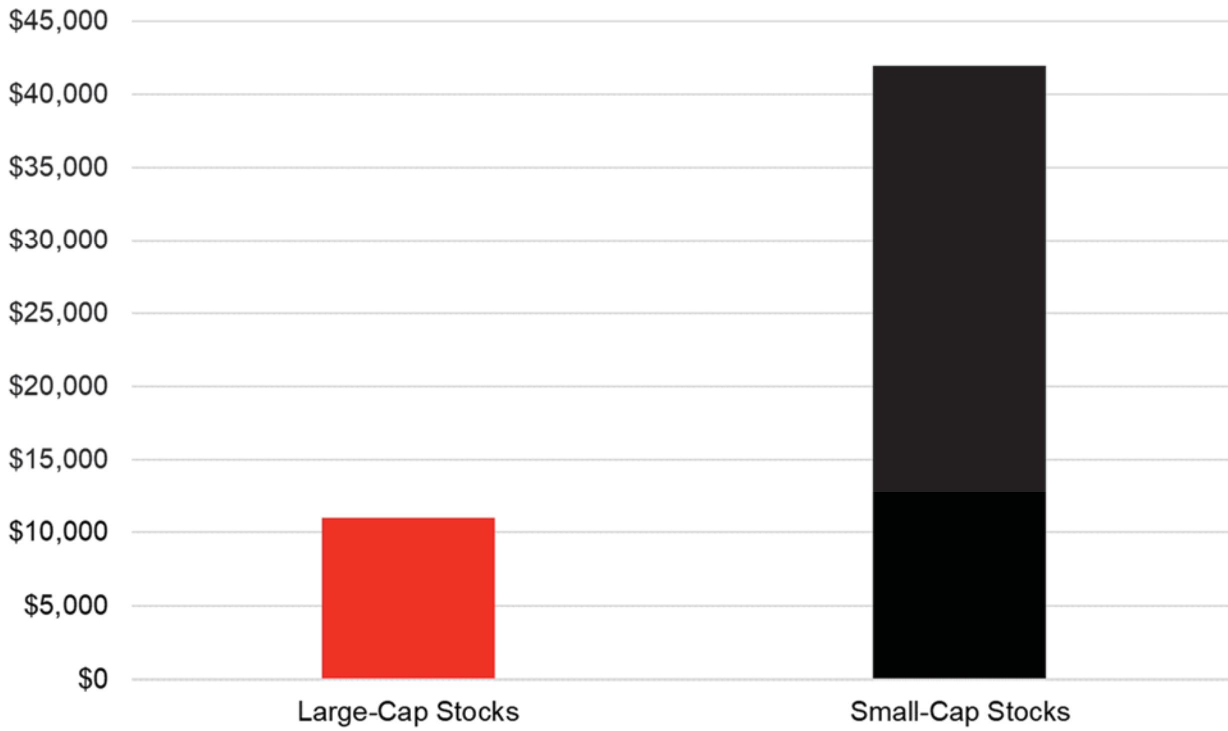
86 For detailed descriptions of how to use monthly SBBI® data to calculate index values over any time horizon, see Chapter 5, "Annual Returns and Indexes".

87 Each hypothetical $1 investment is made on December 31, 1925, at midnight.

88 Pre-calculated index values for the growth of $1 as of each month-end over the January 1926–December 2020 time horizon for each of the seven SBBI® series are presented in table format in the *2021 SBBI® Yearbook* at: dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

89 "Size Premia" are often used in the development of discount rates for use in discounted cash-flow (DCF) models to value a business, business ownership interest, security, or intangible asset. The CRSP Deciles Size Study and Risk Premium Report Study, both of which provide size premia and other risk premia based upon data licensed from the Center for Research in Security Prices (CRSP) at the University of Chicago Booth School of Business, are fully available in the D&P/Kroll online Cost of Capital Navigator platform at dpcostofcapital.com. CRSP® is a registered trademark and service mark of Center for Research in Security Prices, LLC and has been licensed for use by D&P/Kroll. The D&P/Kroll publications and services are not sponsored, sold or promoted by CRSP®, its affiliates or its parent company. To learn more about CRSP, visit www.crsp.com.

**Exhibit 2.2:** Using Index Values to Measure the Relative Performance of the SBBI® Large-Cap Stock and SBBI® Small-Cap Stock Series; 1926–2020 (Year-end 1925 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

Exhibit 2.3 illustrates the growth of $1 invested in each of the SBBI® long-term corporate bonds (approximately 20 years maturity), long-term government bonds (approximately 20 years maturity), intermediate-term government bonds (approximately 5 years maturity), 30-day U.S. Treasury bills, and inflation series over the over the time horizon 1926 through 2020 (95 years total).[90]

A few observations about the relationships in Exhibit 2.3:

- All of the bond and bill series in Exhibit 2.3 *underperformed* the equity series in Exhibit 2.2.

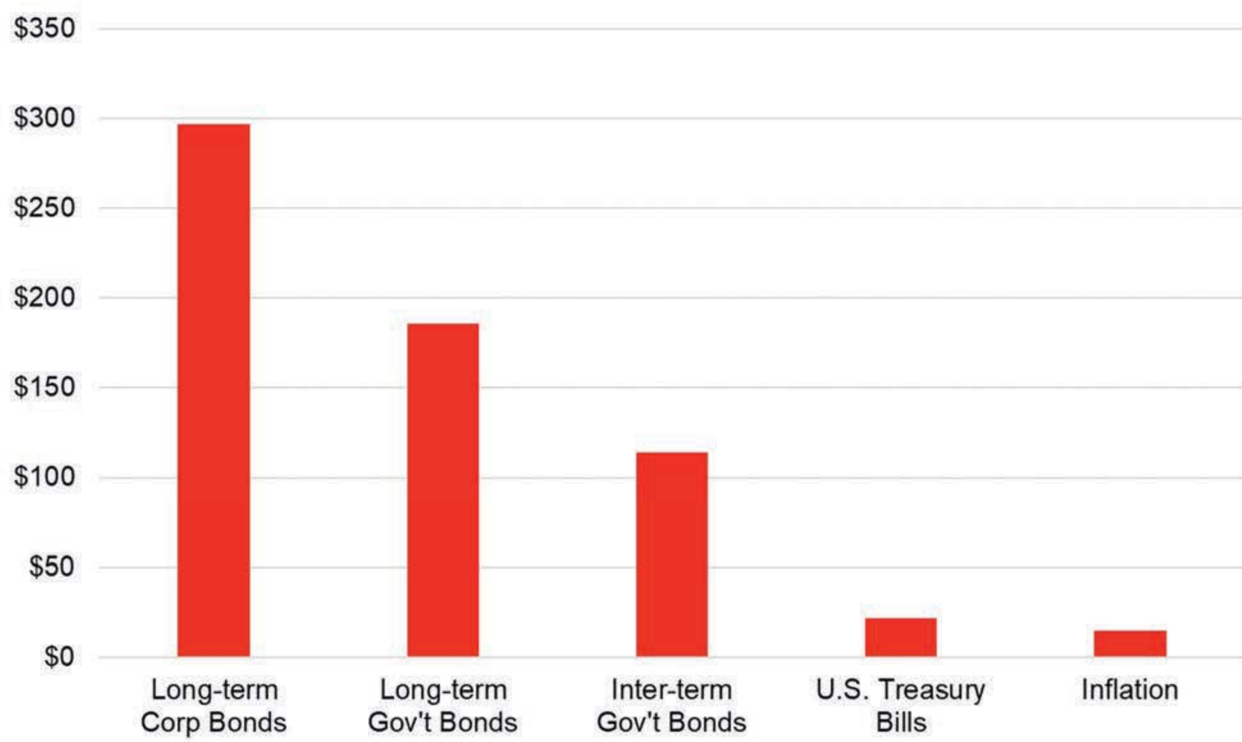
- Long-term corporate bonds *outperformed* long-term government bonds: a hypothetical investment of $1 in long-term corporate bonds grew to nearly $300 by the end of 2020, and a hypothetical investment of $1 in long-term government bonds grew to just a little

90 Each hypothetical $1 investment is made on December 31, 1925, at midnight.

over $185 by the end of 2020.[91] The bond default premium is defined as the net return from investing in long-term corporate bonds rather than long-term government bonds of equal maturity. Because there is a possibility of default on a corporate bond, bondholders receive a premium that reflects this possibility.[92]

- Long-term government bonds *outperformed* intermediate-term government bonds, which *outperformed* U.S. Treasury bills. The bond horizon premium is the extra return investors demand for holding long-term bonds instead of shorter-term fixed income securities.[93]

**Exhibit 2.3:** Using Index Values to Measure the Relative Performance of the SBBI® Long-Term Corporate Bond, Long-Term Government Bond, Intermediate Government Bond, U.S. Treasury Bills, and Inflation Series; 1926–2020 (Year-end 1925 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (ii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (iii) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (iv) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (v) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

---

91 Pre-calculated index values for the growth of $1 as of each month-end over the January 1926–December 2020 time horizon for each of the seven SBBI® series are presented in table format in the full-version *2021 SBBI® Yearbook* at: dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

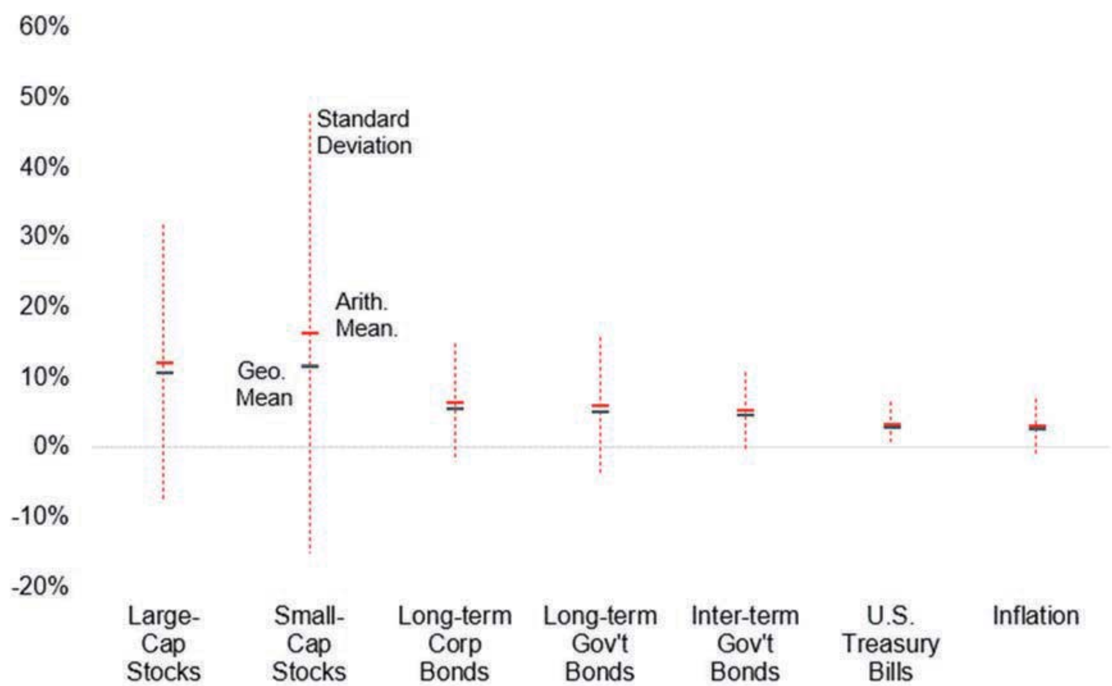
92 To learn more about the bond default premium, see Chapter 4, "Description of the Derived Series".

93 To learn more about the bond horizon premium, see Chapter 4, "Description of the Derived Series".

## Summary Statistics of Total Returns

Exhibit 2.4 is a visual presentation of selected summary statistics (geometric mean, arithmetic mean, and standard deviation) of the annual total returns on each asset class over the entire 95-year period of 1926–2020. The data presented in these exhibits are described in detail in Chapters 3 and 6.

**Exhibit 2.4:** Basic Series, Summary Statistics of Annual Total Returns (%) 1926–2020[94]




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

A few observations about Exhibit 2.4 are as follows:

- The returns of riskier assets, such as large- and small-cap stocks, tend to have larger standard deviations (standard deviation is shown as the vertical, red, dashed lines), reflecting the broad distribution of returns from very poor to very good. The returns of less

---

94 *Pre-calculated* annual summary statistics (geometric mean, arithmetic mean, standard deviation, etc.) are presented in table format for each of the SBBI® series over various time periods (including 1926–2020) in the full-version *2021 SBBI® Yearbook* at dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook. Annualized monthly returns can be used to compare the performance of each asset class for the same year.

risky assets, such as bonds, tend to have smaller standard deviations, indicating the tightness of the return distribution around the mean of the series.[95]

- The arithmetic mean returns (the short, red, horizontal lines) are all *higher* than the geometric mean returns (the short, dark gray, horizontal lines). The difference between these two means is related to the standard deviation, or variability, of the series. Specifically, the *higher* the standard deviation of the returns, the *larger* the difference will be between geometric and arithmetic averages. Alternatively, if there were no variability in the returns (i.e., the same return is experienced in each period being examined), then the geometric and arithmetic averages would be identical.

## Capital Appreciation, Income, Reinvestment, and Total Returns

Total annual returns are shown as the sum of three components: capital appreciation returns, income returns, and reinvestment returns. The capital appreciation and income components are explained in Chapter 3. The third component, reinvestment return, reflects monthly income reinvested in the total return index in subsequent months in the year. Thus, for a single month the reinvestment return is zero, but over a longer period of time it is nonzero.

The annual total return formed by compounding the monthly total returns does not equal the sum of the annual capital appreciation and income components; the difference is reinvestment return. A simple example illustrates this point (see Exhibit 2.5). In 1995, an "up" year on a total return basis, the total annual return on large-cap stocks was 37.58%. The annual capital appreciation was 34.11% and the annual income return was 3.04%, totaling 37.15%. The remaining 0.43% (37.58% minus 37.15%) of the 1995 total return came from the reinvestment of dividends in the market.[96] For more information on calculating annual total and income returns, see Chapter 5, "Annual Returns and Indexes."[97]

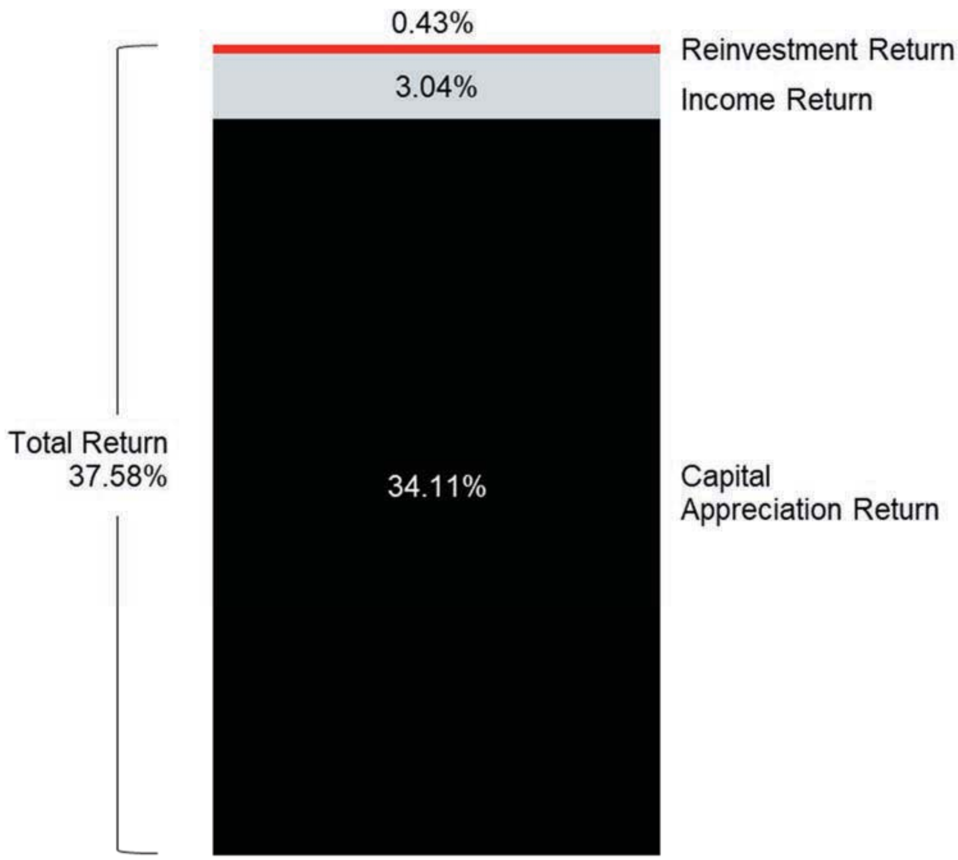
---

[95] The distribution of the returns of Treasury bills is one-sided, lying almost entirely above zero; that is, Treasury bills almost never experienced negative returns on a yearly basis over the 1926–2020 period (the only negative year was 1938). Although a few deflationary months and quarters have occurred recently, the last negative annual inflation rate (on a calendar year basis) occurred in 1954.

[96] *Pre-calculated* annualized capital appreciation returns, income returns, reinvestment returns, and total returns for the SBBI® Large-Cap Stock series, Long-term Government Bond series, and Intermediate-term Government Bond series for each year over the 1926–2020 time horizon are presented in table format in the full-version *2021 SBBI® Yearbook*. For more information, visit: dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

[97] *Precalculated* annualized monthly returns for each of the six SBBI series plus inflation, for each year over the 1926–2020 time horizon, are presented in table format in the full-version *2021 SBBI® Yearbook* in that book's appendices. For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook .

**Exhibit 2.5:** Illustration of the Decomposition of the 1995 Total Return of Large-Cap Stocks into Capital Appreciation Return, Income Return, and Reinvestment Return




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Large-Cap Stocks represented by the following series: IA SBBI® US Large Stock TR USD Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## Rolling Period Returns

Rolling period returns can be obtained by rolling a data window of fixed length along each time series. They are useful for examining the behavior of returns for holding periods similar to those actually experienced by investors and show the effects of time diversification. Holding assets for long periods of time has the effect of lowering the risk of experiencing a loss in asset value.

If you wanted to calculate 12-month (1 year) rolling period geometric (i.e. compound) returns, you could use the framework illustrated in Exhibit 2.6.[98,99]

98 Geometric (i.e. compound) returns are used here for example purposes only; rolling period analysis can be performed on any calculable statistic desired.

99 To learn more about calculating period returns (including annualized monthly returns), see Chapter 6, "Statistical Analysis of Returns".

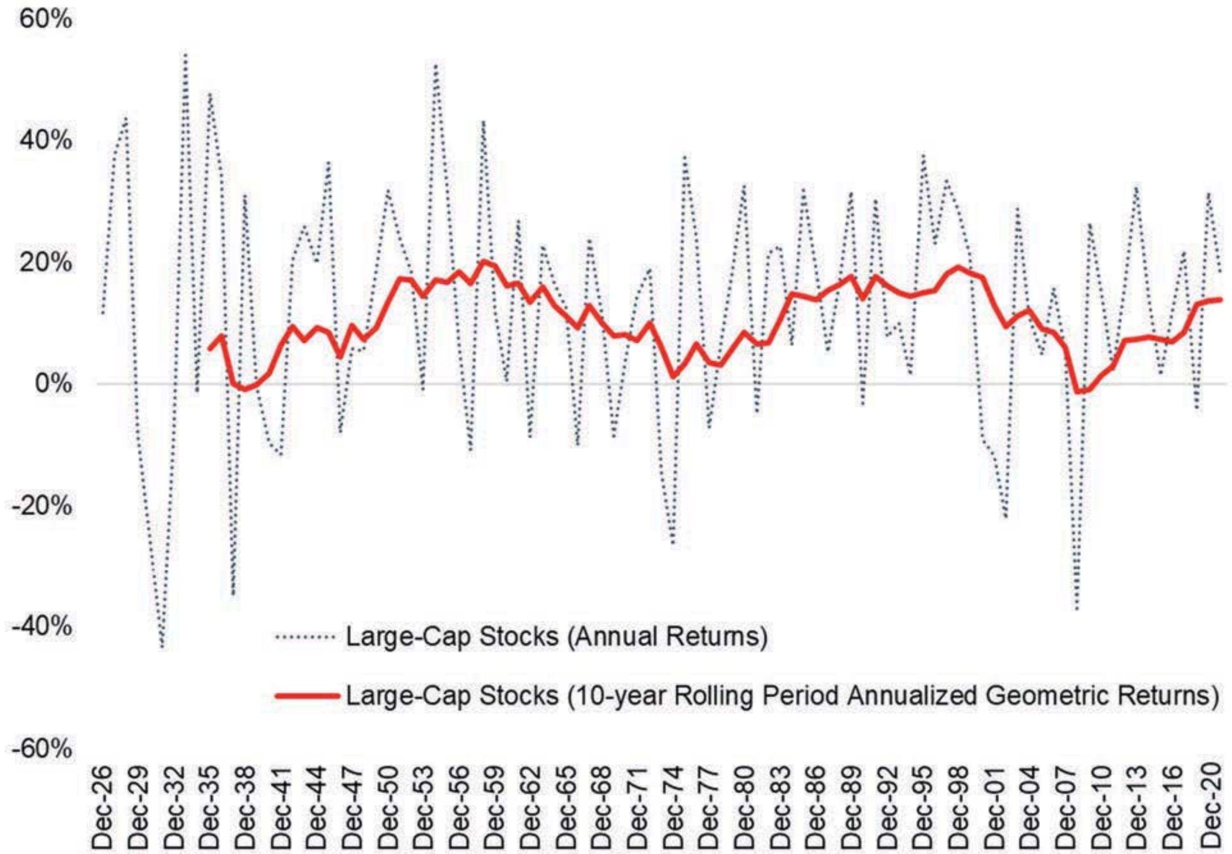
**Exhibit 2.6:** Framework for Calculating Rolling Period Statistics (geometric return over 12-month periods used for example purposes).

| Time Period | Calculations |
|---|---|
| Month 1 | – |
| Month 2 | – |
| Month 3 | – |
| Month 4 | – |
| Month 5 | – |
| Month 6 | – |
| Month 7 | – |
| Month 8 | – |
| Month 9 | – |
| Month 10 | – |
| Month 11 | – |
| Month 12 | Calculate geometric (i.e., compound) return over months 1–12 |
| Month 13 | Calculate geometric (i.e., compound) return over months 2–13 |
| Month 14 | Calculate geometric (i.e., compound) return over months 3–14 |
| | Etc… |

The same framework illustrated in Exhibit 2.6 could be used for any rolling period desired (e.g., 60 months, 120 months, etc.).

Exhibit 2.7 provides a visual representation of annual Large-Cap Stock returns compared to rolling 10-year annual geometric returns.

**Exhibit 2.7:** Large-Cap Stocks: Annual and 10-year Rolling Period Annual Geometric Returns (1926–2020)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Large-Cap Stocks represented by the following series: IA SBBI® US Large Stock TR USD Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

A few observations about Exhibit 2.7 are as follows:

- The 10-year rolling period geometric returns are *less* volatile than the annual returns.

- The 10-year rolling period geometric returns rarely drop below 0%. This implies that holding assets for longer periods of time has the effect of lowering the risk of experiencing a loss in asset value.[100]

---

[100] *Precalculated* annualized monthly returns (essentially a 1-year holding period) and 5-, 10-, and 20-year rolling geometric (i.e., compound) returns for each of the six SBBI® series plus inflation, and for each year and applicable holding period (from 1926–2020), are presented in table format in the full-version *2021 SBBI® Yearbook*. Precalculated statistics for each year and applicable holding period presented in these tables include: (i) maximum return for each holding period, (ii) the year (or period *ending* in year) of the maximum return for each holding period, (iii) the minimum return for each holding period, (iv) the year, or period *ending* in year of the minimum return for each holding period, (v) the number of times the holding period return was positive over the 1926–2020 period, and (vi) the number of times each of the six SBBI® series plus inflation had the *highest* return for each

## Portfolio Performance Returns

A portfolio is a group of assets, such as stocks and bonds, held by an investor. Because stocks, bonds, and cash generally do not react identically to the same economic or market stimulus, combining these assets can often produce a better risk-adjusted return. There were plenty of years in which stock returns were up at times when bond returns were down and vice versa. These offsetting movements can assist in reducing portfolio volatility. Some recent examples include the years 2000 through 2002: Large-cap stocks posted returns of -9.10%, -11.89%, and 22.10%, respectively, while long-term government bonds posted positive returns of 21.48%, 3.70%, and 17.84%, respectively. This illustrates the low correlation of stocks and bonds; that is, they tend to (but not always) move independently of each other.[101,102]

While bond prices tend to fluctuate less than stock prices, they are still subject to price movement. Investing in a mix of asset classes, such as stocks, bonds, and Treasury bills (cash), may protect a portfolio from major downswings in a single asset class. One of the main advantages of diversification is that it makes investors less dependent on the performance of any single asset class.

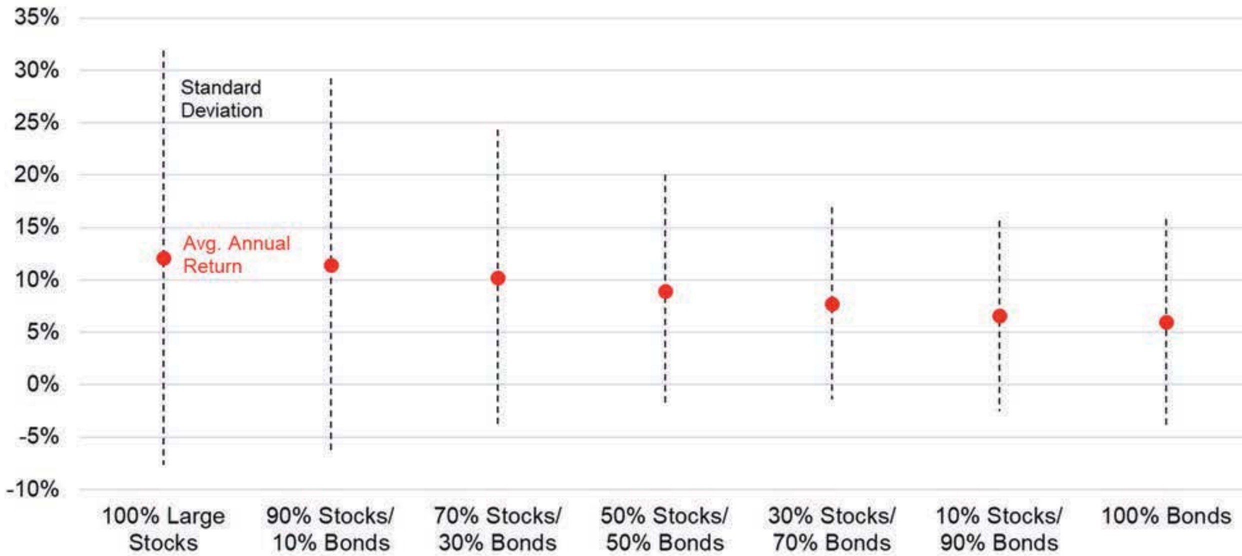
There is generally a tradeoff. As bonds are added to a stock portfolio, the risk as measured by standard deviation tends to decline, but so does return. In Exhibit 2.8 this concept is illustrated for various portfolio mixes over the 1926–2020 time horizon.

holding period over the 1926–2020 period. For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

101 See Chapter 6, "Statistical Analysis of Returns", for a more detailed discussion of correlation.

102 See Chapter 10, "Using Historical Data in Wealth Forecasting and Portfolio Optimization" for a discussion about forecasting portfolio returns.

**Exhibit 2.8:** Average Annual Return and Standard Deviation of Large-Cap Stock and Long-Term Government Bond Portfolios (1926–2020)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD. Portfolios were rebalanced monthly over the January 1926–December 2020 time horizon. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

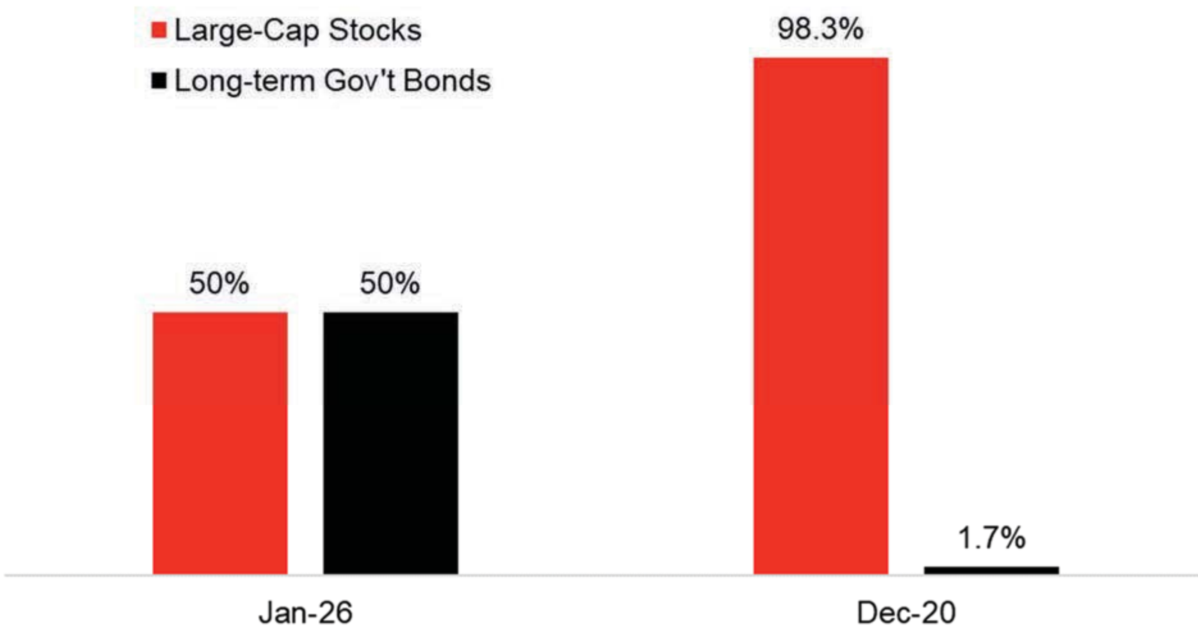
## Portfolio Rebalancing

Without periodic rebalancing, a portfolio's asset mix can change from its original percentages as a result of differing returns among the various asset classes held in the portfolio. Thus, asset allocation percentages can change over time without the investor's input.

For example, if a hypothetical investor invests $1.00 in stocks and $1.00 in bonds, the portfolio that he holds is initially 50% stocks and 50% bonds. If the returns of stocks and bonds over the next year are 20% and 5%, respectively, the "stock" portion of his portfolio has now increased to $1.20, and the "bond" portion of his portfolio has now increased to $1.05. The investor's portfolio mix has also changed: it is now approximately 53.3% stocks ($1.20/($1.20 + $1.05)) and 46.7% bonds ($1.05/($1.20 + $1.05)).

This concept is illustrated in Exhibit 2.9. The portfolio mix of a portfolio originally comprised of 50% stocks and 50% bonds at the end of 1925 would (if never rebalanced) become a portfolio comprised of 98.3% stocks and 1.7% bonds by the end of 2020.

**Exhibit 2.9:** Ending Asset Mix of a Never Rebalanced 50-50 Stock/Bond Portfolio
January 1926–December 2020[103]




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

103 *Precalculated* annualized monthly returns (essentially a 1-year holding period) and 5-, 10-, and 20-year rolling geometric (i.e., compound) returns for each year and applicable holding period (from 1926–2020) are presented in table format in the full-version *2021 SBBI® Yearbook* for the following portfolios of SBBI® large-cap stocks and SBBI® long-term (i.e., 20-year) government bonds: (i) 100% Large Stocks, (ii) 90% Stocks/10% Bonds, (iii) 70% Stocks/30% Bonds, (iv) 50% Stocks/50% Bonds, (v) 30% Stocks/70% Bonds, (vi) 10% Stocks/90% Bonds, and (vii) 100% LT Gov't Bonds. The *precalculated* statistics for each year and applicable holding period presented in these tables include: (i) maximum return for each holding period, (ii) the year (or period *ending* in year) of the maximum return for each holding period, (iii) the minimum return for each holding period, (iv) the year, or period *ending* in year of the minimum return for each holding period, (v) the number of times the holding period return was positive over the 1926–2020 period, and (vi) the number of times each of the portfolio mixes had the *highest* return for each holding period over the 1926–2020 period. For more information about the full-version *2021 SBBI® Yearbook*, visit: dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

## Real Estate Investment Trusts (REITs)[104]

Real estate properties can be *directly* owned by individuals (i.e., sole proprietorship), partnerships, corporations (either subchapter C or subchapter S), limited liability company (LLCs) or trusts. An equity investment in real estate can also be made indirectly by purchasing shares of an entity holding real property interests. Real estate entities exist substantially for the purpose of holding, directly or indirectly, title to or beneficial interest in real property. The value of a real estate entity includes many components, such as land, buildings, furniture, fixtures and equipment, intangible assets, and often the business operation.

A real estate investment trust, or REIT, "is a company dedicated to owning, and in most cases, operating income-producing real estate, such as apartments, shopping centers, offices and warehouses. Some REITs also engage in financing real estate."[105,106] Most REITs trade on major stock exchanges. To qualify as a REIT a company must:[107]

- Invest at least 75% of its total assets in real estate
- Derive at least 75% of its gross income from rents from real property, interest on mortgages financing real property or from sales of real estate
- Pay at least 90% of its taxable income in the form of shareholder dividends each year
- Be an entity that is taxable as a corporation
- Be managed by a board of directors or trustees
- Have a minimum of 100 shareholders
- Have no more than 50% of its shares held by five or fewer individuals

REITs can be classified in two broad categories, (i) equity REITs, and (ii) mortgage REITs:[108]

---

104 Data presented throughout this section comes from the National Association of Real Estate Investment Trusts® (Nareit®). Nareit® is the worldwide representative voice for real estate investment trusts – REITs – and publicly traded real estate companies with an interest in U.S. real estate and capital markets. Nareit® advocates for REIT-based real estate investment with policymakers and the global investment community. To learn more, visit: https://www.reit.com/.

105 Glossary of REIT Terms, National Association of Real Estate Investment Trusts® (Nareit®), available at: https://www.reit.com/what-reit/glossary-reit-terms.

106 For more information, visit the U.S. Securities and Exchange Commission (SEC) website at: https://www.investor.gov/introduction-investing/basics/investment-products/real-estate-investment-trusts-reits.

107 Source: National Association of Real Estate Investment Trusts® (Nareit®) website at https://www.reit.com/what-reit.

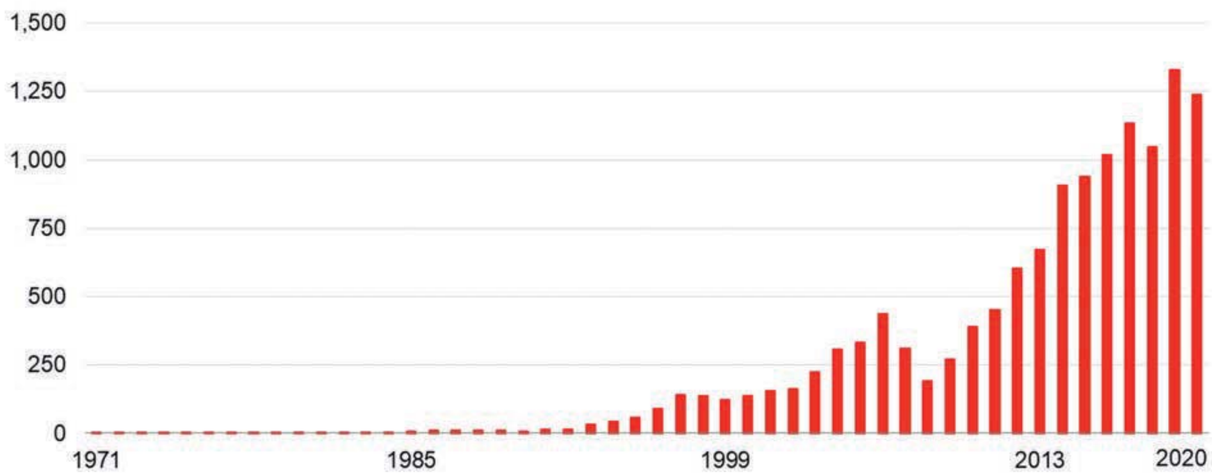
108 Smaller categories of REITs include: (i) Public, non-listed REITs (PNLRs). PNLRs are registered with the SEC, but do not trade on national stock exchanges. (ii) Private REITs are offerings that are exempt from SEC registration and whose shares do not trade on national stock exchanges. Source: National Association of Real Estate Investment Trusts® (Nareit®) at: https://www.reit.com/what-reit.

- **Equity REIT**: A REIT which owns, or has an "equity interest" in, rental real estate (rather than making loans secured by real estate collateral). The majority of REITs are publicly traded equity REITs.[109]

- **Mortgage REIT**: A REIT that makes or owns loans and other obligations that are secured by real estate collateral. Mortgage REITs are commonly referred to as mREITs.[110]

The number of REITs in the U.S. grew dramatically in the last several decades from 34 in 1971 to over 200 at the end of 2020.[111] This growth enabled a broader group of investors to add real estate to their portfolios and enjoy greater liquidity than they would otherwise be able. Exhibit 2.10 displays the growth in market cap of U.S. REITs between 1971 and 2020.

**Exhibit 2.10:** REITs Market Cap ($ Billions) 1971–2020




**Source of underlying data**: (i) U.S. REIT Industry Equity Market Cap is available at the National Association of Real Estate Investment Trusts (Nareit®) website at: https://www.reit.com/data-research/reit-market-data/us-reit-industry-equity-market-cap

## Historical Returns on Equity REITs

Exhibit 2.11 depicts the growth of $1.00 invested in equity REITs, U.S. small-cap and large-cap stocks, long-term government bonds, Treasury bills, and a hypothetical asset returning the inflation rate over the period from the end of 1971 to the end of 2020. Of the asset classes shown, small-cap stocks accumulated the highest ending wealth. An investment of $1.00 in small-cap stocks at year-end 1971 would have grown to $345.72 by year-end 2020, a compound return of 12.7%. Notice, however, that the same investment in equity REITs would have returned $201.01,

---

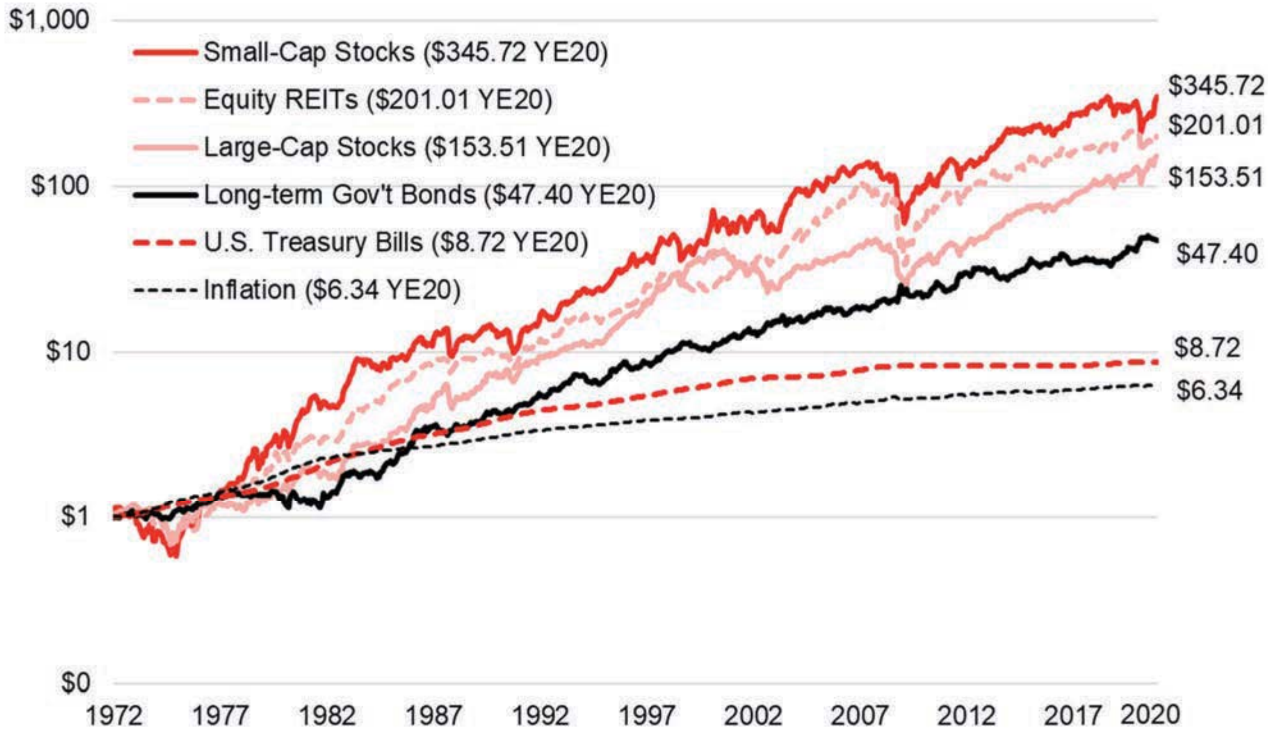
109 Glossary of REIT Terms, National Association of Real Estate Investment Trusts® (Nareit®), available at: https://www.reit.com/what-reit/glossary-reit-terms.

110 For more information, visit the U.S. Securities and Exchange Commission (SEC) website at: https://www.investor.gov/introduction-investing/basics/investment-products/real-estate-investment-trusts-reits.

111 Source: National Association of Real Estate Investment Trusts® (Nareit®) website at https://www.reit.com/what-reit.

a compound return of 11.4%. Equity REITs outperformed all the remaining asset classes and inflation during the period.

**Exhibit 2.11:** Wealth Indices of Investments in Equity REITs and Basic Series Index (Year-end 1971 = $1.00) 1972–2020



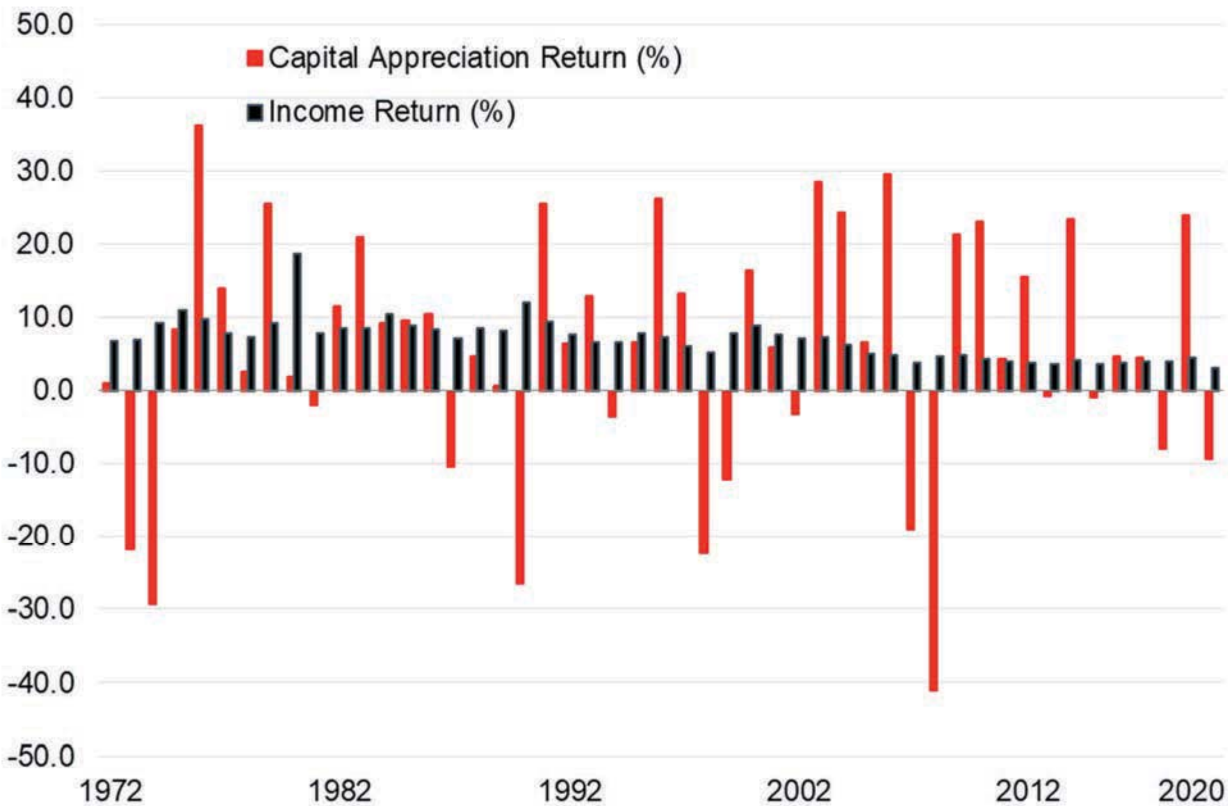

**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (iv) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (v) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. Performance measured by compound annual rates of return. **Source 2 of underlying data:** National Association of Real Estate Investment Trusts® (Nareit®) at https://www.reit.com/. FTSE Nareit U.S. Real Estate Index Series used: "All Equity REITs" total return series.

## Income Returns on Equity REITs

REITs must pay to shareholders at least 90% of their taxable income each year. As a result, income generated from REITs has proven to be steady and reasonably predictable.

Exhibit 2.12 shows both the income return and capital appreciation return of REITs annually from 1972 to 2020. REITs, similar to equity, can be quite volatile but offer the potential for price appreciation. However, price appreciation is by no means guaranteed (note the large negative price returns of 2007 and 2008). On the other hand, the income produced by REITs has been relatively stable since 1972. Equity REITs posted an average annual income return during that period of 7.0%. The highest annual income return was 18.8% in 1980 while the lowest was 3.7% in 2015.[112]

**Exhibit 2.12:** Annual Returns on Equity REITs (%) 1972–2020




**Source of underlying data:** National Association of Real Estate Investment Trusts® (Nareit®) at https://www.reit.com/. FTSE Nareit U.S. Real Estate Index Series used: (i) "All Equity REITs" price return series, (ii) "All Equity REITs" income return series.

112 The annualized monthly income returns in Exhibit 2.12 are calculated in accordance with the methodology outlined in Chapter 5, Annual Returns and Indexes."

## Correlation of U.S. REITs Compared to Other U.S. Asset Classes

Diversification is “spreading a portfolio over many investments to avoid excessive exposure to any one source of risk.”[113] Put simply, diversification is “not putting all your eggs in one basket.” Diversification offers the potential of higher returns for the same level of risk or lower risk for the same level of return.

REITs have been an attractive investment vehicle to investors because they have traditionally had a relatively low and declining correlation to stocks and bonds. Though the reasons are not quite clear, this relationship changed in the early 2000s when REITs became increasingly correlated with both stocks and bonds, though correlation levels remain fairly low. A low correlation between assets in a portfolio allows for the possibility of an increase in returns without a corresponding increase in risk, or alternatively, a reduction in risk without a corresponding decrease in return. For example, from 1972 to 2020, a portfolio (rebalanced annually) with a mix of 75% stocks and 25% bonds returned 10.5% annually with a standard deviation of 13.2%. Adding a 20% allocation to REITs to the portfolio increases returns to 10.9% annually and at the same time decreases standard deviation to 13.1%.

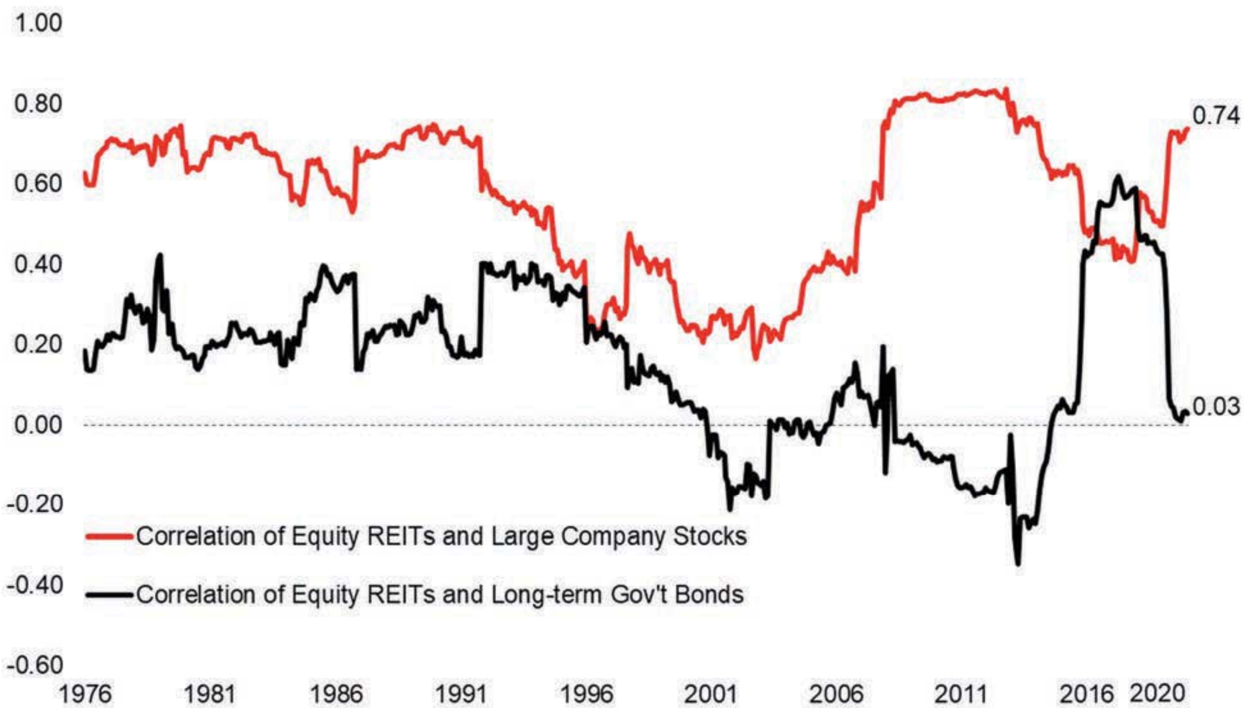
In Exhibit 2.13, the correlation of U.S. REITs and (i) U.S. large company stocks and (ii) long-term U.S. government bonds is shown. Correlation is a measure of how alternative investments “move” relative to each other and is thus a measure of potential diversification benefit. The *higher* the correlation (the more investments “move” together), the *less* potential diversification benefit, whereas the *lower* the correlation (the less investments “move” together), the *greater* the potential diversification benefit. The thinking is that by holding a portfolio of assets that do not have high correlation with each other, as some investments decrease in value, others will increase (and vice versa) and thus potentially mitigate overall portfolio losses.

The correlation of U.S. REITs with both stocks and bonds declined during the 1990s, thus increasing the potential diversification benefit.

In the immediate years leading up to and following the 2008–2009 Financial Crisis, the correlation of U.S. REITs with stocks generally *increased*, decreasing the potential diversification benefit between these two asset classes. This increase in the correlation was likely due, at least in part, to the Federal Reserve’s tamping down of interest rates through various market interventions and moral suasion. The publicly-stated intention of these interventions by the Federal Reserve was to boost asset prices (e.g., stocks, housing).

113 Cara Griffith, “Practical Tax Considerations for Working with REITs”, State Tax Notes (October 31, 2011): 315–320, quoting Jennifer Weiss: 316. In 2009, the IRS issued guidance that indicates that the distributions may be in the form of cash or stock in certain instances.

**Exhibit 2.13:** Rolling 60-month Correlation of U.S. Equity REITs and (i) U.S. Large Company Stocks, and (ii) U.S. Long-term Government Bonds 1972–2020




**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** National Association of Real Estate Investment Trusts® (Nareit®) at https://www.reit.com/. FTSE Nareit U.S. Real Estate Index Series used: "All Equity REITs" total return series.

From 2014 through 2019 these trends seemed to reverse, with the correlation of U.S. REITs with stocks generally decreasing and the correlation of U.S. REITs with long-term U.S. government bonds generally increasing. These correlations' regression to values more typical of pre-financial-crisis levels is likely due, at least in part, to the Federal Reserve's stated public desire to normalize U.S. interest rates.[114,115]

---

114 During the 2008 Financial Crisis and subsequent recession, total assets increased significantly from $870 billion in August 2007 to $4.5 trillion in early 2015. Then, reflecting the Federal Open Market Committee's (FOMC) balance sheet normalization program that took place between October 2017 and August 2019, total assets declined to under $3.8 trillion. Beginning in September 2019, total assets again began to increase. The Federal Reserve's balance sheet increased from approximately $4.2 trillion on February 10, 2020 to approximately $7.4 trillion by the end of 2020 in response to the outbreak of COVID-19 and the surrounding economic upheaval that accompanied it. To learn more about the Federal Reserve's "Monetary Policy Implementation and Balance Sheet Normalization", visit: https://www.federalreserve.gov/monetarypolicy/bst_recenttrends.htm.

115 The Federal Reserve maintained a target federal funds range of 0.00%–0.25% through open market operations from December 16, 2008 through December 16, 2015. On December 17, 2015, the Federal Reserve raised the target federal funds range 25 bps to 0.25% –0.50%, and again raised the target range an additional 25 bps in each of December 2016, and March 2017, June 2017, December of 2017, March 2018, June 2018, September 2018, and December 2018 to a level of 2.25%–2.50% as of December 31, 2018. In the second half of 2019 the Federal Reserve reversed course and lowered the target federal funds range 25 bps in each of August 2019, September 2019, and October 2019, ending at a level of 1.5%–1.75%. On March 3, 2020 and

In 2020 the rolling 60-month correlation of REITs with stocks increased dramatically while the correlation of REITs with long-term U.S. government bonds decreased dramatically. This was primarily driven by a strong de-coupling in the first quarter of 2020 of the returns of REITs and stocks with havens of safety like U.S. treasuries likely due to the spread of COVID-19 and risks associated with that. REITs moved even more strongly contra to bonds than stocks did in this period[116] and then failed to recover to the degree that stocks did after the first quarter. The respective total return of REITs, stocks (as measured by the SBBI Large Company Stocks series), and long-term U.S. government bonds (as measure by the SBBI Long-term Government Bond series) in 2020 was –5.9%, 18.4%, and 16.7%, respectively.

## Summary Statistics for Equity REITs and Basic Series

Exhibit 2.14 shows summary statistics of annual total returns for REITs and the SBBI® basic series from 1972 to 2020. The summary statistics presented are geometric mean, arithmetic mean, and standard deviation. While small-cap stocks posted the highest geometric mean over the period analyzed, they also had the highest amount of risk. Equity REITs produced a higher return than large -cap stocks with only slightly higher risk.

**Exhibit 2.14:** Summary Statistics of Annual Returns (%) 1972–2020

| | Geometric Average | Arithmetic Average | Standard Deviation |
|---|---|---|---|
| Equity REITs | 11.4 | 12.9 | 17.7 |
| Large-Cap Stocks | 10.8 | 12.3 | 17.2 |
| Small-Cap Stocks | 12.7 | 14.9 | 22.2 |
| Long-term Corp Bonds | 8.5 | 9.0 | 10.1 |
| Long-term Gov't Bonds | 8.2 | 8.8 | 12.0 |
| Inter-term Gov't Bonds | 6.8 | 7.0 | 6.4 |
| U.S. Treasury Bills | 4.5 | 4.6 | 3.5 |
| Inflation | 3.8 | 3.9 | 3.0 |

**Source1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** National Association of Real Estate Investment Trusts® (Nareit®) at https://www.reit.com/. FTSE Nareit U.S. Real Estate Index Series used: "All Equity REITs" total return series.

on March 16, 2020 the Federal Reserve lowered the target federal funds range (again, in response to the outbreak of COVID-19) an additional 50 bps and 100 bps, respectively, bringing the range to 0.00%–0.25%, where it remained through the end of 2020. This marks the lowest the target federal funds range has been since 2015. For a list of Federal Reserve open market operations, visit: https://www.federalreserve.gov/ monetarypolicy/openmarket.htm. For a detailed discussion of monetary policy and interest rates, see the Cost of Capital Navigator's Resources (subscription required) at dpcostofcapital.com.

[116] By the end of March 2020 REITs and stocks experience large losses (down 25.4% and 19.6% through March 2020 YTD, respectively) while long-term U.S. government bonds showed large gains (up 19.6% through March 2020 YTD).

Exhibit 2.15 presents annual serial correlations and cross-correlations from 1972 to 2020 for equity REITs and the six basic SBBI® asset classes plus inflation. The serial correlation, or the extent to which the return in one period is related to the return in the next period (discussed in greater detail in Chapter 6) of equity REITs suggests no strong pattern; it can best be interpreted as mostly random or unpredictable.

**Exhibit 2.15:** Serial and Cross-Correlations of Annual Returns 1972–2020

| | Equity REITs | Large-Cap Stocks | Small-Cap Stocks | Long-term Corp Bonds | Long-term Gov't Bonds | Inter-term Gov't Bonds | U.S. Treasury Bills | Inflation |
|---|---|---|---|---|---|---|---|---|
| Equity REITs | 1.00 | | | | | | | |
| Large-Cap Stocks | 0.54 | 1.00 | | | | | | |
| Small-Cap Stocks | 0.74 | 0.72 | 1.00 | | | | | |
| Long-term Corp Bonds | 0.28 | 0.29 | 0.12 | 1.00 | | | | |
| Long-term Gov't Bonds | 0.05 | 0.04 | -0.12 | 0.89 | 1.00 | | | |
| Inter-term Gov't Bonds | 0.04 | 0.05 | -0.04 | 0.82 | 0.86 | 1.00 | | |
| U.S. Treasury Bills | 0.05 | 0.03 | 0.06 | 0.03 | 0.08 | 0.42 | 1.00 | |
| Inflation | 0.00 | -0.11 | 0.08 | -0.32 | -0.26 | -0.03 | 0.70 | 1.00 |
| Serial Correlation | 0.07 | -0.02 | 0.00 | -0.12 | -0.28 | 0.09 | 0.89 | 0.75 |

**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** National Association of Real Estate Investment Trusts® (Nareit®) at https://www.reit.com/. FTSE Nareit U.S. Real Estate Index Series used: "All Equity REITs" total return series.

In conclusion, equity REITs have historically offered an attractive risk/return trade-off for investors. They have provided a current income stream along with the potential for long-term capital appreciation. The recent increase in correlation of REIT returns with other investments may lead to a decrease in the overall diversification benefit to investors, but they remain an attractive option.

# Chapter 3
# Description of the Basic Series

## Large-Cap Stocks

Large-cap stocks are represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series "IA SBBI® US Large Stock TR USD Ext."[117] This series is essentially the S&P 500 Index. Large-cap stock total returns ranged from a high of 54.0% in 1933 to a low of -43.3% in 1931.

### Total Returns

From February 1970 to the present, the large-cap stock total return is provided by S&P Dow Jones Indices which calculates the total return based on the daily reinvestment of dividends on the ex-dividend date. S&P uses closing pricing from stock exchanges in its calculation. Prior to February 1970, the total return for a given month was calculated by summing the capital appreciation return and the income return as described below.

The large-cap stock total return index is based upon the S&P Composite Index. This index is a readily available, carefully constructed, market-capitalization-weighted benchmark of large-cap stock performance. Market-capitalization-weighted means that the weight of each stock in the index, for a given month, is proportionate to its market capitalization (price times the number of shares outstanding) at the beginning of that month. Currently, the S&P Composite includes 500 of the largest stocks (in terms of stock market value) in the U.S.; prior to March 1957 it consisted of 90 of the largest stocks.

### Capital Appreciation Return

The capital appreciation component of the large-cap stock total return is the change in the S&P 500 index as reported by S&P Dow Jones Indices from March 1928 to December 2020 and in Standard & Poor's *Trade and Securities Statistics* from January 1926 to February 1928.

### Income Return

From February 1970 to December 2020, the income return was calculated as the difference between the total return and the capital appreciation return. From January 1926 to January 1970, quarterly dividends were extracted from rolling yearly dividends reported quarterly in S&P's *Trade and Securities Statistics*, then allocated to months within each quarter using proportions taken from the 1974 actual distribution of monthly dividends within quarters.

[117] This is the formal name of the series in Morningstar Direct.

## Contributors to Total Return

As discussed previously in Chapter 2, total return is the sum of three components: capital appreciation return, income return, and reinvestment return. When investors invest in equities (i.e., stocks), they typically tend to focus on the "capital appreciation" component of total return. In other words, investors focus on buying Stock ABC at, say, $10 and then selling it at some higher value.

But is the capital appreciation component actually the largest contributor to investors' total return over the long-term? To investigate this, the growth of $1 hypothetically invested at the end of 1925 in each of the SBBI® large-cap stocks total return series and the SBBI® large-cap stocks capital appreciation return series is compared in Exhibit 3.1. The terminal index value of $1 invested over the 1926–2020 time horizon in the total return series far outstrips the terminal index value of $1 invested over the 1926–2020 time horizon in the capital appreciation return series.

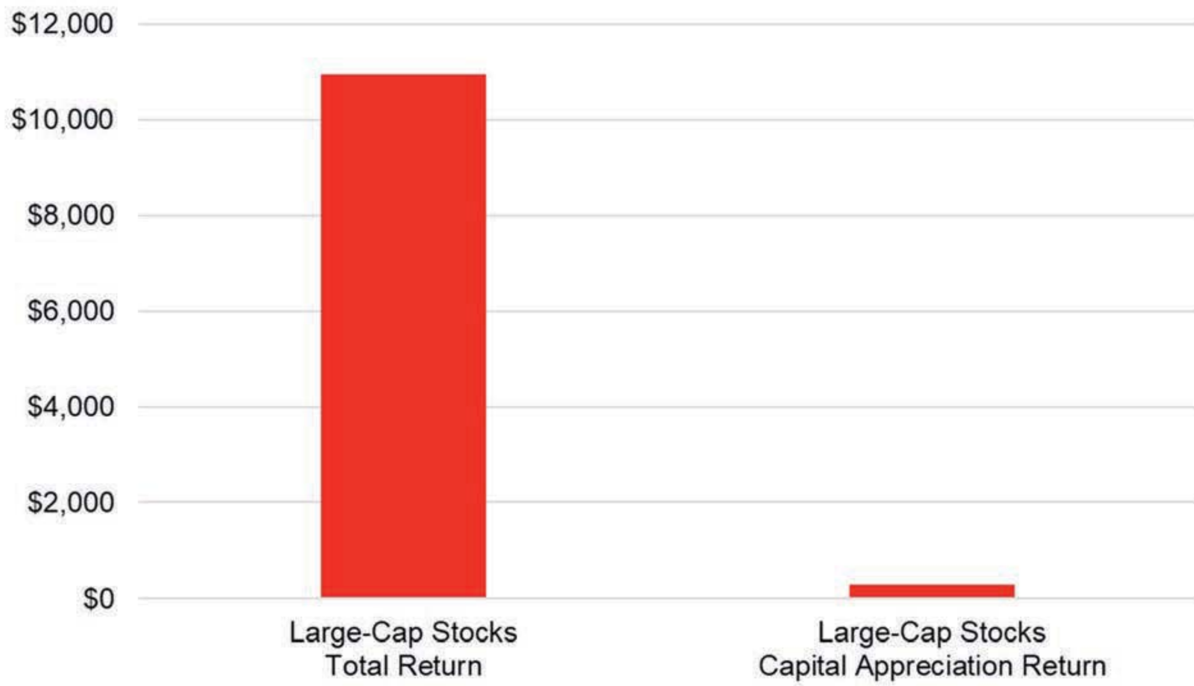
This implies that over the long-term, capital appreciation's contribution to total return is relatively *small* compared to the other two components of total return, income return and reinvestment return. [118,119]

---

[118] To learn more about calculating index values, see Chapter 5, "Annual Returns and Indexes".

[119] Pre-calculated index values at each month-end over the January 1926–December 2020 time horizon are presented in table format in the *2021 SBBI® Yearbook* available at dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook: Large-Capitalization Stocks: Total Return Index, Large-Capitalization Stocks: Capital Appreciation Index, Small-Capitalization Stocks: Total Return Index, Long-term Corporate Bonds: Total Return Index, Long-term Government Bonds: Total Return Index, Long-term Government Bonds: Capital Appreciation Index, Intermediate-term Government Bonds: Total Return Index, Intermediate-term Government Bonds: Capital Appreciation Index, U.S. Treasury Bills: Total Return Index, Inflation Index.

**Exhibit 3.1:** Large-Cap Stocks Total Return and Capital Appreciation Return; Terminal Index Value as of December 31, 2020 (Year-End 1925 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks total return series: IA SBBI® US Large Stock TR USD Ext, (ii) Large-Cap Stocks capital appreciation return series: IA SBBI US Large Stock Cap App Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## Small-Cap Stocks

Small-cap stocks are represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, "Small-Cap Stocks: IA SBBI® US Small Stock TR USD".[120] Small-cap stock total returns ranged from a high of 142.9% in 1933 to a low of -58.0% in 1937.

### DFA U.S. Micro Cap Portfolio (April 2001 December 2020)

For April 2001 to December 2020, the small-cap stock return series is the total return achieved by the DFA U.S. Micro Cap Portfolio net of fees and expenses. In April 2001, Dimensional Fund Advisors renamed the DFA U.S. 9–10 Small Company Portfolio (see next page) the DFA U.S. Micro Cap Portfolio and changed some of the criteria. The fund is designed to capture the returns and diversification benefits of a broad cross-section of U.S. small companies on a market-cap weighted basis. The fund's target buy range includes those companies whose market cap falls in the lowest 5% of the market universe defined as the aggregate of the NYSE, NYSE AMEX, and

[120] This is the formal name of the series in Morningstar Direct.

NASDAQ National Market System or companies smaller than the 1,500th largest U.S. company in the same market universe, whichever results in a higher market cap break.

The market universe is examined on a dynamic basis to determine which stocks are eligible for purchase or sale based on market capitalization. To minimize turnover, a hold or buffer range is created for stocks that migrate above the buy range. The upper bound of the hold range is the fifth percentile of the market universe. Stocks that grow above the hold range are eligible for sale and proceeds are reinvested into the portfolio.

At year-end 2020, the DFA U.S. Micro Cap Portfolio contained 1,618 stocks with a weighted average market cap of $1.763 billion.

**DFA U.S. 9–10 Small Company Portfolio (January 1982–March 2001)**

For January 1982 to March 2001, the small-cap stock return series was the total return achieved by the DFA U.S. Small Company 9–10 (for ninth and 10th deciles) Portfolio. The fund's target buy range was a market-cap-weighted universe of the ninth and 10th deciles of the New York Stock Exchange, plus stocks listed on the NYSE Amex (now the NYSE MKT) and NASDAQ National Market with the same or less capitalization as the upper bound of the NYSE ninth decile. Because the lower bound of the 10th decile is near zero, stocks were not purchased if they were smaller than $10 million in market cap (although they were held if they fell below that level).

**NYSE Fifth Quintile Returns (1926–1981)**

The equities of smaller companies from 1926 to 1980 are represented by the historical series developed by Professor Rolf W. Banz (see Acknowledgements). This is composed of stocks making up the fifth quintile (i.e., the ninth and 10th deciles) of the New York Stock Exchange (NYSE); the stocks on the NYSE are ranked by capitalization, and each decile contains an equal number of stocks at the beginning of each formation period. The ninth and 10th decile portfolio was first ranked and formed as of December 31, 1925. This portfolio was "held" for five years with value weighted portfolio returns computed monthly. Every five years the portfolio was rebalanced (i.e., all of the stocks on the NYSE were re-ranked, and a new portfolio of those falling in the ninth and 10th deciles was formed) as of December 31, 1930 and every five years thereafter through December 31, 1980. This method avoided survivorship bias by including the return after the delisting or failure of a stock in constructing the portfolio returns. (Survivorship bias is caused by studying only stocks that have survived events such as bankruptcy and acquisition.)

For 1981, Dimensional Fund Advisors updated the returns using Professor Banz's methods. The data for 1981 are significant to only three decimal places (in decimal form) or one decimal place when returns are expressed in percent.

## Long term Corporate Bonds

Long-term (i.e., 20-year) corporate bonds are represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, "IA SBBI® US LT Corp TR USD."[121] Long-term corporate bond total returns ranged from a high of 42.6% in 1982 to a low of -8.1% in 1969.

### Total Returns

For 1969 to 2020, corporate bond total returns are represented by the FTSE USBIG Corp AAA/AA 10+ Yr (formerly Citigroup Long-Term High-Grade Corporate Bond Index). Because most large corporate bond transactions take place over the counter, a major dealer is the natural source of these data. The index includes nearly all Aaa- and Aa-rated bonds. If a bond is downgraded during a particular month, its return for the month is included in the index before removing the bond from future portfolios.

For 1926 to 1968, total returns were calculated by summing the capital appreciation returns and the income returns. For the period 1946 to 1968, Ibbotson and Sinquefield (1976) backdated the Salomon Brothers index, using Salomon Brothers' monthly yield data; a methodology similar to that used by Salomon was used for 1969 to 2016. Capital appreciation returns were calculated from yields assuming (at the beginning of each monthly holding period) a 20-year maturity, a bond price equal to par, and a coupon equal to the beginning-of-period yield.

For the period 1926 to 1945, Standard & Poor's monthly High Grade Corporate Composite yield data were used, assuming a 4% coupon and a 20-year maturity. The conventional present-value formula for bond price was used for the beginning and end-of-month prices.[122] The monthly income return was assumed to be one-twelfth the coupon.

## Long-term Government Bonds

Long-term (i.e., 20-year) government bonds are represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, "IA SBBI® US LT Govt TR USD."[123] Long-term government bond total returns ranged from a high of 40.4% in 1982 to a low of -14.9% in 2009.

### Total Returns

The total returns on long-term government bonds from 1977 to 2020 are constructed with data from The Wall Street Journal. The data for 1926 to 1976 is obtained from the Government Bond File at the Center for Research in Security Prices at the University of Chicago Booth School of Business.

121 This is the formal name of the series in Morningstar Direct.

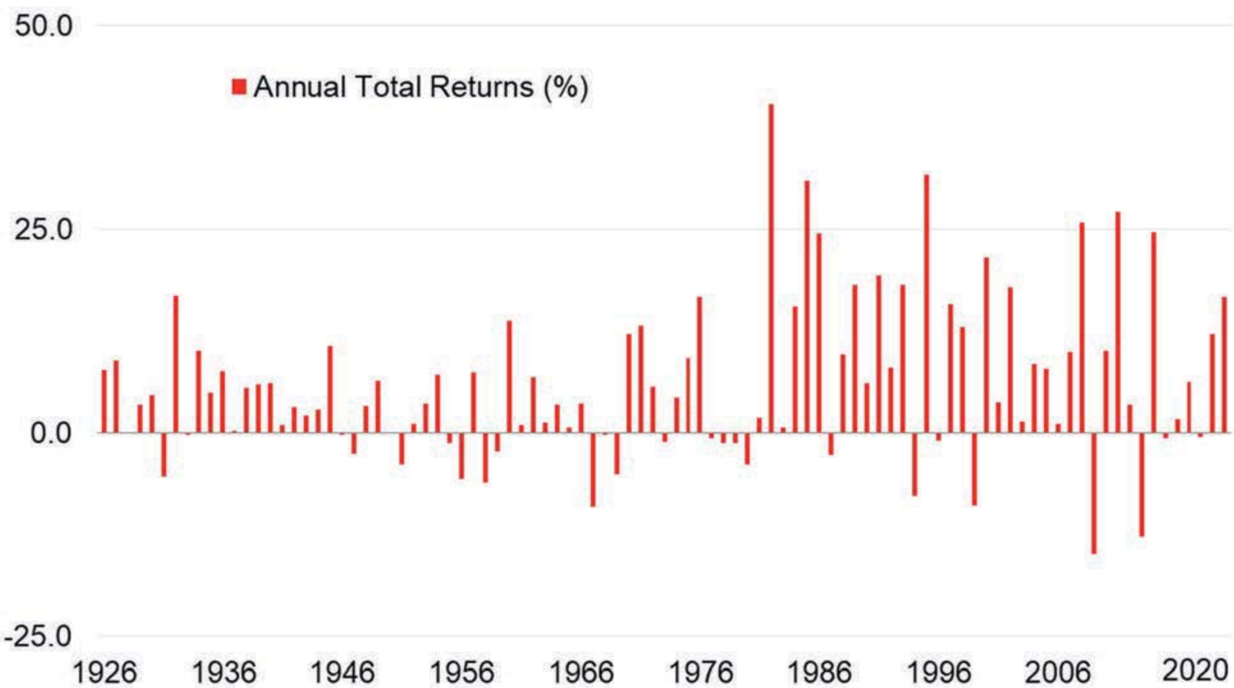
122 This formula is presented in Ross, S. A., & Westerfield, R.W. 1988. "Level-Coupon Bonds." P. 97 in *Corporate Finance* (St. Louis: Times Mirror/Mosby).

123 This is the formal name of the series in Morningstar Direct.

The bonds used to construct the index from 1926–2020 are shown in Exhibit 3.3. The bond used in 2020 is the 4.25% issue that matures on November 15, 2040. To the greatest extent possible, a one-bond portfolio with a term of approximately 20 years and a reasonably current coupon – whose returns did not reflect potential tax benefits, impaired negotiability, or special redemption or call privileges – was used each year. Where "flower" bonds (tenderable to the Treasury at par in payment of estate taxes) had to be used, the bond with the smallest potential tax benefit was chosen. Where callable bonds had to be used, the term of the bond was assumed to be a simple average of the maturity and first call dates minus the current date. The bond was "held" for the calendar year and returns were computed.

The annual total returns for the long-term government bond series from 1926–2020 is illustrated in Exhibit 3.2.

**Exhibit 3.2:** Long-term Government Bonds Annual Total Returns (%) 1926–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

The U.S. Treasury has periodically changed the maturities that it issues. For example, in April 1986 the U.S. Treasury stopped issuing 20-year Treasuries, and from October 2001 through January 2006 the U.S. Treasury did not issue 30-year bonds (it resumed issuing 30-year Treasury bonds in February 2006), making the 10-year bond the longest-term Treasury security issued over the October 2001–January 2006 period. Most recently, on January 16, 2020, the U.S. Department of the Treasury announced its plans to issue a 20-year nominal coupon bond in the first half of calendar year 2020, the first time a 20-year maturity will be offered since March 1986.[124,125]

Total returns for 1977 to 2020 are calculated as the change in the flat (or and-interest) price.[126] The flat price is the average of the bond's bid and ask prices plus the accrued coupon.[127] The accrued coupon is equal to zero on the day a coupon is paid and increases over time until the next coupon payment according to the formula below:

$$A = fC$$

Where:

$A$ = Accrued coupon

$C$ = Semiannual coupon rate

$f$ = (number of days since last coupon payment)/(number of days from last coupon payment to next coupon payment)

### Income Return

For 1977 to 2020, the income return for the long-term government bond series is calculated as the change in flat price plus any coupon actually paid from one period to the next, holding the yield constant over the period. As in the total return series, the exact number of days composing the period is used. For 1926 to 1976, the income return for a given month is calculated as the total return minus the capital appreciation return.

### Capital Appreciation or Return in Excess of Yield

For 1977 to 2020, capital appreciation is taken as the total return minus the income return for each month. For 1926 to 1976, the capital appreciation return (also known as the return in excess of yield) is obtained from the CRSP Government Bond File.

A bond's capital appreciation is defined as the total return minus the income return; that is, the return in excess of yield. This definition omits the capital gain or loss that comes from the movement of a bond's price toward par (in the absence of an interest-rate change) as it matures.

124 To learn more, visit the U.S. Department of the Treasury website at: https://home.treasury.gov/news/press-releases/sm878

125 See Kate Davidson, "Treasury to Issue New 20-Year Bond in First Half of 2020", *The Wall Street Journal*, January 16, 2020 at: https://www.wsj.com/articles/treasury-to-issue-new-20-year-bond-in-first-half-of-2020-11579217450

126 "Flat price" is used here to mean the unmodified economic value of the bond, i.e., the and-interest price, or quoted price plus accrued interest. In contrast, some sources use flat price to mean the quoted price.

127 For the purpose of calculating the return in months when a coupon payment is made, the change in the flat price includes the coupon.

Capital appreciation, as defined here, captures changes in bond prices caused by changes in the interest rate.

### Yields

The yield on the long-term government bond series is defined as the internal rate of return that equates the bond's price (the average of bid and ask, plus the accrued coupon) with the stream of cash flows (coupons and principal) promised to the bondholder. The yields reported for 1977 to 2020 were calculated from *The Wall Street Journal* prices for the bonds listed in Exhibit 3.3. For non-callable bonds, the maturity date is shown. For callable bonds, the first call date and the maturity dates are shown as in the following example: 10/15/47–52 refers to a bond that is first callable on Oct. 15, 1947, and matures on Oct. 15, 1952. Dates from 47–99 refer to 1947 to 1999; 00–12 refers to 2000 to 2012. For callable bonds trading below par, the yield to maturity is used; for those trading above par, the yield to call is used. The yields for 1926 to 1976 were obtained from the CRSP Government Bond File.

**Exhibit 3.3:** Long-term and Intermediate-term Government Bond Issues

**Long-term Gov't Bonds**

| Period Bond Is Held In Index | Coupon (%) | Call/Maturity Date |
|---|---|---|
| 1926–1931 | 4.250 | 10/15/47–52 |
| 1932–1935 | 3.000 | 9/15/51–55 |
| 1936–1941 | 2.875 | 3/15/55–60 |
| 1942–1953 | 2.500 | 9/15/67–72 |
| 1954–1958 | 3.250 | 6/15/78–83 |
| 1959–1960 | 4.000 | 2/15/1980 |
| 1961–1965 | 4.250 | 5/15/75–85 |
| 1966–1972 | 4.250 | 8/15/87–92 |
| 1973–1974 | 6.750 | 2/15/1993 |
| 1975–1976 | 8.500 | 5/15/94–99 |
| 1977–1980 | 7.875 | 2/15/95–00 |
| 1981 | 8.000 | 8/15/96–01 |
| 1982 | 13.375 | 8/15/2001 |
| 1983 | 10.750 | 2/15/2003 |
| 1984 | 11.875 | 11/15/2003 |
| 1985 | 11.750 | 2/15/05–10 |
| 1986–1989 | 10.000 | 5/15/05–10 |
| 1990–1992 | 10.375 | 11/15/07–12 |
| 1993–1996 | 7.250 | 5/15/2016 |
| 1997–1998 | 8.125 | 8/15/2019 |
| 1999–2001 | 8.125 | 8/15/2021 |
| 2002 | 6.250 | 8/15/2023 |
| 2003–2004 | 7.500 | 11/15/2024 |
| 2005 | 6.875 | 8/15/2025 |
| 2006 | 6.750 | 8/15/2026 |
| 2007 | 6.375 | 8/15/2027 |
| 2008 | 5.500 | 8/15/2028 |
| 2009 | 5.250 | 2/15/2029 |
| 2010–2012 | 5.375 | 2/15/2031 |
| 2013 | 4.500 | 2/15/2036 |
| 2014 | 4.500 | 2/15/2036 |
| 2015 | 4.500 | 2/15/2036 |
| 2016 | 4.500 | 2/15/2036 |
| 2017 | 4.750 | 2/15/2037 |
| 2018 | 4.500 | 5/15/2038 |
| 2019 | 4.250 | 5/15/2039 |
| 2020 | 4.250 | 11/15/2040 |

**Intermediate-term Gov't Bonds**

| Period Bond Is Held In Index | Coupon (%) | Call/Maturity Date |
|---|---|---|
| 1934–1936 | 3.250 | 8/1/1941 |
| 1937 | 3.375 | 3/15/1943 |
| 1938–1940 | 2.500 | 12/15/1945 |
| 1941 | 3.000 | 1/1/1946 |
| 1942 | 3.000 | 1/1/1947 |
| 1943 | 1.750 | 6/15/1948 |
| 1944–1945 | 2.000 | 3/15/1950 |
| 1946 | 2.000 | 6/15/1951 |
| 1947 | 2.000 | 3/15/1952 |
| 1948 | 2.000 | 9/15/1953 |
| 1949 | 2.500 | 3/15/1954 |
| 1950 | 2.250 | 6/15/1955 |
| 1951–1952 | 2.500 | 3/15/1958 |
| 1953 | 2.375 | 6/15/1958 |
| 1954 | 2.375 | 3/15/1959 |
| 1955 | 2.125 | 11/15/1960 |
| 1956 | 2.750 | 9/15/1961 |
| 1957–1958 | 2.500 | 8/15/1963 |
| 1959 | 3.000 | 2/15/1964 |
| 1960 | 2.625 | 2/15/1965 |
| 1961 | 3.750 | 5/15/1966 |
| 1962 | 3.625 | 11/15/1967 |
| 1963 | 3.875 | 5/15/1968 |
| 1964 | 4.000 | 2/15/1969 |
| 1965 | 4.000 | 8/15/1970 |
| 1966 | 4.000 | 8/15/1971 |
| 1967 | 4.000 | 2/15/1972 |
| 1968 | 4.000 | 8/15/1973 |
| 1969 | 5.625 | 8/15/1974 |
| 1970 | 5.750 | 2/15/1975 |
| 1971 | 6.250 | 2/15/1976 |
| 1972 | 1.500 | 10/1/1976 |
| 1973 | 6.250 | 2/15/1978 |
| 1974 | 6.250 | 8/15/1979 |
| 1975 | 6.875 | 5/15/1980 |
| 1976 | 7.000 | 2/15/1981 |
| 1977 | 6.375 | 2/15/1982 |
| 1978 | 8.000 | 2/15/1983 |
| 1979 | 7.250 | 2/15/1984 |

**Intermediate-term Gov't Bonds (cont.)**

| Period Bond Is Held In Index | Coupon (%) | Call/Maturity Date |
|---|---|---|
| 1980 | 8.000 | 2/15/1985 |
| 1981 | 13.500 | 2/15/1986 |
| 1982 | 9.000 | 2/15/1987 |
| 1983 | 12.375 | 1/1/1988 |
| 1984 | 14.625 | 1/15/1989 |
| 1985 | 10.500 | 1/15/1990 |
| 1986 | 11.750 | 1/15/1991 |
| 1987 | 11.625 | 1/15/1992 |
| 1988 | 8.750 | 1/15/1993 |
| 1989 | 9.000 | 2/15/1994 |
| 1990 | 8.625 | 10/15/1995 |
| 1991–1992 | 7.875 | 7/15/1996 |
| 1993 | 6.375 | 1/15/1999 |
| 1994 | 5.500 | 4/15/2000 |
| 1995 | 8.500 | 2/15/2000 |
| 1996 | 7.750 | 2/15/2001 |
| 1997 | 6.375 | 8/15/2002 |
| 1998 | 5.750 | 8/15/2003 |
| 1999 | 7.250 | 8/15/2004 |
| 2000 | 6.500 | 8/15/2005 |
| 2001 | 6.500 | 10/15/2006 |
| 2002 | 6.125 | 8/15/2007 |
| 2003 | 5.625 | 5/15/2008 |
| 2004 | 5.500 | 5/15/2009 |
| 2005 | 5.750 | 8/15/2010 |
| 2006 | 5.000 | 8/15/2011 |
| 2007 | 4.875 | 2/15/2012 |
| 2008 | 3.625 | 5/15/2013 |
| 2009 | 4.250 | 8/15/2014 |
| 2010 | 4.125 | 5/15/2015 |
| 2011 | 3.250 | 7/31/2016 |
| 2012 | 2.750 | 5/31/2017 |
| 2013 | 2.375 | 5/31/2018 |
| 2014 | 3.125 | 5/15/2019 |
| 2015 | 3.500 | 5/15/2020 |
| 2016 | 3.125 | 5/15/2021 |
| 2017 | 2.125 | 6/30/2022 |
| 2018 | 2.500 | 8/15/2023 |
| 2019 | 2.125 | 11/30/2024 |
| 2020 | 2.125 | 5/15/2025 |

## Intermediate-term Government Bonds

Intermediate-term (i.e., 5-year) government bonds are represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, "IA SBBI® US IT Govt TR USD."[128] Intermediate-term government bond total returns ranged from a high of 29.1% in 1982 to a low of -5.1% in 1994.

Capital appreciation caused $1.00 to increase to $1.84 over the 95-year period, representing a compound annual growth rate of 0.6%. This increase was unexpected; because yields rose on average over the period, capital appreciation on a hypothetical intermediate-term government bond portfolio with a constant five-year maturity should have been negative. An explanation of the positive average return is given at the end of this chapter.

### Total Returns

Total returns of the intermediate-term government bonds for 1987 to 2020 are calculated from *The Wall Street Journal* prices using the coupon accrual method described above for long-term government bonds (see equation in previous section). The bond used in 2020 is the 2.125% issue maturing on May 15, 2025. Returns for 1934 to 1986 are obtained from the CRSP Government Bond File. The bonds used to construct the index for 1934 to 2020 are shown in Exhibit 3.3.

As with long-term government bonds, one-bond portfolios are used to construct the intermediate-term government bond index. The bond chosen each year is the shortest non-callable bond with a maturity not less than five years, and it is "held" for the calendar year. Monthly returns are computed. Bonds with impaired negotiability or special redemption privileges are omitted, as are partially or fully tax-exempt bonds starting with 1943.

For 1934 to 1942, almost all bonds with maturities near five years were partially or fully tax-exempt and selected using the rules described above. Personal tax rates were generally low in that period so that yields on tax-exempt bonds were similar to yields on taxable bonds.

For 1926 to 1933, there are few bonds suitable for construction of a series with a five-year maturity. For this period, five-year bond yield estimates are used. These estimates are obtained from Thomas S. Coleman, Lawrence Fisher, and Roger G. Ibbotson, *Historical U.S. Treasury Yield Curves*: 1926– 1992 with 1995 update (Ibbotson Associates, Chicago, 1995). The estimates reflect what a "pure play" five-year Treasury bond, selling at par and with no special redemption or call provisions, would have yielded had one existed. Estimates are for partially tax-exempt bonds for 1926 to 1932 and for fully tax-exempt bonds for 1933. Monthly yields are converted to monthly total returns by calculating the beginning and end-of-month flat prices for the hypothetical bonds. The bond is "bought" at the beginning of the month at par (i.e., the coupon equals the previous month-end yield), assuming a maturity of five years. It is "sold" at the end of the month, with the flat price calculated by discounting the coupons and principal at the end-of-month yield,

[128] This is the formal name of the series in Morningstar Direct.

assuming a maturity of four years and 11 months. The flat price is the price of the bond including coupon accruals so that the change in flat price represents total return. Monthly income returns are assumed to be equal to the previous end-of month yield, stated in monthly terms. Monthly capital appreciation returns are formed as total returns minus income returns.

### Income Return and Capital Appreciation

For 1987 to 2020, the income return is calculated according to the methodology stated under "Long-term Government Bonds." Monthly capital appreciation (return in excess of yield) over this same period is the difference between total return and income return.

For 1934 to 1986, capital appreciation (return in excess of yield) is taken directly from the CRSP Government Bond File. The income return is calculated as the total return minus the capital appreciation return. Prior to 1934, the income and capital appreciation components of total return are generated from yield estimates as described earlier for total returns.

### Yields

The yield on an intermediate-term government bond is the internal rate of return that equates the bond's price with the stream of cash flows (coupons and principal) promised to the bondholder. The yields reported for 1987 to 2020 are calculated from *The Wall Street Journal* bond prices listed in Exhibit 3.3. For 1934 to 1986, yields were obtained from the CRSP Government Bond File. Yields for 1926 to 1933 are estimates from Coleman, Fisher, and Ibbotson, *Historical U.S. Treasury Yield Curves*: 1926–1992 with 1995 update.

## U.S. Treasury Bills

U.S. Treasury bills are represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, "IA SBBI® US 30 Day TBill TR USD."[129] Treasury bill total returns ranged from a high of 14.7% in 1981 to a low of 0.0% in 1938.[130]

In Exhibit 3.4, all years in which the annual total return of U.S. Treasury Bills was less than 0.5% are shown. The years in which Treasury bills had less than 0.5% annual total return primarily occurred during the following periods: (i) the Great Depression (the 1930s), (ii) World War II (the 1940s), (iii) in the period after the 2008 financial crisis (2009–2016), and (iv) in 2020 during the COVID-19 pandemic.[131] The U.S. Treasury Bill annual total return in 2020 was 0.4%, a significant decrease when compared to U.S. Treasury Bills total returns in 2018 and 2019 which were 1.8% and 2.1%, respectively (see Exhibit 3.6).

---

129 This is the formal name of the series in Morningstar Direct.

130 At a 4-decimal level, the low in 1938 was -0.0162%.

131 The first three "periods" are multi-year. The last of the four periods (2020) is a single year as of date of publication

**Exhibit 3.4:** Years in which Annual Total Returns of U.S. Treasury Bills Were Less than 0.5% 1926–2020

| Year | Total Return | Year | Total Return |
|---|---|---|---|
| 1933 | 0.3% | 2009 | 0.1% |
| 1934 | 0.2% | 2010 | 0.1% |
| 1935 | 0.2% | 2011 | 0.0% |
| 1936 | 0.2% | 2012 | 0.1% |
| 1937 | 0.3% | 2013 | 0.0% |
| 1938 | 0.0% | 2014 | 0.0% |
| 1939 | 0.0% | 2015 | 0.0% |
| 1940 | 0.0% | 2016 | 0.2% |
| 1941 | 0.1% | 2020 | 0.4% |
| 1942 | 0.3% | | |
| 1943 | 0.3% | | |
| 1944 | 0.3% | | |
| 1945 | 0.3% | | |
| 1946 | 0.4% | | |

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## Total Returns

For the U.S. Treasury bill index, data from *The Wall Street Journal* is used for 1977 to 2020; the CRSP U.S. Government Bond File is the source until 1976. Each month a one-bill portfolio containing the shortest-term bill having not less than one month to maturity is constructed. (The bill's original term to maturity is not relevant.) To measure holding-period returns for the one-bill portfolio, the bill is priced as of the last trading day of the previous month end and as of the last trading day of the current month. The price of the bill ($P$) at each time ($t$) is given as:

$$P_t = \left[1 - \frac{rd}{360}\right]$$

Where:

$P_t$ = Price of the bill at time $t$

$r$ = decimal yield (the average of bid and ask quotes) on the bill at time $t$

$d$ = number of days to maturity as of time $t$

The total return on the bill is the month-end price divided by the previous month-end price, minus one.

### Negative Returns on Treasury Bills

*Monthly* Treasury bill returns were negative in February 1933, and in 12 months during the 1938 to 1941 period. More recently since July 2011, monthly Treasury bill returns have been negative in 8 months. Annual total returns have been negative only once, in 1938. Because negative Treasury bill returns seem to contradict logic, an explanation is in order.

Negative yields observed in the data do not imply that investors purchased Treasury bills with a guaranteed negative return. Rather, Treasury bills of that era were exempt from personal property taxes in some states, while cash was not. Further, for a bank to hold U.S. government deposits, Treasury securities were required as collateral. These circumstances created excessive demand for the security, and thus bills were sold at a premium. Given the low interest rates during the period, owners of the bills experienced negative returns.

In 2008, yields on U.S. Treasury bills fell from a little over 3.0% at the beginning of the year to approximately zero percent by the end of the year, but the dynamics were different from those for 1938 to 1941. In the wake of the 2008 financial crisis, investors' behavior could be described as an extreme flight to safety; investors were willing to accept little (if anything) in return for the assurance that they would get their principal back. In other words, the return *of* capital took precedence over the return *on* capital.

From 2009 to 2016, U.S. Treasury bill yields remained close to historical lows near zero percent. These low yields can be at least partially explained by the Federal Funds target rate, which was actually a range 0% to 0.25% from December 16, 2008 through December 16, 2015.

In 2017, 2018, and 2019 the annual total return of U.S. Treasury bills was 0.8%, 1.8%, and 2.1%, respectively, a significant increase over the annual total return seen over the 2009–2016 period. These higher yields can be at least partially explained by the accompanying general increase in the Fed Funds target rate. On December 17, 2015, the Federal Reserve raised the target federal funds range 25 basis points ("bps") to 0.25%–0.50% and again raised the target range an additional 25 bps in each of December 2016, March 2017, June 2017, December of 2017, March 2018, June 2018, September 2018, and December 2018 to a level of 2.25%–2.50% as of December 31, 2018. In the second half of 2019 the Federal Reserve reversed course and lowered the target federal funds range 25 bps in each of August 2019, September 2019, and October 2019, ending at a level of 1.5%–1.75%.[132]

In response to the outbreak of COVID-19, on March 3, 2020 and on March 16, 2020 the Federal Reserve lowered the target federal funds range an additional 50 bps and 100 bps, respectively,

[132] For a list of Federal Reserve open market operations, visit https://www.federalreserve.gov/monetarypolicy/openmarket.htm. For a detailed discussion of monetary policy and interest rates, see the Cost of Capital Navigator's Resources Section (subscription required) at dpcostofcapital.com.

bringing the range to 0.00%–0.25%, where it remained through the end of 2020. The low yields of U.S. Treasury bills in 2020 (0.4%) can at least be partially explained by the Fed's return to a 0.00%–0.25% Federal Funds target range for most of 2020.[133]

## Inflation

Inflation is represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, "IA SBBI® US Inflation."[134] Inflation rates ranged from a high of 18.1% in 1946 to a low of -10.3% in 1932.[135]

### Consumer Price Index

The Consumer Price Index for All Urban Consumers, or CPI-U, non-seasonally adjusted, is used to measure inflation, which is the rate of change of consumer goods prices. Unfortunately, the CPI is not measured over the same period as the other asset returns. All the security returns are measured from one month-end to the next month-end. CPI commodity prices are collected during the month. Thus, measured inflation rates lag the other series by about one-half month. Prior to January 1978, the CPI (rather than the CPI-U) was used. For 1978 to 1987, the index uses the year 1967 in determining the items composing the basket of goods. After 1987, a three-year period, 1982 to 1984, was used to determine the items making up the basket of goods. All inflation measures are constructed by the U.S. Department of Labor, Bureau of Labor Statistics, Washington.

## Bond Capital Appreciation Despite Rising Yields

The capital appreciation component of intermediate-term government bond returns caused $1.00 invested at year end 1925 to grow to $1.84 by the end of 2020, representing a compound annual growth rate of 0.6%. This is surprising because yields, on average, rose over the period.

An investor in a hypothetical five-year constant maturity portfolio, with continuous rebalancing, suffered a capital loss (that is, excluding coupon income) over 1926 to 2020. An investor who rebalanced yearly, choosing bonds according to the method set forth above, fared better. This investor would have earned the 0.6% annualized capital gain recorded here.

This performance relates to the construction of the intermediate-term bond series. For 1926 to 1933, the one bond portfolio was rebalanced monthly to maintain a constant maturity of five years. For 1934 to 2020, one bond (the shortest bond not less than five years to maturity) was chosen at the beginning of each year and priced monthly. New bonds were not picked each month to maintain a constant maturity intra-year.

---

133 This marks the lowest the target federal funds range has been since 2015.

134 This is the formal name of the series in Morningstar Direct.

135 Revised to 18.1% from 18.2% (as reported in previous editions). On February 26, 2021 Morningstar revised the "IA SBBI US Inflation" series. The revisions were applied to various dates from February 1926 through December 2020. The revisions were small and did not affect long-term averages materially.

There are several possible reasons for the positive capital appreciation return. Chief among these reasons are convexity of the bond portfolio and the substitution of one bond for another at each year-end.

**Convexity**

Each year, we “bought” a bond with approximately five years to maturity and held it for one year. During this period, the market yield on the bond fluctuates. Because the duration of the bond shortens (the bond becomes less interest-rate sensitive) as yields rise and the duration lengthens as yields fall, more is gained from a fall in yield than is lost from a rise in yield. This characteristic of a bond is known as convexity.

For example, suppose an 8% coupon bond is bought at par at the beginning of a year; the yield fluctuates (but the portfolio is not rebalanced) during the year; and the bond is sold at par at the end of the year. The price of the bond at both the beginning and end of the year is $100; the change in bond price is zero. However, the fluctuations will have caused the gains during periods of falling yields to exceed the losses during periods of rising yields. Thus, the total return for the year exceeds 8%. Because our measure of capital appreciation is the return in excess of yield, rather than the change in bond price, capital appreciation for this bond (as measured) will be greater than zero.

In 1992, the yield for intermediate-term government bonds started the year at 5.97%, rose, fell, and finally rose again to end at 6.11%, slightly higher than the starting point. In the absence of convexity, the capital appreciation return for 1992 would be negative. Because of the fluctuation of yields during the year, however, the capital appreciation return on the intermediate-term government bond index was positive 0.64%.

It should be noted that the return in excess of yield, or capital gain, from convexity is caused by holding, over the year, a bond whose yield at purchase is different from the current market yield. If the portfolio were rebalanced each time the data were sampled (in this case, monthly), by selling the old bond and buying a new five-year bond selling at par the portfolio would have no convexity. That is, over a period where yields ended where they started, the measured capital appreciation would be zero. However, this is neither a practical way to construct an index of actual bonds nor to manage a bond portfolio.

**Bond Substitution**

Another reason the intermediate term government bond series displays positive capital appreciation despite rising yields is the way in which bonds were removed from the portfolio and replaced with other bonds. In general, it was not possible to replace a bond “sold” by buying one with exactly the same yield. This produces a spurious change in the yield of the series – one that should not be associated with a capital gain or loss.

For example: Suppose a five-year bond yielding 8% is bought at par at the beginning of the year; at that time, four-year bonds yield 7%. Over the year, the yield curve rises in parallel by 1 percentage point so that when it comes time to sell the bond at year-end, it yields 8% and has four years to maturity. Therefore, at both the beginning and end of the year, the price of the bond is $100.

The proceeds from the sale are used to buy a new five-year bond yielding 9%. While the bond price change was zero over the year, the yield of the series has risen from 8% to 9%. Thus, it is possible because of the process of substituting one bond for another for the yield series to contain a spurious rise that is not, and should not be expected to be, associated with a decline in the price of any particular bond. This phenomenon is likely to be the source of some of the positive capital appreciation in our intermediate-term government bond series.

**Other Issues**

Although convexity and bond substitution may explain the anomaly of positive capital appreciation in a bond series with rising yields, there are other incomplete-market problems that may also help explain the capital gain. For example, intermediate-term government bonds were scarce in the 1930s and 1940s. As a result, the bonds chosen for this series occasionally had maturities longer than five years, ranging as high as eight years when bought. The 1930s and the first half of the 1940s were bullish for the bond market. Longer bonds included in this series had higher yields and substantially higher capital gain returns than bonds with exactly five years to maturity might have had if any existed. This upward bias is particularly noticeable in 1934, 1937, and 1938.

In addition, callable and fully or partially tax-exempt bonds were used when necessary to obtain a bond for some years. The conversion of the Treasury bond market from tax-exempt to taxable status produced a one-time upward jump in stated yields but not a capital loss on any given bond. Therefore, part of the increase in stated yields over 1926 to 2020 was a tax effect that did not cause a capital loss on the intermediate-term bond index. Further, the callable bonds used in the early part of the period may have commanded a return premium for taking this extra risk.

# Chapter 4
# Description of the Derived Series

Historical data suggests that investors are rewarded for taking risks and that returns are related to inflation rates. The risk/return and the real/nominal relationships in the historical data are revealed by looking at the risk-premium and inflation-adjusted series derived from the basic asset series.

## Derived Series Calculated Using Geometric Differences

Derived series are calculated as the geometric differences between two basic asset classes. Returns on basic series A and B and derived series C are related as follows:

$$(1+C)=\left[\frac{1+A}{1+B}\right]$$

where the series returns for A, B, and C are in decimal form (e.g., 5% is indicated by 0.05). Thus, C is given by:

$$C=\left[\frac{1+A}{1+B}\right]-1\approx A-B$$

As an example, suppose return A equals 15%, or 0.15; and return B is 5%, or 0.05; then C equals (1.15 / 1.05) — 1 = 0.0952, or 9.52%. This result, while slightly different from the simple arithmetic difference of 10%, is conceptually the same.

## Definitions of the Derived Series[136]

From the seven basic asset classes (large-cap stocks, small-cap stocks, long-term corporate bonds, long-term government bonds, intermediate-term government bonds, U.S. Treasury bills, and Inflation), 10 additional series are derived that represent the component or elemental parts of the asset returns.

## Two Categories of Derived Series

The 10 derived series are categorized as (i) risk premiums, or payoffs for taking various types of risk and (ii) as inflation-adjusted asset returns. The risk premiums are (i) the bond horizon

136 Precalculated annual statistics for each of the derived series in Exhibit 4.1 for each year over the 1926–2020 time horizon are presented in table format in the full-version *2021 SBBI® Yearbook*. For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

premium, (ii) the bond default premium, (iii) the equity risk premium, and (iv) the small-stock premium. The inflation-adjusted asset return series are constructed by geometrically subtracting inflation from each of the six asset total return series. The 10 derived series are summarized in Exhibit 4.1.

**Exhibit 4.1:** The Derived Series

| Risk Premia Series | Derivation |
| --- | --- |
| Equity Risk Premium | $\frac{(1 + \textit{Large Stock TR})}{(1 + \textit{Treasury Bill TR})} - 1$ |
| Small-Stock Premium | $\frac{(1 + \textit{Small Stock TR})}{(1 + \textit{Large Stock TR})} - 1$ |
| Bond Default Premium | $\frac{(1 + \textit{LT Corp Bond TR})}{(1 + \textit{LT Govt Bond})} - 1$ |
| Bond Horizon Premium | $\frac{(1 + \textit{LT Govt Bond TR})}{(1 + \textit{Treasury Bill TR})} - 1$ |

| Inflation–Adjusted Series | Derivation |
| --- | --- |
| Large-Cap Stock Returns | $\frac{(1 + \textit{Large Stock TR})}{(1 + \textit{Inflation})} - 1$ |
| Small-Cap Stock Returns | $\frac{(1 + \textit{Small Stock TR})}{(1 + \textit{Inflation})} - 1$ |
| Corporate Bond Returns | $\frac{(1 + \textit{LT Corp Bond TR})}{(1 + \textit{Inflation})} - 1$ |
| Long-term Government Bond Returns | $\frac{(1 + \textit{LT Govt Bond TR})}{(1 + \textit{Inflation})} - 1$ |
| Intermediate-term Government Bond Returns | $\frac{(1 + \textit{IT Govt Bond TR})}{(1 + \textit{Inflation})} - 1$ |
| Treasury Bill Returns (Real Riskless Rate of Returns) | $\frac{(1 + \textit{Treasury Bill TR})}{(1 + \textit{Inflation})} - 1$ |

## Equity Risk Premium

Large-cap stock returns are composed of inflation, the real riskless rate, and the equity risk premium. The equity risk premium is the geometric difference between large-cap stock total returns and U.S. Treasury bill total returns. Because large-cap stocks are not strictly comparable with bonds, horizon and default premiums are not used to analyze the components of equity returns (large-cap stocks have characteristics that are analogous to horizon and default risk, but they are not equivalent).

The monthly equity risk premium is given by:

$$\frac{(1+\textit{Large Stock TR})}{(1+\textit{Treasury Bill TR})}-1$$

## Small-Stock Premium

The small-stock premium is the geometric difference between small-cap stock total returns and large-cap stock total returns. The monthly small-stock premium is given by:

$$\frac{(1+\textit{Small Stock TR})}{(1+\textit{Large Stock TR})}-1$$

## Bond Default Premium

The bond default premium is defined as the net return from investing in long-term corporate bonds rather than long-term government bonds of equal maturity. Because there is a possibility of default on a corporate bond, bondholders receive a premium that reflects this possibility, in addition to inflation, the real riskless rate, and the horizon premium. The monthly bond default premium is given by:

$$\frac{(1+\textit{LT Corp Bond TR})}{(1+\textit{LT Govt Bond TR})}-1$$

### Components of the Bond Default Premium

Bonds susceptible to default have higher returns (when they do not default) than those of riskless bonds. Default on a bond may be a small loss, such as a late or skipped interest payment, or it may be a larger loss, such as the loss of any or all principal as well as interest. In any case, part of the default premium on a portfolio of bonds is consumed by the losses on those bonds that do default.

The remainder of the default premium (the portion not consumed by defaults) is a pure risk premium, which the investor demands and, over the long run, receives for taking on the risk of default. The expected return on a corporate bond, or portfolio of corporate bonds, is less than the bond's or portfolio's yield. The portion of the yield that is expected to be consumed by defaults must be subtracted. The expected return on a corporate bond is equal to the expected return on a government bond of similar maturity, plus the pure risk premium portion of the bond default premium.

### Callability Risk Is Captured in the Default Premium

Callability risk is the risk that a bond will be redeemed (at or near par) by its issuer before maturity, at a time when market interest rates are lower than the bond's coupon rate. The possibility of redemption is risky because it would prevent the bondholder of the redeemed issue from reinvesting the proceeds at the original (higher) interest rate. The bond default premium, as measured here, also inadvertently captures any premium investors may demand or receive for this risk.

## Bond Horizon Premium

Long-term government bonds behave differently from short-term bills in that their prices (and hence returns) are more sensitive to interest-rate fluctuations. The bond horizon premium is the extra return investors demand for holding long-term bonds instead of U.S. Treasury bills.

The monthly bond horizon premium is given by:

$$\frac{\left(1+LT\ Govt\ Bond\ TR\right)}{\left(1+Treasury\ Bill\ TR\right)}-1$$

Long-term rather than intermediate-term government bonds are used to derive the bond horizon premium so as to capture a "full unit" of price fluctuation risk. Intermediate-term government bonds may display a partial horizon premium, which is smaller than the difference between long-term bonds and short-term bills.

### Determining the Bond Premium: Maturity vs. Duration

Duration is the present-value weighted average time to receipt of cash flows (coupons and principal) from holding a bond, and can be calculated from the bond's yield, coupon rate, and term to maturity. The duration of a given bond determines the amount of return premium arising from differences in bond life. The bond horizon premium is also referred to as the "maturity premium," based on the observation that bonds with longer maturities command a return premium over shorter-maturity bonds. However, duration, not term to maturity, is the bond characteristic that determines this return premium.

**Why a "Horizon" Premium?**

Investors often strive to match the duration of their bond holdings (cash inflows) with the estimated duration of their obligations (cash outflows). Consequently, investors with short time horizons regard long-duration bonds as risky (due to price fluctuation risk) and short-term bills as riskless. Conversely, investors with long time horizons regard short-term bills as risky (due to the uncertainty about the yield at which bills can be reinvested) and long-duration bonds as riskless or less risky.

Empirically, long-duration bonds bear higher yields and greater returns than short-term bills; that is, the yield curve slopes upward on average over time. This indicates that investors are more averse to the price fluctuation risk of long-duration bonds than to the reinvestment risk of bills.

Bond duration risk is thus in the eye of the beholder, or bondholder. Therefore, rather than identifying the premium as a payoff for long-bond risk (which implies a judgment that short-horizon investors are "right" in their risk perceptions), it is better to go directly to the source of the return differential (the differing time horizons of investors) and use the label "horizon premium."

## Large-Cap Stock Real Returns

**Construction**

The inflation-adjusted return is a geometric difference and is approximately equal to the arithmetic difference between the large-cap stock total return and the inflation rate. The monthly inflation adjusted large-cap stock return is given by:

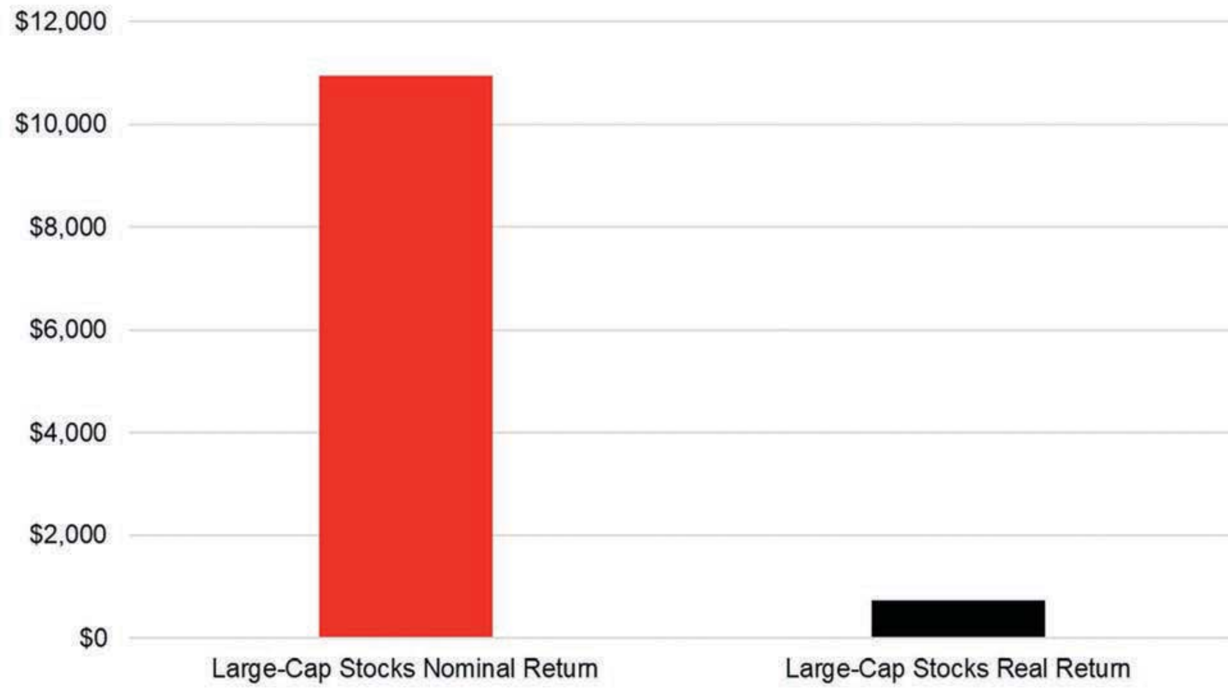
$$\frac{(1+\mathit{Large\,Stock\,TR})}{(1+\mathit{Inflation})}-1$$

The inflation-adjusted large-cap stock return may also be expressed as the geometric sum of the real riskless rate and the equity risk premium:

$$\left[(1+\mathit{Real\,Riskless\,Rate})\times(1+\mathit{Equity\,Risk\,Premium})\right]-1$$

Exhibit 4.2 depicts (i) what $1.00 invested at the end of December 1925 in large cap stocks would have grown to by the end of 2020, and (ii) what $1.00 invested at the end of December 1925 in large-cap stocks would have grown to by the end of 2020 if large-cap stock returns were adjusted for inflation.

**Exhibit 4.2:** Large-cap Stocks, Real and Nominal Return Terminal Index Value 1926–2020 (Year-end 1925 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset class represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext., and (ii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission

## Small-Cap Stock Real Returns

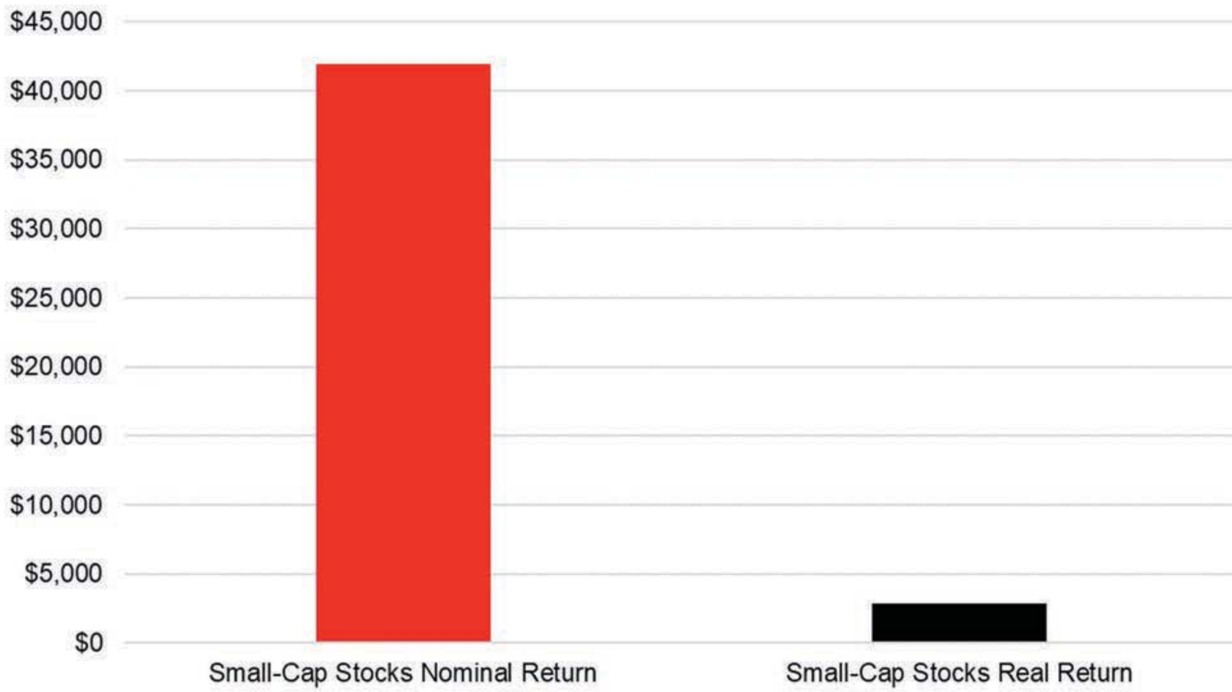
### Construction

The inflation-adjusted return is a geometric difference and is approximately equal to the arithmetic difference between the small-cap stock total return and the inflation rate. The monthly inflation-adjusted small-cap stock return is given by:

$$\frac{(1+\mathit{Small\ Stock\ TR})}{(1+\mathit{Inflation})}-1$$

Exhibit 4.3 depicts (i) what $1.00 invested at the end of December 1925 in small-cap stocks would have grown to by the end of 2020 and (ii) what $1.00 invested at the end of December 1925 in small-cap stocks would have grown to by the end of 2020 if small-cap stock returns were adjusted for inflation.

**Exhibit 4.3:** Small-cap Stocks, Real and Nominal Return Terminal Index Value 1926–2020 (Year-end 1925 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset class represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Small-Cap Stocks: IA SBBI® US Small Stock TR USD., and (ii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission

## Long-term Corporate Bond Real Returns

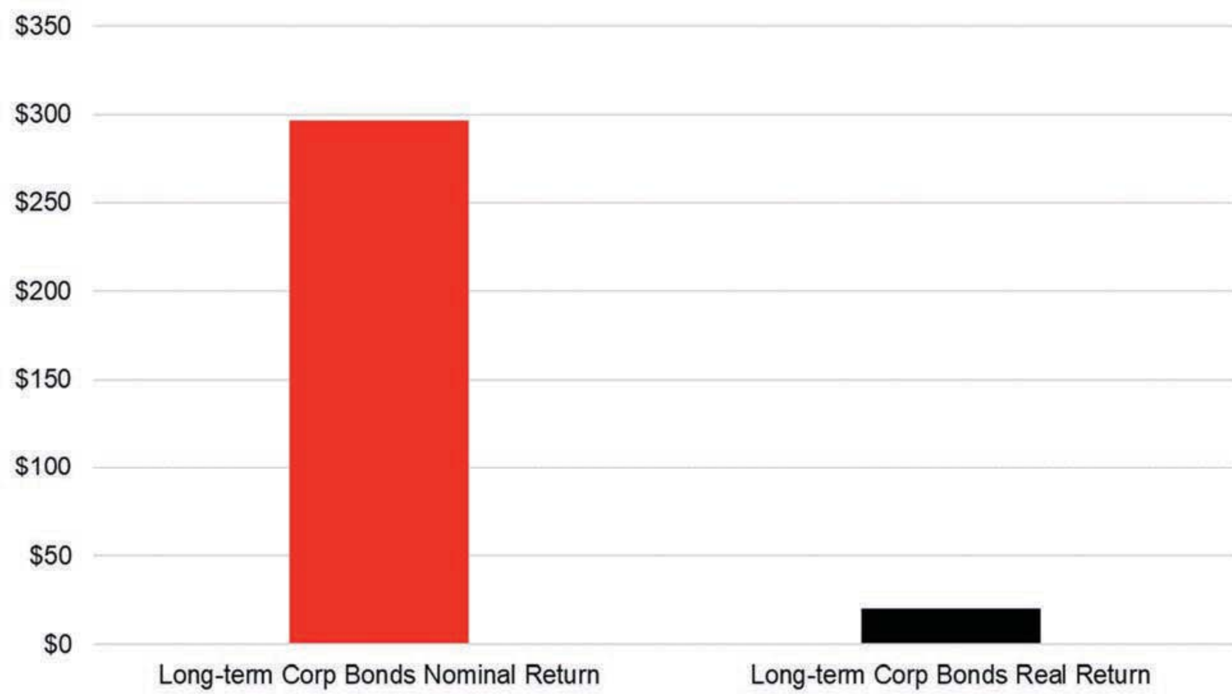
### Construction

The inflation-adjusted return is a geometric difference and is approximately equal to the arithmetic difference between the long-term corporate bond total return and the inflation rate. The monthly inflation-adjusted corporate bond total return is given by:

$$\frac{(1+Corp\,Bond\,TR)}{(1+Inflation)}-1$$

Exhibit 4.4 depicts (i) what $1.00 invested at the end of December 1925 in long-term corporate bonds would have grown to by the end of 2020, and (ii) what $1.00 invested at the end of December 1925 in long term corporate bonds would have grown to by the end of 2020 if long-term corporate bond returns were adjusted for inflation.

**Exhibit 4.4:** Long-Term Corporate Bonds, Real and Nominal Return Terminal Index Value 1926–2020 (Year-end 1925 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset class represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, and (ii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission

## Long-term Government Bond Real Returns

### Construction

The inflation-adjusted return is a geometric difference and is approximately equal to the arithmetic difference between the long-term government bond total return and the inflation rate. The monthly inflation-adjusted long-term government bond total return is given by:

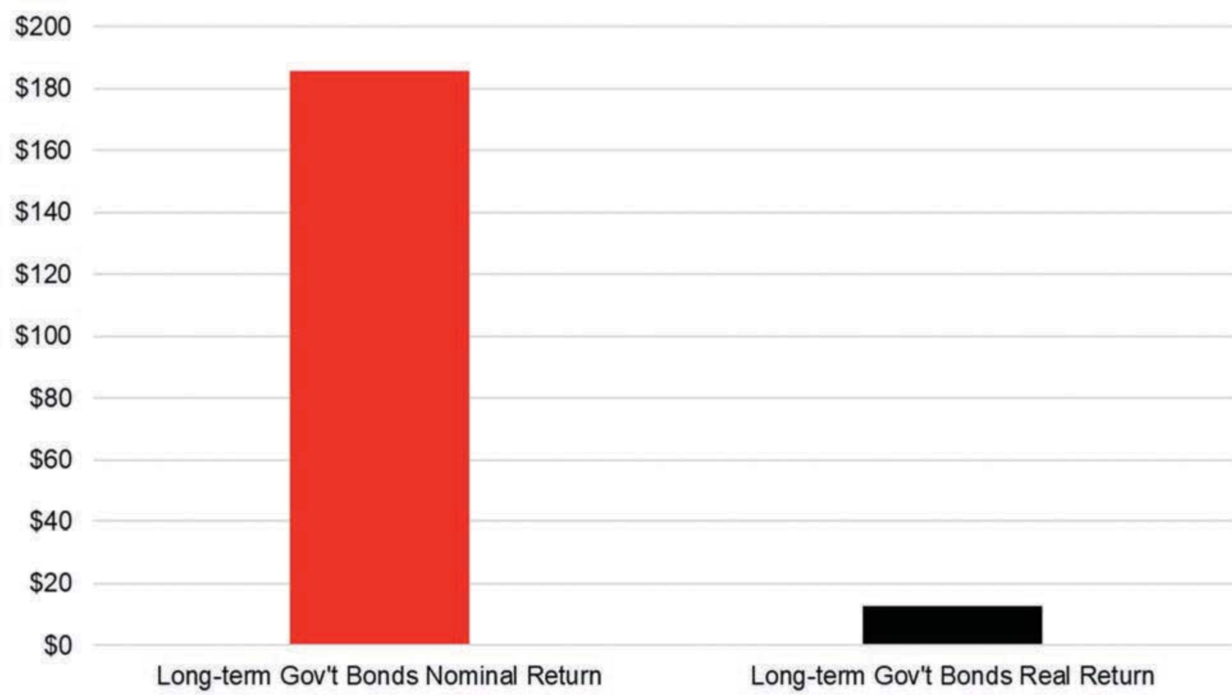
$$\frac{(1+LT\ Govt\ Bond\ TR)}{(1+Inflation)}-1$$

Because government bond returns are composed of inflation, the real riskless rate, and the horizon premium, the inflation-adjusted government bond returns may also be expressed as:

$$[(1+Real\ Riskless\ Rate)\times(1+Horizon\ Premium)]-1$$

Exhibit 4.5 depicts (i) what $1.00 invested at the end of December 1925 in long-term government bonds would have grown to by the end of 2020 and (ii) what $1.00 invested at the end of December 1925 in long-term government bonds would have grown to by the end of 2020 if long-term government bond returns were adjusted for inflation.

**Exhibit 4.5:** Long-term Government Bonds, Real and Nominal Return Terminal Index Value 1926–2020 (Year-end 1925 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset class represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, and (ii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission

## Intermediate-term Government Bond Real Returns

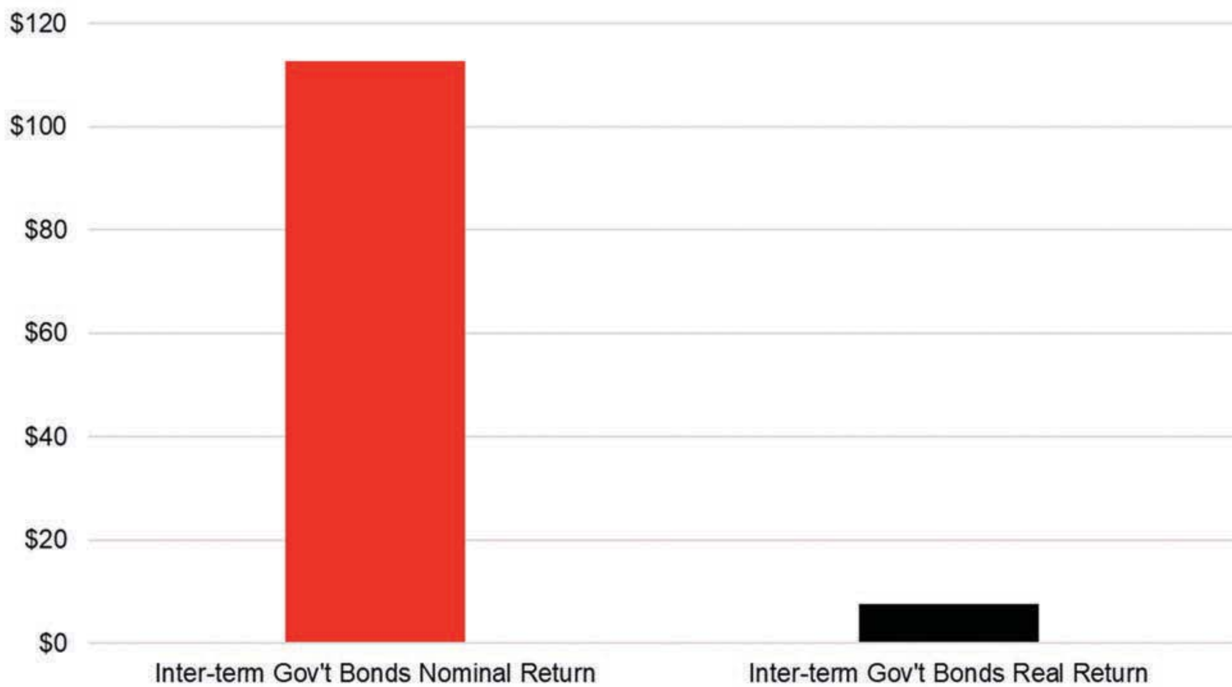
### Construction

The inflation adjusted return is a geometric difference and is approximately equal to the arithmetic difference between the intermediate-term government bond total return and the inflation rate. The monthly inflation-adjusted intermediate-term government bond return is given by:

$$\frac{(1+IT\,Govt\,Bond\,TR)}{(1+Inflation)}-1$$

Exhibit 4.6 depicts (i) what $1.00 invested at the end of December 1925 in intermediate-term government bonds would have grown to by the end of 2020, and (ii) what $1.00 invested at the end of December 1925 in intermediate-term government bonds would have grown to by the end of 2020 if intermediate-term government bond returns were adjusted for inflation.

**Exhibit 4.6:** Intermediate-term Government Bonds, Real and Nominal Return Terminal Index Value 1926–2020 (Year-end 1925 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset class represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, and (ii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, “Description of the Basic Series”. “Stocks, Bonds, Bills, and Inflation” and “SBBI” are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission

## Real Riskless Rates of Return (U.S. T-Bill Real Returns)

Treasury bills returned 3.3% compounded annually over the 1926–2020 period, in nominal terms, but only a 0.4% compound annual return in real (inflation-adjusted) terms. Thus, an investor in Treasury bills would have barely beaten inflation (or retained purchasing power) over the 95-year period.

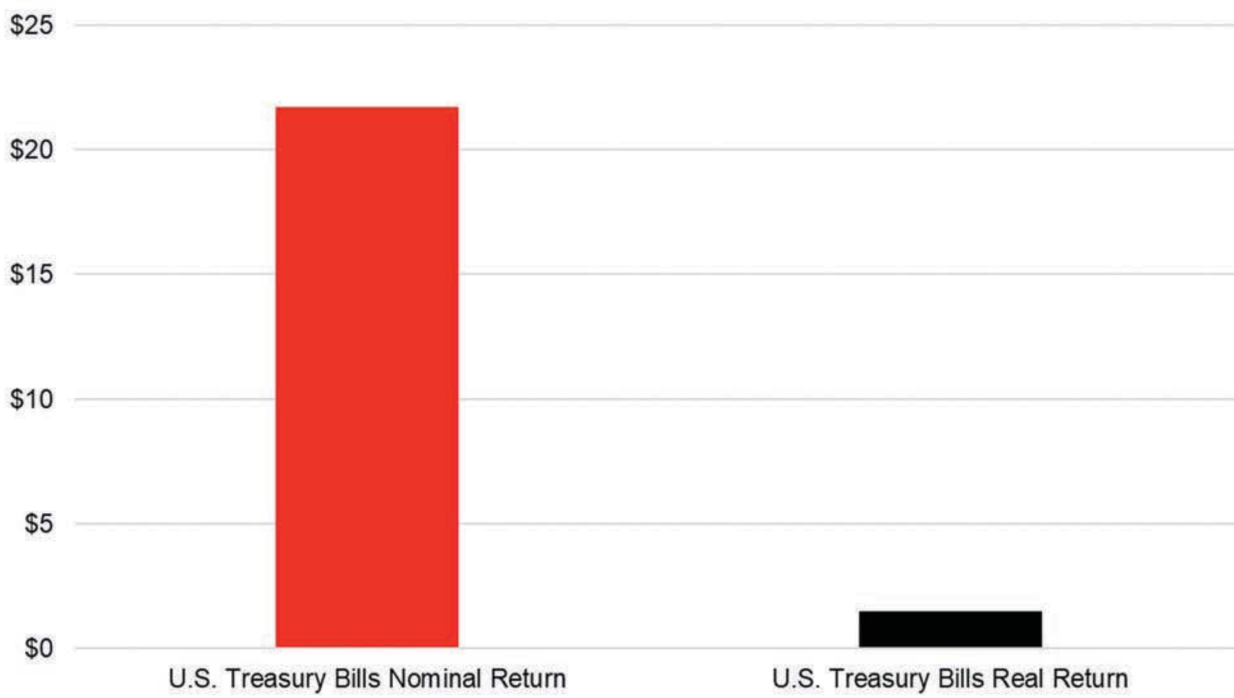
## Construction

The real riskless rate of return is the difference in returns between U.S. Treasury bills and inflation. This is given by:

$$\frac{(1+Treasury\ Bill\ TR)}{(1+Inflation)}-1$$

Exhibit 4.7 depicts (i) what $1.00 invested at the end of December 1925 in U.S. Treasury bills would have grown to by the end of 2020 and (ii) what $1.00 invested at the end of December 1925 in U.S. Treasury bills would have grown to by the end of 2020 if U.S. Treasury bill returns were adjusted for inflation.

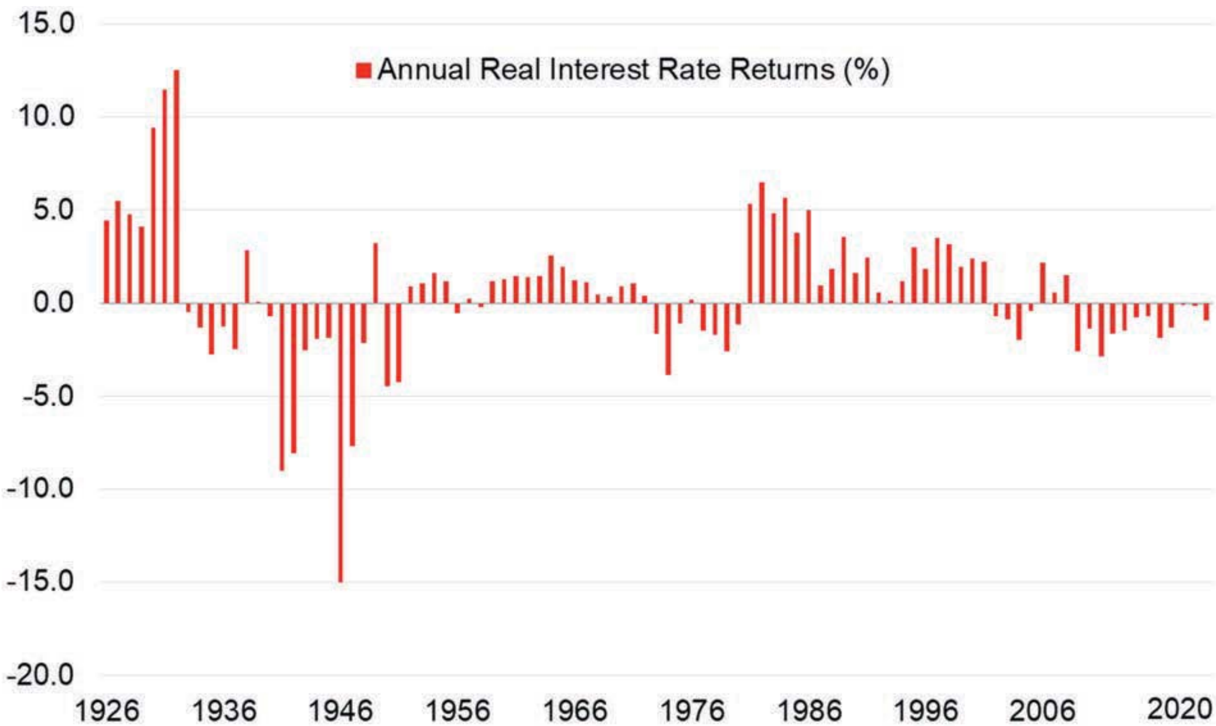
**Exhibit 4.7:** U.S. Treasury Bills, Real and Nominal Return Terminal Index Value 1926–2020 (Year-end 1925 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset class represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (ii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission

Exhibit 4.8 shows the levels, volatility, and patterns of real interest rates (inflation-adjusted U.S. Treasury Bill return) over the last 95 years.

**Exhibit 4.8:** Annual Real Interest Rate Returns (%) 1926–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset class represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission

# Chapter 5
# Annual Returns and Indexes

Returns and benchmark indexes are used to measure the rewards investors earn for holding an asset class. Indexes represent levels of wealth or prices while returns represent changes in levels of wealth. Total returns for specific asset classes can be divided into two primary components: income and capital appreciation. The income return measures the cash stream earned by holding the security, such as coupon interest for bonds or dividend payments for stocks. In contrast, the capital appreciation return results from a change in the price of the security. The method for computing a return varies with the nature of the payment (income or capital appreciation) and the period of measure (monthly or annual). Indexes are computed by establishing a base period and base value and increasing that value by the returns in successive periods. Indexes are used to illustrate the cumulative growth of wealth from holding an asset class. This chapter describes the computation of the annual returns and indexes.

The first generation of stock indexes was created to assess the market's general direction. One of the oldest and most recognizable market indexes is the Dow Jones Industrial Average, or DJIA, first published on May 26, 1896. When Charles Dow initially calculated the DJIA, which originally consisted of only 12 stocks, the process was simple: Add up the share prices of the stocks in the index and then divide by the number of stocks in the index.[137,138] In this type of index, known as a price weighted index, higher-priced stocks have a greater influence than lower-priced stocks.

However, most modern indexes are market-capitalization weighted, meaning companies with greater overall market capitalization (share price times number of shares outstanding) have a larger influence than companies with lesser market capitalization. Market-cap weighting has a strong theoretical motivation because the capital asset pricing model, or CAPM, implies in its simplest form that every investor should hold every security in proportion to its market capitalization. In contrast, price weighting lacks any theoretical motivation, so it is rarely used outside of the Dow Jones Averages (S&P Dow Jones Indices uses market-cap weighting for most of its other indexes).

Market-cap weighting is widely considered to be the central organizing principle of good index construction. Its practical advantage is that the weights adjust automatically as share prices fluctuate, eliminating the need for the frequent and expensive rebalancing that can occur with other weighting schemes. So-called strategic beta or smart beta, or other alternative weighting

[137] None of the original 12 companies listed in the DJIA remain as a component of the average. The total number of companies listed in the DJIA has not changed since 1928, when the number of companies in the index was increased to 30. For more information on the historical makeup of the DJIA, please visit the S&P Dow Jones Indices website at https://us.spindices.com/indices/equity/dow-jones-industrial-average.

[138] In June 2018, General Electric (GE) was the last company (out of the original 12) to be removed from the DIJA. See: https://www.businessinsider.com/dow-original-companies-2016-12.

schemes, seek to outperform market-cap weighted indexes but deviate from CAPM and are not macro consistent.

Market-cap weighting is usually implemented with a “float” adjustment that subtracts the number of closely-held and illiquid shares from the number of shares outstanding. A float-adjusted market-cap weighted portfolio is macro-consistent, meaning that if all investors held such a portfolio, all available shares of its constituent stocks would be held, with none left over. Accepting this is an extension of Roll’s critique, which states in part that no portfolio can capture shares of all assets (e.g., jewelry, fine wine, automobile collections, etc.) and instruments with market value, so indexes can only approximate a true diversified CAPM portfolio.[139]

With all other weighting schemes, it is mathematically impossible for all investors to hold the index portfolio. While there is wide agreement on the general principles of equity index construction, index providers differ in their methodologies that determine which stocks are selected for inclusion, the number of stocks to include, and other details. Exhibit 5.2 (at the end of the chapter) summarizes the construction methodologies of the major broad indexes of the U.S. equity market.

## Annual and Monthly Returns

### Returns on the Basic Asset Classes

Summary statistics of annual total returns of the seven basic SBBI® asset classes are discussed in Chapter 2.

### Calculating Annual Returns

Annual returns are formed by compounding the 12 monthly returns. Compounding, or linking, monthly returns is multiplying together the return relatives, or one plus the return, then subtracting one from the result. The equation is denoted as the geometric sum as follows:

$$r_{year} = \left[(1+r_{Jan})\times(1+r_{Feb})\ldots\times(1+r_{Dec})\right]-1$$

Where:

$r_{year}$ = The compound total return for the year

$r_{Jan}, r_{Feb}, \ldots, r_{Dec}$ = The returns for the 12 months of the year

139 See Roll, R. 1977. “A critique of the asset pricing theory’s tests Part I: On past and potential testability of the theory.” *Journal of Financial Economics*, Vol. 4, No. 2, P. 129.

The compound return reflects the growth of funds invested in an asset. The following example illustrates the compounding method for a hypothetical year:

| Month | Return (%) | Return (Decimal) | Return (Relative) |
|---|---|---|---|
| January | 1 | 0.01 | 1.01 |
| February | 6 | 0.06 | 1.06 |
| March | 2 | 0.02 | 1.02 |
| April | 1 | 0.01 | 1.01 |
| May | -3 | -0.03 | 0.97 |
| June | 2 | 0.02 | 1.02 |
| July | -4 | -0.04 | 0.96 |
| August | -2 | -0.02 | 0.98 |
| September | 3 | 0.03 | 1.03 |
| October | -3 | -0.03 | 0.97 |
| November | 2 | 0.02 | 1.02 |
| December | 1 | 0.01 | 1.01 |

The return for this hypothetical year is the geometric sum:

(1.01 x 1.06 x 1.02 x 1.01 x 0.97 x 1.02 x 0.96 x 0.98 x 1.03 x 0.97 x 1.02 x 1.01) – 1 = 1.0567 – 1 = 0.0567

or a gain of 5.67%. One dollar invested in this hypothetical asset at the beginning of the year would have grown to slightly less than $1.06. Note that this is different from the simple addition result, (1 + 6 + 2 + 1 – 3 + 2 – 4 – 2 + 3 – 3 + 2 + 1) = 6%

## Calculation of Returns from Index Values

Equivalently, annual returns, $r_t$, can be formed by dividing index values according to:

$$r_t = \left[\frac{V_t}{V_{t-1}}\right] - 1$$

Where:

$r_t$ = The annual return in period $t$

$V_t$ = The index value as of year-end $t$

$V_{t\text{-}1}$ = The index value as of the pervious year-end, $t$-1

The construction of index values is discussed later in this chapter in the section entitled "Calculation of Index Values."

## Calculation of Annual Income Returns

The conversion of monthly income returns to annual income returns is calculated by adding all the cash flows (income payments) for the period, then dividing the sum by the beginning period price:

$$r_I = \frac{(I_{Jan} + I_{Feb} \ldots + I_{Dec})}{P_0}$$

Where:

| | | |
|---|---|---|
| $r_I$ | = | The income return for the year |
| $(I_{Jan}, I_{Feb} \ldots I_{Dec})$ | = | The income payments for the 12 months of the year |
| $P_0$ | = | The price of the security at the beginning of the year |

The following example illustrates the method for a hypothetical year:

| Month | Beginnin of Month Price ($) | Income Return (Decimal) | Income Payment ($) |
|---|---|---|---|
| January | 100 | 0.006 | 0.60 |
| February | 102 | 0.004 | 0.41 |
| March | 105 | 0.002 | 0.21 |
| April | 101 | 0.001 | 0.10 |
| May | 99 | 0.005 | 0.50 |
| June | 103 | 0.004 | 0.41 |
| July | 105 | 0.003 | 0.32 |
| August | 103 | 0.002 | 0.21 |
| September | 105 | 0.003 | 0.32 |
| October | 103 | 0.004 | 0.41 |
| November | 106 | 0.001 | 0.11 |
| December | 105 | 0.002 | 0.21 |

Sum the income payments (not the returns), and divide by the price at the beginning of the year:(0.60 + 0.41 + 0.21 + 0.10 + 0.50 + 0.41 + 0.32 + 0.21 + 0.32 + 0.41 + 0.11 + 0.21) / 100 = 0.0381 or an annual income return of 3.81%.

Annual income and capital appreciation returns do not sum to the annual total return. The difference may be viewed as a reinvestment return, which is the return from investing income from a given month into the same asset class in subsequent months within the year.

## Index Values

Index values represent the *cumulative* (i.e., compound) effect of investment returns. For example, in 1926 the total return (i.e., with dividends reinvested) of large-cap stocks was approximately 11.6%. A hypothetical investor investing $1.00 as of December 31, 1925, in large-cap stocks would have seen their investment grow to approximately $1.12 ($1.00 x (1+ 0.116)) by the end of 1926. During the following year (1927), the total return of large-cap stocks was approximately 37.5%. The $1.12 our hypothetical investor began 1927 with would have grown to approximately $1.53 ($1.12 x (1 + 0.375)) by the end of 1927. This can also be calculated as:

$1.53 = $1 x (1+ 0.116) x (1 + 0.375)

Of course, the cumulative effect over the entire 1926–2020 period can also be calculated as follows:

$\$1.00 \times (1+ r_{1926}) \times (1 + r_{1927}) \ldots \times (1 + r_{2020})$

Where "$r_{year}$" is the total return in a given year.

Following this methodology, the $1.00 invested in large-cap stocks at year-end 1925 by our hypothetical investor would have grown to nearly $11,000 by the end of 2020. Such growth reveals the power of compounding (reinvesting) one's investment returns.

Year-end index levels (based upon $1.00 invested at the end of 1925) for all six SBBI® asset classes plus inflation are displayed in Exhibit 5.1 as of year-end (i.e., December 31) 2010, 2011, and 2012.[140] Exhibit 5.1 also includes year-end index levels for the capital appreciation ("Capital App") component of total return for large-cap stocks, long-term government bonds, and intermediate term government bonds.

Note that the capital appreciation component of total return is generally a small contributor to total return over the longer term. For example, $1.00 invested at the end of 1925 in large-cap stocks would have grown to $3,532.55 by the end of 2012, but capital appreciation contributed only $111.77 to total return, or about 3.2% ($111.77 ÷ $3,532.55). This implies that the other two

---

140 2010–2012 was selected for example purposes only. *Precalculated* monthly and annual index values from January 1926 through December 2019 are presented in table format in the full-version *2020 SBBI® Yearbook* for the seven SBBI® total return series, and for all accompanying SBBI® capital appreciation series, as follows: Large-Capitalization Stocks: Capital Appreciation Index, Small-Capitalization Stocks: Total Return Index, Long-term Corporate Bonds: Total Return Index, Long-term Government Bonds: Total Return Index, Long-term Government Bonds: Capital Appreciation Index, Intermediate-term Government Bonds: Total Return Index, Intermediate-term Government Bonds: Capital Appreciation Index, U.S. Treasury Bills: Total Return Index, Inflation Index. For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

components of total return (dividend return and reinvestment return) contributed approximately 96.8% to the total return of large-cap stocks over the 1926–2012 period.

**Exhibit 5.1:** SBBI® Series Terminal Index Values as of Year- 2010, 2011, and 2012 (Year-end 1925 = $1.00)

| | Large-Cap Stocks | | Small-Cap Stocks | Long-term Corp Bonds | Long-term Gov't Bonds | |
|---|---|---|---|---|---|---|
| Year | Total Returns | Capital App | Total Returns | Total Returns | Total Returns | Capital App |
| 2010 | 2,982.23 | 98.56 | 16,054.70 | 133.38 | 92.94 | 1.12 |
| 2011 | 3,045.21 | 98.56 | 15,532.07 | 157.32 | 118.13 | 1.38 |
| 2012 | 3,532.55 | 111.77 | 18,364.60 | 174.12 | 122.18 | 1.39 |
| etc… | | | | | | |

| | Inter-term Gov't Bonds | | U.S. Treasury Bills | Inflation* |
|---|---|---|---|---|
| Year | Total Returns | Capital App | Total Returns | Inflation |
| 2010 | 84.12 | 1.60 | 20.55 | 12.24 |
| 2011 | 91.53 | 1.71 | 20.56 | 12.61 |
| 2012 | 93.05 | 1.73 | 20.57 | 12.83 |
| etc… | | | | |

* NOTE: On February 26, 2021 Morningstar revised the "IA SBBI US Inflation" series. The revisions were applied to various dates from February 1926 through December 2020. The revisions were small and did not materially affect long-term averages.

### Calculation of Index Values

It is possible to mathematically describe the nature of the index values in Exhibit 5.1 precisely. These indexes are initialized as of December 31, 1925, at $1.00 (represented by $V_0$ in the equation below). At the end of each month, a cumulative wealth index ($V_n$) for each of the monthly return series is formed. This index is formed for month *n* by taking the product of one plus the returns each period, in the following manner:

$$V_n = V_0 \left[ \prod_{t=1}^{n} (1 + r_t) \right]$$

Where:

| | | |
|---|---|---|
| $V_n$ | = | The index value at end of period *n* |
| $V_0$ | = | The initial index value at time 0 |
| $r_t$ | = | The return in period *t* |

**Using Index Values for Performance Measurement**

Index values can be used to determine whether an investment portfolio accumulated more wealth over a period than another portfolio would have done, or whether the investment performed as well as an industry benchmark. In the following example, which produced more wealth: the "investor portfolio" or a hypothetical S&P 500 index fund returning exactly the S&P total return?[141]

| | Investor Portfolio (%) | S&P 500 (%) |
|---|---|---|
| January 1990 | -5.35 | -6.71 |
| February 1990 | 0.65 | 1.29 |
| March 1990 | 0.23 | 2.65 |
| Accumulated Wealth | $0.955 | $0.970 |

Taking December 1989 as the base period (i.e., $1.00 invested at the end of 1989) and using the computation method described above, the S&P 500 *outperformed* the investor portfolio.

**Computing Returns for Non-Calendar Periods**

Index values are also useful for computing returns for non-calendar-year periods. For example, using the index values in Exhibit 5.1 for the "Investor Portfolio," the return over the January 1990 to March 1990 period can be calculated by dividing the index value at the end of March 1990 ($0.955) by the starting index value as of December 1998 ($1.00), and subtracting 1. This yields:

($0.955 ÷ $1.00) – 1 = -4.5%.

The same calculation can be performed for the S&P 500 Index over the January 1990 to March 1990 period:

($0.970 ÷ $1.00) – 1 = -3.0%.

[141] In this example, each index measures total return and assumes monthly reinvestment of dividends.

## Inflation-Adjusted Returns and Indexes

Inflation-adjusted returns and indexes can be used to measure investors' real returns and real changes in wealth over time.[142,143]

For example, a hypothetical investment of $1.00 in SBBI® large-cap stocks over the most recent 10-year period (2011–2020) would have grown to $3.67 by December 31, 2020 in nominal terms, and $3.09 in real (i.e., inflation-adjusted) terms. This demonstrates that investors in large-cap stocks multiplied their nominal wealth over the 2011–2020 period by a factor of 3.67 ($3.67 ÷ $1.00) and their real wealth (i.e., purchasing power) over the 2011–2020 period by a factor of 3.09 ($3.09 ÷ $1.00).

## Overview of Major Broad Market U.S. Equity Indexes

The "market" is typically represented by a broad-based equity index that includes a wide range of industries and arguably behaves like the market as a whole. The SBBI large-cap stock series (essentially the S&P 500 index) is one example of this. Exhibit 5.2 provides an overview of some additional major broad market U.S. equity indexes.

142 The calculation of inflation-adjusted returns is discussed in Chapter 4, "Description of the Derived Series".

143 *Precalculated* annual inflation-adjusted index values (Year-end 1925 = $1.00) for all years 1926–2020 are presented in table format in the full-version *2021 SBBI® Yearbook* for the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, and (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD. For more information, visit: dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

**Exhibit 5.2:** Major Broad Market U.S. Equity Indexes*

| | Index Family | | | |
|---|---|---|---|---|
| | **Morningstar** | **MSCI** | **Russell** | **S&P Dow Jones** |
| **Broad Market Index** | Morningstar U.S. Market Index | MSCI Investable Market | Russell 3000 | S&P Composite 1500** |
| **Percent U.S. Market Cap Coverage** | 97% | >99% | 98% | 90% |
| **Total Number of Stocks** | 1441 | 2375 | 3000 | 1500 |
| **Transparent, Rules-Based Methodology** | Yes | Yes | Yes | Yes |
| **Eligibility** | Stocks of companies domiciled in the U.S. listed on the NYSE, NYSE MKT, or NASDAQ | Stocks of companies domiciled in the U.S. listed on the NYSE, NYSE MKT, NYSE Arca, or NASDAQ | Stocks of the largest 3000 companies domiciled in the U.S. listed on a U.S. exchange | Stocks of companies domiciled in the U.S. listed on the NYSE, NYSE MKT, NYSE Arca, or NASDAQ chosen for market size, liquidity, and industry group representation by the S&P Index Committee |
| **Exclusion Criteria** | ADRs Limited Partnerships, Investment Trusts (except REITs), and Holding Companies | ADRs, Fixed Dividend Shares, Convertible Notes, Warrants, and Rights, Limited Partnerships, Master Limited Partnerships, LLCs, Business Development Companies, Pooled Investment Vehicles, Royalty Trusts and Investment Trusts (except REITs) | ADRs Limited Partnerships, Closed-end Mutual Funds, Price < $1, and Royalty Trusts and LLCs | ADRs Limited Partnerships, Investment Trusts (except REITs), Tracking Stocks and Holding Companies, and Royalty Trusts and LLCs |
| **Market Cap Cutoff Method** | Market Cap Percent | Fixed Number of Stocks | Fixed Number of Stocks | Fixed Number of Stocks |
| **Unique Cap Cutoff Method** | Yes | Yes | Yes | Yes |
| **Unique Style Classification** | Yes | No, stocks may be included in more than one style index | No, stocks may be included in more than one style index | No, stocks may be included in more than one style index |
| **Core Style Index** | Yes | No | No | No |
| **Reconstitution Frequency** | Semiannual | Semiannual | Annual | Ad hoc |

*The broad market indices show n in Exhibit 5.3 can be disaggregated into capitalization and style indices. For example, the S&P Composite 1500 can be disaggregated into the S&P 500 (large-cap stocks), S&P 400 (mid-cap stocks), and the S&P 600 (small-cap stocks).

**The market for U.S. large-cap stocks is represented by the S&P 500 throughout the Ibbotson® SBBI® Yearbook series.

# Chapter 6
# Statistical Analysis of Returns

Statistical analysis of historical asset returns can reveal the growth rate of wealth invested in an asset or portfolio, the riskiness or volatility of asset classes, the co-movement of assets, and the random or cyclical behavior of asset returns. This chapter focuses on arithmetic and geometric mean returns, standard deviations, and serial and cross-correlation coefficients, and discusses the use of each statistic to characterize the various asset classes by growth rate, variability, and safety.

## Calculating Arithmetic Mean Return

The arithmetic mean of a series is the simple average of the elements in the series. The arithmetic mean return equation is:

$$r_A = \frac{1}{n}\sum_{t=1}^{n} r_t$$

Where:

$r_A$ = The arithmetic mean return

$r_t$ = The series return in period $t$, that is, from time $t$ - 1 to time $t$

$n$ = The inclusive number of periods

## Calculating Geometric Mean Return

The geometric mean of a return series over a period is the compound rate of return over the period. The geometric mean return equation is:

$$r_G = \left[\prod_{t=1}^{n}(1+r_t)\right]^{\frac{1}{n}} - 1$$

Where:

$r_G$ = The geometric mean return

$r_t$ = The series return in period $t$

= The inclusive number of periods

The geometric mean return can be restated using beginning and ending period index values. The equation is:

$$r_G = \left[\frac{V_n}{V_0}\right]^{\frac{1}{n}} - 1$$

Where:

$r_G$ = The geometric mean return

$V_n$ = The ending period index value at time $n$

$V_0$ = The initial index value at time 0

$n$ = The inclusive number of periods

The annualized geometric mean return over any period of months can also be computed by expressing $n$ as a fraction. For example, the beginning of 2020 to the end of May 2020 is equivalent to five twelfths of a year, or 0.4167. $V_n$ would be the index value at the end of May 2020, $V_0$ would be the index value at the beginning of 2020, and $n$ would be 0.4167.

## Geometric Mean Versus Arithmetic Mean

A simple example illustrates the difference between geometric and arithmetic means. Suppose $1.00 was invested in a large-cap stock portfolio that experiences successive annual returns of 50% and negative 50%. At the end of the first year, the portfolio is worth $1.50 and at the end of the second year, it is worth $0.75. The annual arithmetic mean is 0.0%, whereas the annual geometric mean is -13.4%. Both are calculated as follows:

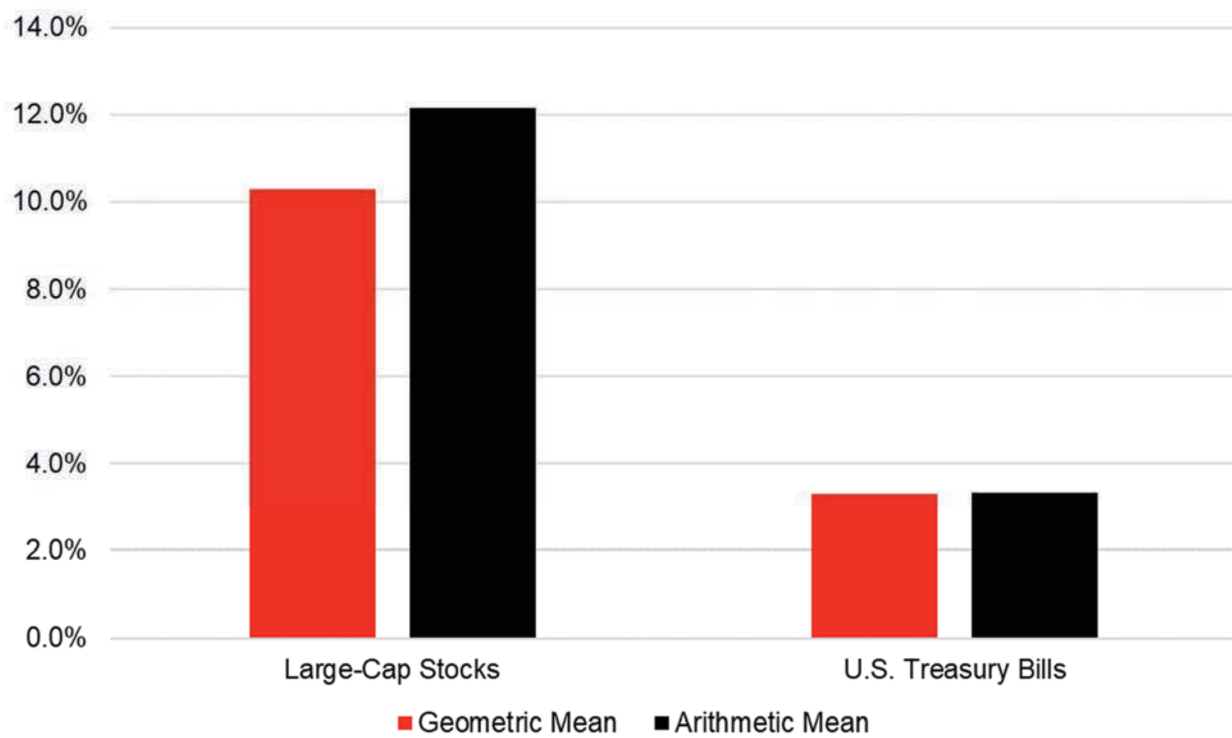
$$r_A = \frac{1}{2}(0.50 - 0.50) = 0.00$$

$$r_G = \left[\frac{0.75}{1.00}\right]^{\frac{1}{2}} - 1 = -0.134$$

The geometric mean is backward-looking, measuring the change in wealth over more than one period. On the other hand, the arithmetic mean better represents a typical performance over a single period.

In general, the geometric mean for any period is less than or equal to the arithmetic mean. The two means are equal only for a return series that is constant (i.e., the same return in every period). For a non-constant series, the difference between the two is positively related to the variability or standard deviation of the returns. For example, in Exhibit 6.1 the difference between the arithmetic and geometric mean is much larger for risky large-cap stocks than it is for nearly riskless Treasury bills. This is because the "variability" (as measured by annual standard deviation) of large-cap stock returns is nearly 20%, while the standard deviation of Treasury bill returns is much lower, at just over 3%.

**Exhibit 6.1:** Comparison of the Annual Geometric (i.e., Compound) Mean Return and Annual Average Return of Large-Cap Stocks and U.S. Treasury Bills (%)
1926–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes are represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, and (ii) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## Calculating Standard Deviation

The standard deviation of a series is a measure of the extent to which observations in the series differ from the arithmetic mean of the series. For a series of asset returns, the standard deviation is a measure of the volatility, or risk, of the asset.

In a normally distributed series, about two-thirds of the observations lie within one standard deviation of the arithmetic mean; about 95% of the observations lie within two standard deviations; and more than 99% lie within three standard deviations.

For example, the standard deviation for large-cap stock returns from 1926 to 2020 was 19.7% with an annual arithmetic mean of 12.2%. Therefore, roughly two-thirds of the observations have annual returns between -7.5% and 31.9% (12.2% plus or minus 19.7%); approximately 95% of the observations are between -27.2% and 51.6% (12.2% plus or minus 39.4%).

The equation for the standard deviation of a series of returns ($\sigma_r$) is:

$$\sigma_r = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - r_A)^2}$$

Where:

$r_t$ = The return in period $t$

$r_A$ = The arithmetic mean of the return series $r$

$n$ = the number of periods

The scaling of the standard deviation depends on the frequency of the data; therefore, a series of monthly returns produces a monthly standard deviation. For example, using the monthly returns for the hypothetical year on page 5-3, a monthly standard deviation of 2.94% calculated as follows:

$$0.0294 = \frac{1}{12-1} \times \begin{bmatrix} (0.01-0.005)^2 + (0.06-0.005)^2 + (0.02-0.005)^2 + \\ (0.01-0.005)^2 + (\text{-}0.03-0.005)^2 + (0.02-0.005)^2 + \\ (\text{-}0.04-0.005)^2 + (\text{-}0.02-0.005)^2 + (0.03-0.005)^2 + \\ (\text{-}0.03-0.005)^2 + (0.02-0.005)^2 + (0.01-0.005)^2 \end{bmatrix}^{\frac{1}{2}}$$

It is sometimes useful to express the standard deviation of the series in another time scale. To calculate *annualized* monthly standard deviations ($\sigma_n$) one uses the following equation.[144]

$$\sigma_n = \sqrt{[\sigma_1^2 + (1+\mu_1)^2]^n - (1+\mu_1)^{2n}}$$

144 The equation appears in Levy, H. & Gunthorpe, D. 1993. "Optimal Investment Proportions in Senior Securities and Equities Under Alternative Holding Periods." *Journal of Portfolio Management*, Vol. 19, No. 4, P. 33.

Where:

$n$ = The number of periods per year, e.g., 12 for monthly, 4 for quarterly, etc.

$\sigma_1$ = The monthly standard deviation

$\mu_1$ = The months arithmetic mean

Applying this formula to the prior monthly standard deviation of 2.94% results in an annualized monthly standard deviation of 10.78%. The *annualized* monthly standard deviation is calculated as follows:

$$\sqrt{\left[0.0294^2+(1+0.005)^2\right]^{12}-(1+0.005)^{2(12)}}$$

This equation is the exact form of the common *approximation*:

$$\sigma_n \approx \sqrt{n}\sigma_1$$

The "approximation" treats an annual return as if it were the sum of 12 *independent* monthly returns, whereas the "exact form" treats an annual return as the *compound* return of 12 independent monthly returns. While the approximation can be used for "back of the envelope" calculations, the exact formula should be used in applications of quantitative analysis. Forming inputs for mean-variance optimization is one such example. Note that both the exact formula and the approximation assume that there is no monthly autocorrelation.

## Limitations of Standard Deviation[145]

Using the statistical measure of standard deviation of returns is clearly the easiest way to mathematically express the concept of risk. However, practitioners and academics alike have noted that standard deviation misses important and essential qualities of risk from the standpoint of an investor of capital.

One limitation of standard deviation as a measure of risk is the tacit assumption that returns can be described by a measure that assumes a normal distribution of returns, while it is empirically acknowledged that many financial market returns exhibit excess kurtosis relative to the normal (Gaussian) distribution. This characteristic is referred to as a leptokurtic, or "fat-tailed," return distribution. Fat-tailed outcomes reflect market movements far larger than one would reasonably expect from a normal distribution of returns. One of the most extreme examples of a fat-tailed return profile occurred on Oct. 19, 1987, when the Dow Jones Industrial Average declined by 22.68%, or more than 20 standard deviations. The magnitude of the deviation from normal returns

[145] The Limitations of Standard Deviation, the Semi-Variance and Semi-Standard Deviation, and the Issues Regarding Semi-Variance sections were written by Erik Kobayashi-Solomon and Philip Guziec.

can be understood when considering that a normal distribution would predict such a move once in more than 4.5 billion years. More recently, 2008 had 11 days with declines greater than 4 standard deviations, and on May 6, 2010, the Dow Jones Industrial Average declined by 9% in a matter of minutes on an intraday basis, a move that on a daily basis would have been among the top 10 declines in recorded history. Clearly, an awareness of the nature of statistical descriptions of market moves beyond standard deviation is helpful in developing a representative profile of market risk. As Laurence B. Siegel wrote in July 2018:[146]

"Although most of classical finance focuses only on risk and expected return, investors differ in their tastes and preferences and assets differ in their characteristics other than risk and return."

An excellent recent book discusses how investor preferences for various assets and premia, broadly characterized as the "popularity" of these characteristics, influence returns. In 2019, Roger G. Ibbotson and colleagues Thomas M. Idzorek, CFA, Paul D. Kaplan, CFA, and James X. Xiong, CFA, published a new CFA Institute Research Foundation monograph entitled *Popularity: A Bridge Between Classical and Behavioral Finance* (available for download at: https://www.cfainstitute.org/en/research/foundation/2018/popularity-bridge-between-classicaland- behavioral-finance).[147]

## Semivariance and Semistandard Deviation

Given academic and practitioner concerns about variance, various approaches have been suggested to more appropriately measure risk. We take a moment here to briefly discuss investor perception of risk and to review another measure, semi variance.

One criticism of variance and standard deviation is that an investor is less worried about bidirectional variation in value (the essence of the standard deviation measure) than about an ultimately unrecoverable shortfall in investment capital. In considering risk from this point of view, two cases stand out as the most salient: (i) suffering a realized or mark-to-market loss of capital that prevents fulfillment of a goal or mandate over an investment time frame and (ii) allocating capital in investments that appreciate too little to fulfill a goal or mandate over the investment time frame. The former case involves an excess of variation in an unacceptable direction; the latter case involves a paucity of variation to an acceptable magnitude.

Of these two cases, most academic work has focused on developing a framework to accurately measure and analyze directionally-specific variance. Foremost in this attempt has been the concept of semivariance.

Semivariance characterizes the downside risk of a distribution and focuses on the portion of risk that is below (to the left of) the mean or a specific target. For example, for a 4% target return, the

---

146 Laurence B. Siegel is the Gary P Brinson Director of Research at the CFA Institute Research Foundation. The quote is from the Forward of *Popularity – A Bridge between Classical and Behavioral Finance*, by Roger G Ibbotson, Thomas M. Idzorek, CFA, Paul D. Kaplan, CFA, and James X. Xiong, CFA

147 Or, go to the CFA Institute website at cfainstitute.org and search for "popularity".

semivariance describes the variance of the data points below (to the left of) the return of 4%. The semivariance below the mean uses the mean return as the target return. The semistandard deviation is simply the square-root of the semivariance. The semivariance (semistandard deviation) is always lower than the total variance (standard deviation) of the distribution.

$$SV_t = \frac{1}{n} \sum_{r_t < r_T}^{n} (r_T - r_t)^2$$

$$SV_t = \frac{1}{n} \sum_{r_t < r_T}^{n} (r_T - r_t)^2$$

$$SSTD_m = \sqrt{SV_m}$$

$$SSTD_t = \sqrt{SV_t}$$

Where:

| | | |
|---|---|---|
| $SV_m$ | = | The semi-variance below mean |
| $SV_t$ | = | The semi-variance below target |
| $r_A$ | = | The arithmetic mean return |
| $r_t$ | = | The series return in period *t* |
| $r_T$ | = | The target selection return |
| $n$ | = | The inclusive number of periods |
| $SSTD_m$ | = | The semi-standard deviation below mean |
| $SSTD_t$ | = | The semi-standard deviation below target |

**Issues Regarding Semivariance**

While semivariance seems to intuitively address issues regarding directionality, it does have empirical, theoretical, and practical shortcomings. Empirically, when returns are measured over relatively short time frames, distributions tend to be symmetric. As such, using semivariance for short time frames effectively gives no extra explanatory power (because semivariance simply equates to one half of the variance) and, in fact, limits the data available for analysis (because the calculation of semivariance discards any positive return observations). When returns are measured over relatively longer time frames (on the order of a year or more), asset returns tend to follow a distribution that is positively skewed. As such, for investors with longer time horizons, semivariance has less explanatory power because the data set is limited to the less germane case, while the richer part of the data set is discarded.

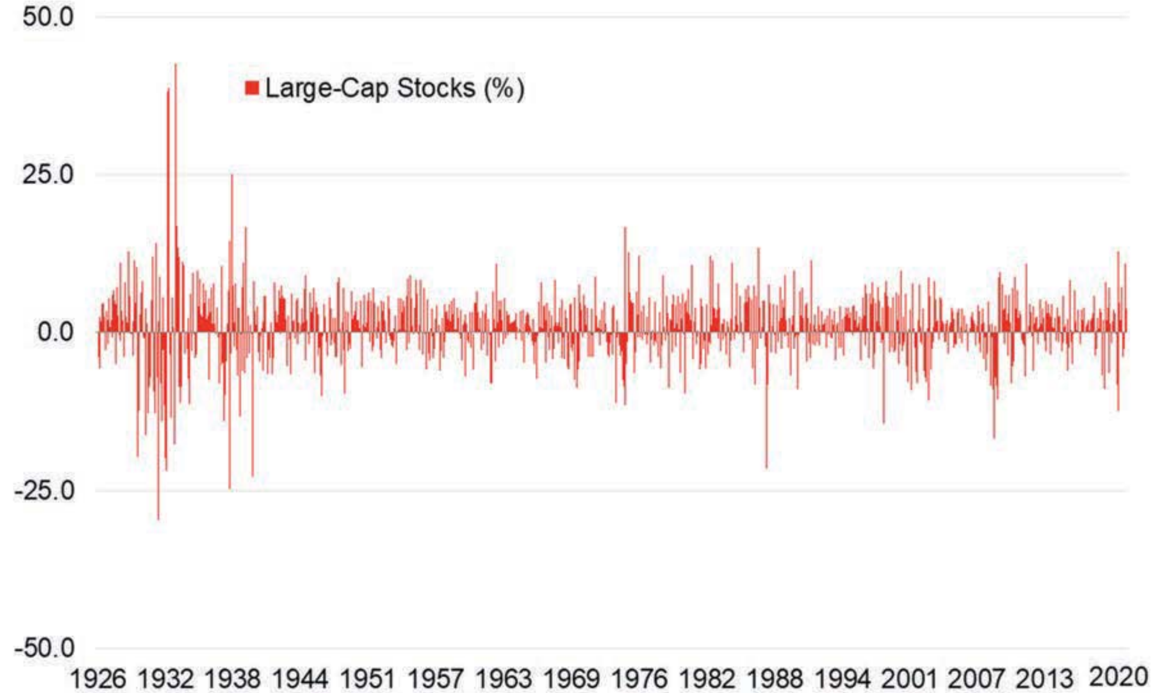
From a theoretical standpoint, the assumption implicit in the calculation of semivariance – investors do not care about positive variance – has repercussions regarding investor utility functions. Namely, ignoring positive variation implies that an investor is indifferent when presented with the choice between making an uncertain but positive return bet and making a bet that is certain to generate the expected payoff. For example, investors would, under the assumptions of semivariance, be agnostic between a 50-50 bet of generating either 5% or 10% and a sure bet paying 7.5%.

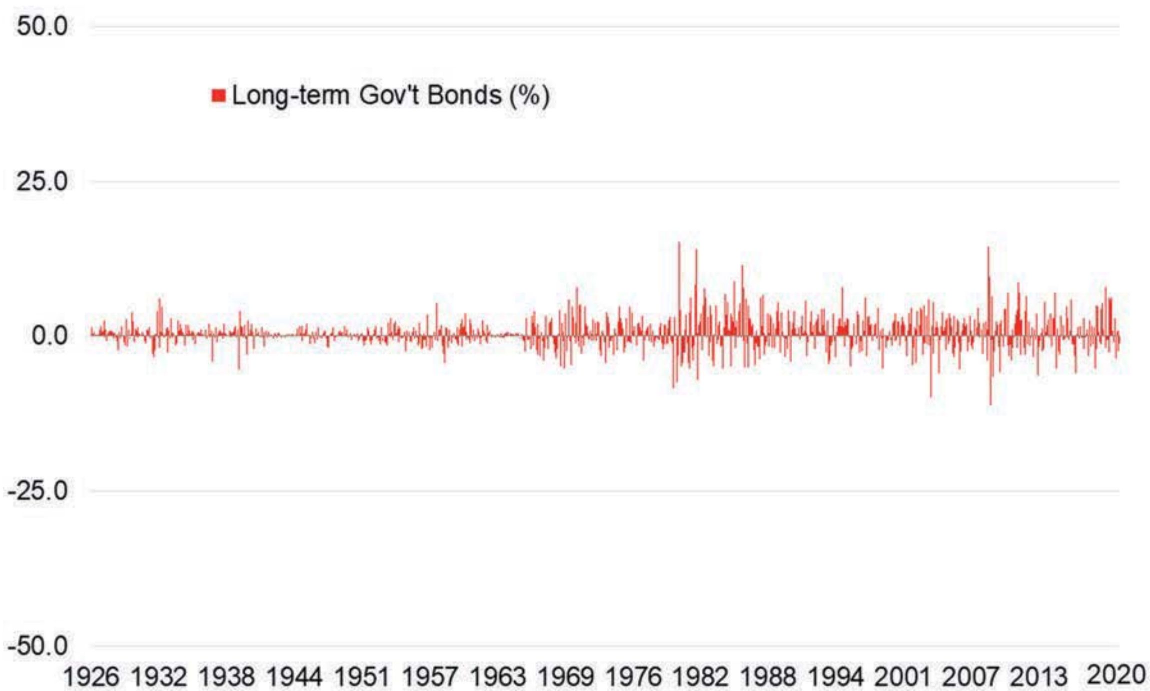
Practically speaking, ignoring upside variation means that we ignore the second aspect of risk mentioned above – a paucity of magnitude. In other words, if one attempts to minimize semivariance, without regard to the degree to which an asset or allocation has upside potential, one runs the risk of generating returns which, while low in downside variance, are also low in upside variance. In this case, one has protected oneself from one class of risk by taking on yet another. Given these issues, semivariance has met with limited acceptance among academics and practitioners alike.

## Volatility of the Markets

The volatility of stocks and long-term government bonds is shown by the bar graphs of monthly returns in Exhibit 6.2. The stock market was tremendously volatile in the first few years studied; this period was marked by the 1920s boom, the crash of 1929–1932, and the Great Depression years. The market seemingly settled after World War II and provided more stable returns in the postwar period. In the 1970s and 1980s, stock market volatility increased, but not to the extreme levels of the 1920s and 1930s. In the 1990s, 2000s, and 2010s, volatility was relatively moderate. Bonds present a mirror image. Long-term government bonds were extremely stable in the 1920s and remained so through the crisis years of the 1930s, providing shelter from the storms of the stock markets. Starting in the late 1960s and early 1970s, however, bond volatility soared; in the 1973–1974 stock market decline, bonds did not provide the shelter they once did. Bond pessimism (i.e., high yields) peaked in 1981 and subsequent returns were sharply positive. While the astronomical interest rates of the 1979–1981 period have passed, the volatility of the bond market remains higher.

**Exhibit 6.2:** Month-by-Month Returns on Stocks and Bonds (%) 1926–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## Changes in the Risk of Assets Over Time

Another time series property of great interest is change in volatility or riskiness over time. Such change is indicated by the standard deviation of the series over different sub-periods. Exhibit 6.3 shows the annualized monthly standard deviations of the basic data series by decade beginning in 1926 and illustrates differences and changes in return volatility. In this exhibit, the 1920s cover the period 1926–1929. Equity returns have been the most volatile of the basic series, with volatility peaking in the 1930s due to the instability of the market following the 1929 market crash. The significant bond yield fluctuations of the 1980s caused the fixed-income series' volatility to soar compared to prior decades. Small cap stocks were the *most* volatile SBBI® asset class in all time periods shown in Exhibit 6.3. Treasury bills were the *least* volatile SBBI® asset class in all time periods shown in Exhibit 6.3.

Exhibit 6.3 displays the *annualized* standard deviation of the *monthly* returns on each of the basic and derived series from January 1926 to December 2020. The estimates in Exhibit 6.3 are not strictly comparable to Exhibits 2.4 or 2.8 where the 95-year period standard deviation of *annual* returns around the 95-year annual arithmetic mean is reported. The arithmetic mean drifts for a series that does not follow a random pattern. A series with a drifting mean will have much higher deviations around its long-term mean than it has around the mean during a particular calendar year.

**Exhibit 6.3:** Annualized Monthly Standard Deviations by Decade (%)

| | 1920s* | 1930s | 1940s | 1950s | 1960s | 1970s | 1980s | 1990s | 2000s | 2010s |
|---|---|---|---|---|---|---|---|---|---|---|
| Large-Cap Stocks | 23.9 | 41.6 | 17.5 | 14.1 | 13.1 | 17.1 | 19.4 | 15.9 | 16.3 | 14.1 |
| Small-Cap Stocks | 24.7 | 78.6 | 34.5 | 14.4 | 21.5 | 30.8 | 22.5 | 20.2 | 26.1 | 19.6 |
| Long-term Corp Bonds | 1.8 | 5.3 | 1.8 | 4.4 | 4.9 | 8.7 | 14.1 | 6.9 | 11.7 | 9.0 |
| Long-term Gov't Bonds | 4.1 | 5.3 | 2.8 | 4.6 | 6.0 | 8.7 | 16.0 | 8.9 | 12.4 | 10.9 |
| Inter-term Gov't Bonds | 1.7 | 3.3 | 1.2 | 2.9 | 3.3 | 5.2 | 8.8 | 4.6 | 5.2 | 3.2 |
| U.S. Treasury Bills | 0.3 | 0.2 | 0.1 | 0.2 | 0.4 | 0.6 | 0.9 | 0.4 | 0.6 | 0.2 |
| Inflation | 2.2 | 2.6 | 3.1 | 1.3 | 0.8 | 1.3 | 1.3 | 0.7 | 1.6 | 1.0 |

*Based on the period 1926–1929

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

Large-cap stocks and equity risk premiums have virtually the same annualized monthly standard deviations because there is very little deviation in the U.S. Treasury bill series (the monthly equity risk premium is calculated as [(1+ Large Stock total return) ÷ (1+Treasury Bill total return) – 1], as described in Exhibit 4.1). The series with drifting means (U.S. Treasury bills, inflation rates, and inflation-adjusted U.S. Treasury bills) all tend to have very low annualized monthly standard deviations since these series are quite predictable from month to month. There is much less predictability for these series over the long term. Because it is difficult to forecast the direction and magnitude of the drift in the long-term mean, these series have higher standard deviations over the long term in comparison to their annualized monthly standard deviations.[148]

Equity investors may have the impression that periods of economic turmoil mainly affect equity markets, but this is not true. For example, during the 2008–2009 global financial crisis the price fluctuation of bonds in general and long-term corporate bonds in particular was dramatic. The annualized standard deviation of long-term corporate bonds recorded an all-time high value of 25.5% in 2008, a year that saw the collapse of storied investment banks Lehman Brothers and Bear Sterns. The previous record annualized standard deviation for long-term corporate bonds of 20.2% (measured during 1981) is more than one-fifth lower than the 2008 value, and more than twice the Depression era record of 11.7%, recorded in 1933. Another mark of the severity of bond price fluctuations can be seen by noting that the annualized standard deviations for long-term corporate bonds in 2008 were only three percentage points shy of the annualized standard deviations for the S&P 500 “Large-Cap Stocks” in 2009.

The annualized monthly standard deviation of all of the SBBI® asset classes increased in 2020 compared to 2019, but long-term corporate and U.S. government bond volatility was still significantly lower than the levels reached in 2008. Equity volatility, however, reached levels not seen for decades. The annualized monthly standard deviation of large-cap stocks in 2020 reached 31.6%, a level not seen since 1987. The price volatility of large-cap stocks in 2020 was the fifth highest on record with 1933’s 99.8% being the highest. The annualized monthly standard deviation of small-cap stocks in 2020 reached 43.2%. The price volatility of small-cap stocks in 2020 was the twelfth highest on record with 1933’s 286.6% being the highest.

## Correlation Coefficients: Serial and Cross-Correlations

The behavior of an asset return series over time reveals its predictability. For example, a series may be random or unpredictable, or it may be subject to trends, cycles, or other patterns, making the series predictable to some degree. The serial correlation coefficient of a series determines its predictability given knowledge of the last observation. The cross-correlation coefficient (often shortened to “correlation”) between two series determines the predictability of one series, conditional on knowledge of the other.

---

148 *Precalculated* annualized monthly standard deviations (1926–2020) are presented in table format in the full-version *2021 SBBI® Yearbook* for the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series and derived series, as follows: Large-Cap Stocks, Small-Cap Stocks, Long-term (i.e., 20-year) Corporate Bonds, Long-term (i.e., 20-year) Government Bonds, Intermediate-term (i.e. 5-year) Gov't Bonds, U.S. (30-day) Treasury Bills, Inflation, Equity Risk Premium, Small Stock Premium, Bond Default Premium, Bond Horizon Premium, Inflation-adjusted T-Bills (i.e., real interest rates), For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

### Serial Correlations

The serial correlation of a return series, also known as the first order autocorrelation, describes the extent to which the return in one period is related to the return in the next period. A return series with a high (near 1) serial correlation is very predictable from one period to the next, while one with a low (near 0) serial correlation is random and unpredictable.

The serial correlation of a series is closely approximated by the equation for the cross correlation between two series. The data, however, are the series and its "lagged" self. For example, the lagged series is the series of one-period-old returns:

| Period | Return Series | Lagged Return Series |
|---|---:|---:|
| 1 | 0.10 | undefined |
| 2 | -0.10 | 0.10 |
| 3 | 0.15 | -0.10 |
| 4 | 0.00 | 0.15 |

### Cross Correlations

The cross-correlation between two series measures the extent to which they are linearly related.[149] The correlation coefficient measures the sensitivity of return on one asset class or portfolio to the return of another. The correlation equation between return series X and Y is:

$$p_{X,Y} = \left[ \frac{Cov(X,Y)}{\sigma_X \sigma_y} \right]$$

Where:

$Cov(X,Y)$ = The covariance of $X$ and $Y$, defined below

$\sigma_X$ = The standard deviation of $X$

$\sigma_y$ = The standard deviation of $Y$

The covariance equation is:

149 Two series can be related in a nonlinear way and have a correlation coefficient of zero. An example is the function $y = x^2$, for which $p_{x,y} = 0$.

$$Cov(X,Y) = \frac{1}{n-1}\sum_{t=1}^{n}(r_{X,t} - r_{X,A})(r_{Y,t} - r_{Y,A})$$

Where:

$r_{X,t}$ = The return for series $X$ in period $t$

$r_{Y,t}$ = The return for series $Y$ in period t

$r_{X,A}$ = The arithmetic mean of series $X$

$r_{Y,A}$ = The arithmetic mean of series $Y$

$n$ = The number of periods

## Correlations of the Basic Series

Exhibit 6.4 presents the annual cross correlations and serial correlations for the seven basic series. Long-term government bond returns and long-term corporate bond returns are highly correlated with each other but negatively correlated with inflation. To the degree that inflation is unanticipated, it has a negative effect on fixed-income securities. In addition, U.S. Treasury bills and inflation are reasonably highly correlated, a result of the post-1951 "tracking" described in Chapter 2. Lastly, both the U.S. Treasury bills and inflation series display high serial correlations.

**Exhibit 6.4:** Basic Series: Serial and Cross-Correlations of Historical Annual Returns 1926–2020

| | Large-Cap Stocks | Small-Cap Stocks | Long-term Corp Bonds | Long-term Gov't Bonds | Inter-term Gov't Bonds | U.S. Treasury Bills | Inflation |
|---|---|---|---|---|---|---|---|
| Large-Cap Stocks | 1.00 | | | | | | |
| Small-Cap Stocks | 0.79 | 1.00 | | | | | |
| Long-term Corp Bonds | 0.17 | 0.05 | 1.00 | | | | |
| Long-term Gov't Bonds | 0.01 | -0.10 | 0.89 | 1.00 | | | |
| Inter-term Gov't Bonds | -0.02 | -0.11 | 0.84 | 0.86 | 1.00 | | |
| U.S. Treasury Bills | -0.02 | -0.08 | 0.14 | 0.17 | 0.47 | 1.00 | |
| Inflation | -0.01 | 0.05 | -0.15 | -0.14 | 0.01 | 0.42 | 1.00 |
| Serial Correlation | 0.01 | 0.06 | 0.02 | -0.15 | 0.15 | 0.91 | 0.63 |

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

### Correlations of the Derived Series

The annual cross-correlations and serial correlations for the four risk premium series and inflation are presented in Exhibit 6.5. Notice that inflation is negatively correlated with the horizon premium. Increasing inflation causes long-term bond yields to rise and prices to fall; therefore, a negative horizon premium is observed in times of rising inflation.

Exhibit 6.6 presents annual cross-correlations and serial correlations for the inflation-adjusted asset return series. It is interesting to observe how the relationship between the asset returns are substantially different when these returns are expressed in inflation adjusted terms (as compared with nominal terms). In general, the cross-correlations between asset classes are higher when one accounts for inflation (i.e., subtracts inflation from the nominal return).

**Exhibit 6.5:** Risk Premia and Inflation: Serial and Cross-Correlations of Historical Annual Returns 1926–2020

| | Equity Risk Premium | Small Stock Premium | Bond Default Premium | Bond Horizon Premium | Inflation |
|---|---|---|---|---|---|
| Equity Risk Premium | 1.00 | | | | |
| Small Stock Premium | 0.26 | 1.00 | | | |
| Bond Default Premium | 0.31 | 0.17 | 1.00 | | |
| Bond Horizon Premium | 0.03 | -0.11 | -0.48 | 1.00 | |
| Inflation | -0.07 | 0.12 | 0.00 | -0.27 | 1.00 |
| Serial Correlation | 0.02 | 0.36 | -0.31 | -0.16 | 0.63 |

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

**Exhibit 6.6:** Inflation-Adjusted Series: Serial and Cross-Correlations of Historical Annual Returns 1926–2020

| Inflation-Adjusted Series | Inflation-Adjusted Large-Cap Stocks | Small-Cap Stocks | Long-term Corp Bonds | Long-term Gov't Bonds | Inter-term Gov't Bonds | U.S. Treasury Bills* | Inflation |
|---|---|---|---|---|---|---|---|
| Large-Cap Stocks | 1.00 | | | | | | |
| Small-Cap Stocks | 0.79 | 1.00 | | | | | |
| Long-term Corp Bonds | 0.23 | 0.08 | 1.00 | | | | |
| Long-term Gov't Bonds | 0.09 | -0.06 | 0.91 | 1.00 | | | |
| Inter-term Gov't Bonds | 0.07 | -0.06 | 0.89 | 0.90 | 1.00 | | |
| U.S. Treasury Bills | 0.09 | -0.06 | 0.51 | 0.49 | 0.70 | 1.00 | |
| Inflation | -0.19 | -0.07 | -0.54 | -0.48 | -0.58 | -0.70 | 1.00 |
| Serial Correlation | 0.00 | 0.03 | 0.13 | -0.06 | 0.20 | 0.66 | 0.63 |

*Real Interest Rates

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## Is Serial Correlation in the Derived Series Random?

The risk/return relationships in the historical data are represented in the equity risk premium, the small stock premium, the bond horizon premium, and the bond default premium. The real/nominal historical relationships are represented in the inflation rates and the real interest rates. The objective is to uncover whether each series is random or is subject to any trends, cycles, or other patterns.

The one-year serial correlation coefficients measure the degree of correlation between returns from each year and the previous year for the same series, as seen in Exhibit 6.6. Highly positive (near 1) serial correlations indicate trends, while highly negative (near -1) serial correlations indicate cycles. Looking to exhibit 6.7, the analysis suggests that both inflation rates and real riskless rates follow trends. Serial correlations near zero suggest no patterns (i.e., random behavior), so the analysis suggests that the equity risk premium and the bond horizon premium are random variables (although this is less-strongly indicated in the case of the bond horizon premium).

The small stock premium and the bond default premium, however, fall into a middle range that makes it more difficult to determine whether the small stock premium is a trend (or is random) and whether the bond default premium is a cycle (or is random). In these two cases, one could argue that the small stock premium is a *possible* trend, and the bond default premium is a *possible* cycle.

**Exhibit 6.7:** Interpretation of the Annual Serial Correlations

| Series | Serial Correlation | Interpretation |
|---|---|---|
| Equity Risk Premium | 0.02 | Random |
| Small Stock Premium | 0.36 | Possible Trend |
| Bond Default Premium | -0.31 | Possible Cycle |
| Bond Horizon Premium | -0.16 | Random/Possible Cycle |
| Real Interest Rates | 0.66 | Trend |
| Inflation | 0.63 | Trend |

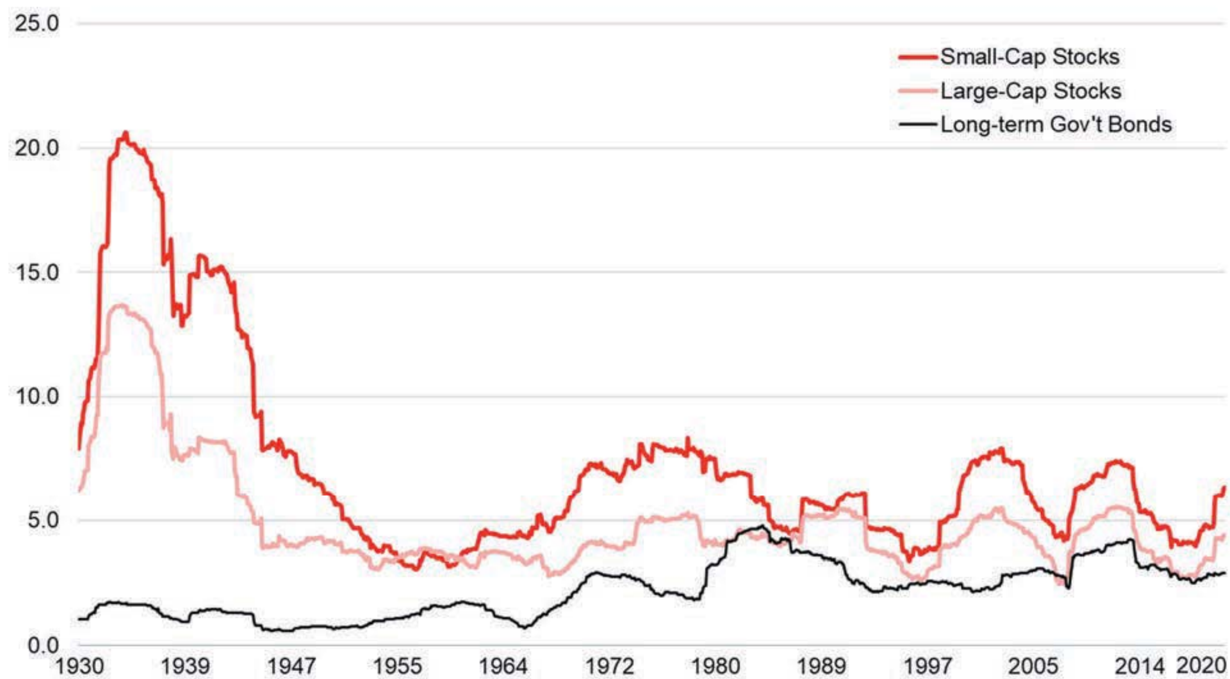
**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## Rolling-Period Standard Deviations

Rolling-period standard deviations are obtained by rolling a window of fixed length along each time series and computing the standard deviation for the asset class for each window of time. They are useful for examining the volatility or riskiness of returns for holding periods similar to those actually experienced by investors. Exhibits 6.8 and 6.9 graphically depict this volatility. Monthly data are used to maximize the number of data points included in the standard deviation computation. The first 60-month (i.e., 5-year) rolling period covered is January 1926–December 1930, so each of the graphs start at 1930.

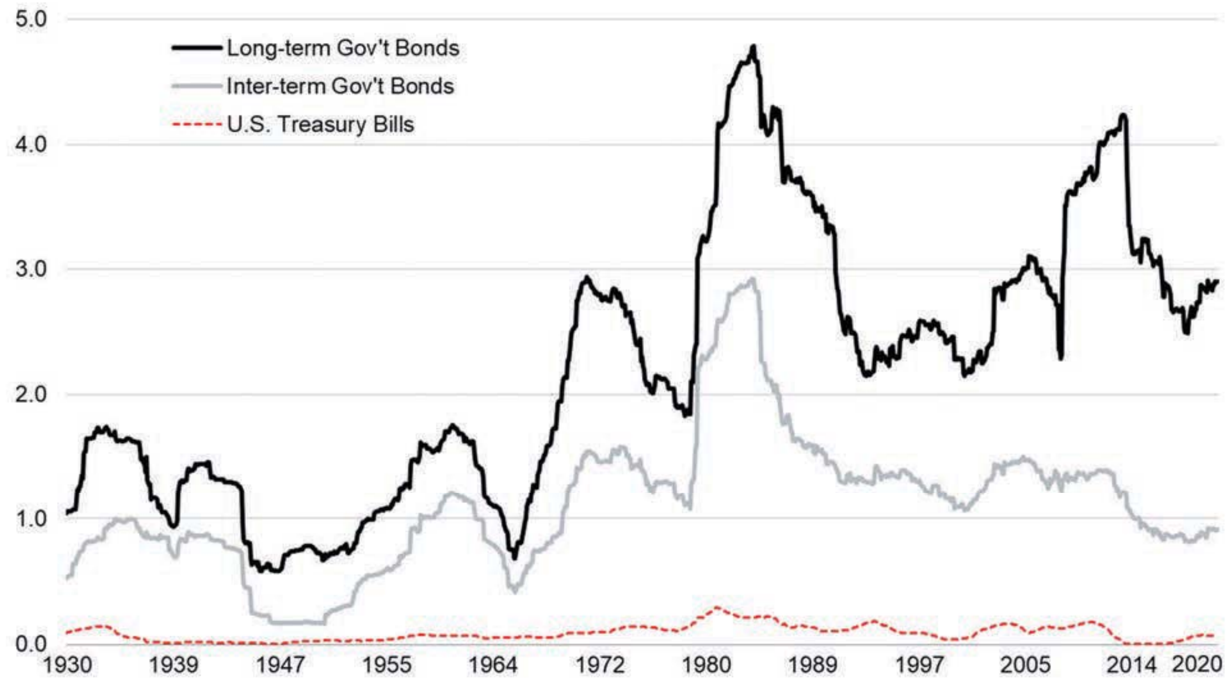
Exhibit 6.8 examines the 60-month rolling standard deviation for large-cap stocks, small-cap stocks, and long-term government bonds. It is interesting to see the relatively high standard deviation for small- and large-cap stocks in the 1930s with an apparent lessening of volatility for 60-month holding periods during the 1980s. Note also how the standard deviation for long-term government bonds reaches the level of both stock asset classes during part of the 1980s. Exhibit 6.9 examines the 60-month rolling standard deviation for long-term and intermediate-term government bonds and U.S. Treasury bills. Note that the vertical scale (from 0.0% to 25.0%) of Exhibit 6.8 is different than the vertical scale (from 0.0% to 5.0%) of Exhibit 6.9.

**Exhibit 6.8:** Rolling 60-month Standard Deviations: Large-cap Stocks, Small-cap Stocks and Long-term Government Bonds (%) 1926–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

**Exhibit 6.9:** Rolling 60-month Standard Deviations: Long-term Government Bonds, Intermediate-term Government Bonds, and Treasury Bills (%) 1926–2020



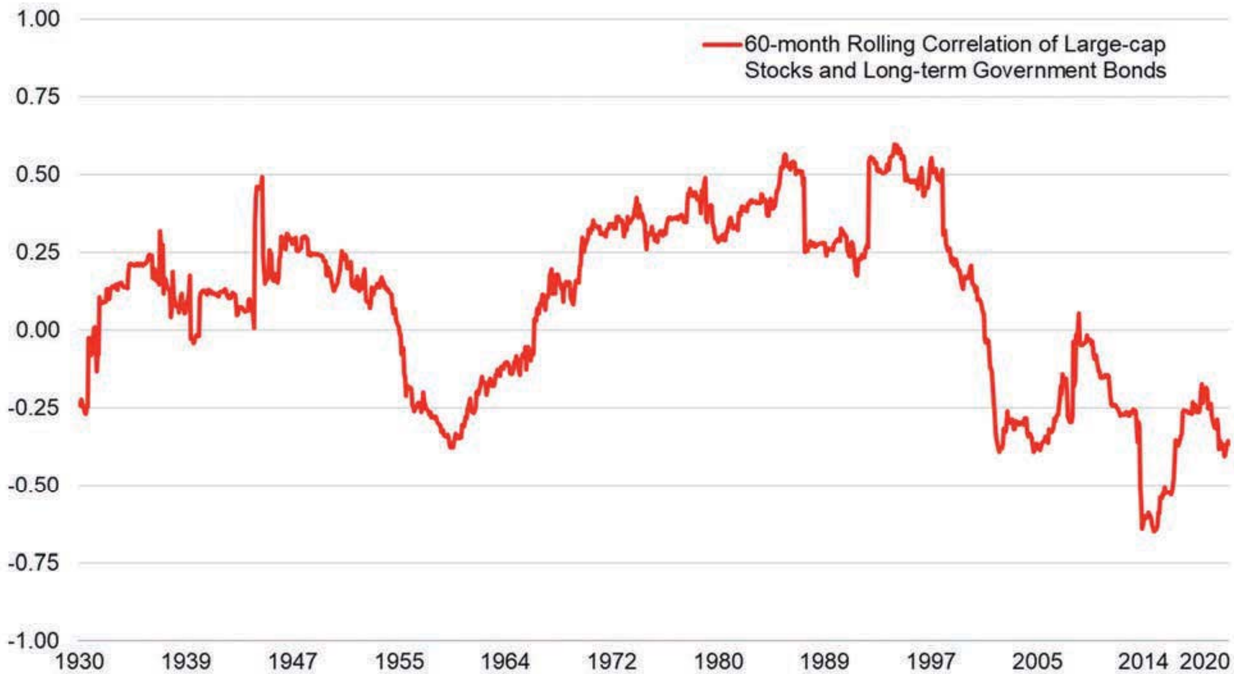

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## Rolling-Period Correlations

Rolling-period correlations are obtained by moving a window of fixed length along time series for two asset classes and computing the cross-correlation between the two asset classes for each window of time. They are useful for examining how asset class returns vary together for holding periods similar to those actually experienced by investors. Exhibits 6.12 and 6.13 graphically depict cross-correlation. Monthly data are used to maximize the number of data points included in the correlation computation. The first 60 month (i.e., 5-year) rolling period covered is January 1926– December 1930, so each of the graphs start at 1930.

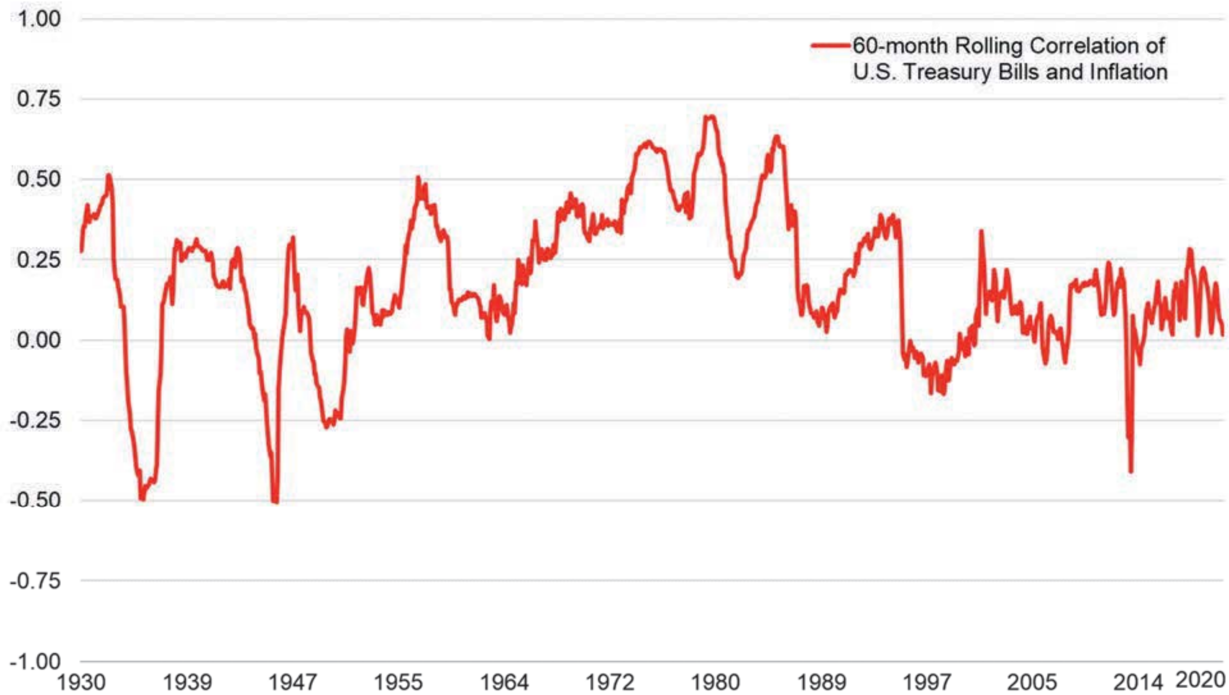
Exhibits 6.10 and 6.11 show cross-correlations between two asset classes for 60-month holding periods. Exhibit 6.10 shows the volatility of the correlations between large-cap stocks and long-term government bonds. There are wide fluctuations between strong positive and strong negative correlations over the past 95 years. Exhibit 6.11 shows the correlation between Treasury bills and inflation. These asset classes also show wide fluctuations in correlation over the past 95 years.

**Exhibit 6.10:** Rolling 60-month Correlation: Large-cap Stocks and Long-term Government Bonds 1926–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD. For a detailed description of the SBBI® series, see Chapter 3, “Description of the Basic Series”. “Stocks, Bonds, Bills, and Inflation” and “SBBI” are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

**Exhibit 6.11:** Rolling 60-month Correlation: U.S. Treasury Bills and Inflation 1926–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## The True Impact of Asset Allocation on Return

The importance of asset allocation has been the subject of considerable debate and misunderstanding for decades. A May 2010 article pinpoints one of the primary sources of confusion surrounding the importance of asset allocation: How much of a portfolio's level of return comes from a fund's asset-allocation policy?[150]

*Note: The following section was written by Thomas Idzorek, president of the Morningstar Investment Management division.*

### The Debate

The seminal work on the importance of asset allocation, the catalyst of a 25-year debate, and unfortunately the source of what is arguably the most prolific misunderstanding among investment professionals, is the 1986 article, "Determinants of Portfolio Performance," by Gary Brinson,

150 Idzorek, T. M. "Asset Allocation is King." *Morningstar Advisor*, April/May, 2010, P. 28.

Randolph Hood, and Gilbert Beebower (BHB).[151] BHB regressed the time series returns of each fund on a weighted combination of indexes reflecting each fund's asset-allocation policy. In one of the many analyses that BHB carried out (and probably one of the least important ones), they found that the policy mix explained 93.6% of the average fund's return variation over time (as measured by the R squared of the regression) – the key word being "variation."

Unfortunately, this 93.6% has been widely misinterpreted. Many practitioners incorrectly believe the number means that 93.6% of a portfolio's return level (for example, a fund's 10-year annualized return) comes from a fund's asset-allocation policy. This is not true. The truth is that, in aggregate, 100% of portfolio return levels comes from asset-allocation policy.

**Return 'Levels' Versus Return 'Variations'**

It is imperative to distinguish between return levels and return variations. In the big picture, investors care far more about return levels than they do return variation. The often-cited 93.6% says nothing about return levels, even though that is what so many practitioners mistakenly believe. It is possible to have a high R-squared, indicating that the return variations in the asset class factors did a good job of explaining the return variations of the fund in question, yet see the weighted-average composite asset-allocation policy benchmark produce a significantly different return level from the fund in question. This is the case in BHB's study. Despite the high average 93.6% R-squared of their 91 separate time-series regressions, the average geometric annualized return of the 91 funds in their sample was 9.01% versus 10.11% for the corresponding policy portfolios.

So even though 93.6% is the number that seems to be stuck in everyone's mind, 112% (10.11% divided by 9.01%) of return levels in the study's sample came from asset-allocation policy. To put it bluntly, when it comes to return levels, asset allocation is king. In aggregate, 100% of return levels come from asset allocation before fees and somewhat more after fees. This is a mathematical truth that stems from the concept of an all-inclusive market portfolio and the fact that active management is a zero-sum game. This fundamental truth is somewhat boring; therefore, it is often lost.

151 Brinson, G.P., Hood, L.R., & Beebower, G.L. 1986. " Determinants of Portfolio Performance." *Financial Analysts Journal,* Vol. 42, No. 4, P. 39.

# Chapter 7
# Company Size and Return[152]

*In previous versions of the Stocks, Bonds, Bills, and Inflation® (SBBI®) Yearbook the discussions in this chapter used data from the Center for Research in Security Prices (CRSP) to demonstrate various concepts about company size and return (i.e., the "size effect"). Starting with the 2020 SBBI® Yearbook®, these concepts are demonstrated using the Morningstar/Ibbotson Associates SBBI Large Stock series and SBBI Small Stock series.*[153,154,155]

One of the most remarkable discoveries of modern finance is the finding of a relationship between company size and return, generally referred to as the "size effect." The size effect is based on the empirical observation that companies of smaller size tend to have higher returns than do larger companies.

A 1981 study by Rolf Banz examined the returns of New York Stock Exchange (NYSE) small-cap companies compared to the returns of NYSE large-cap companies over the period 1926–1975.[156] What Banz found was that the returns of small-cap companies were *greater* than the returns for

152 This chapter is an overview of the relationship between size and return that is limited to analyzing the relative historical performance of "large-cap stocks" and "small-cap stocks," and does not include the much-expanded analyses of the "size effect" as it relates to the development of cost of equity capital found on the D&P/Kroll online Cost of Capital Navigator platform at dpcostofcapital.com. The Cost of Capital Navigator guides the Analyst through the process of estimating the cost of equity capital, a key component of any valuation analysis. The Cost of Capital Navigator includes the critical information and data from the 1999– 2021 CRSP Deciles Size Study and Risk Premium Report Study that were previously published in the *Valuation Handbook – U.S. Guide to Cost of Capital* from 2014 to 2017, and, before that, in the Ibbotson Associates/ Morningstar *Stocks, Bonds, Bills, and Inflation® (SBBI®) Valuation Yearbook* and *Risk Premium Report*, respectively, from 1999 to 2013. The valuation data and information in the Cost of Capital Navigator is the actual "as published" valuation data from those former publications. The 1999–2013 Ibbotson Associates/Morningstar size premia, industry risk premia, and other valuation data that are available within the Cost of Capital Navigator are used with permission from Morningstar, Inc. The Cost of Capital Navigator is web-based, so you can access it from your desktop, laptop, or tablet. To learn more and to purchase the Cost of Capital Navigator, visit dpcostofcapital.com.

153 The focus of the *Stocks, Bonds, Bills, and Inflation® (SBBI®) Yearbook* is to analyze the historical performance data of U.S. asset classes, as represented by the seven Morningstar/Ibbotson Associates "SBBI" series. The seven "SBBI" indices are: (i) SBBI U.S. Large Stocks, (ii) SBBI U.S. Small Stocks, (iii) SBBI U.S. Long-Term Corporate Bonds, (iv) SBBI U.S. Long-Term Government Bonds, (v) SBBI U.S. Intermediate-Term Government Bonds, (vi) SBBI U.S. 30-Day Treasury Bills, and (vii) SBBI U.S. Inflation. For detailed information about the SBBI series, see Chapter 3, "Description of the Basic Series".

154 The CRSP Deciles Size Study and Risk Premium Report Study, both of which provide size premia and other risk premia based upon data licensed from the Center for Research in Security Prices (CRSP) at the University of Chicago Booth School of Business, are fully available in the D&P/Kroll online Cost of Capital Navigator platform at dpcostofcapital.com. CRSP® is a registered trademark and service mark of Center for Research in Security Prices, LLC and has been licensed for use by D&P/Kroll. The D&P/Kroll publications and services are not sponsored, sold or promoted by CRSP®, its affiliates or its parent company. To learn more about CRSP, visit www.crsp.com.

155 A detailed discussion of company size and return (and size premia) based upon CRSP deciles 1-10 and the 10th decile split (10a, 10b, 10w, 10x, 10y, and 10z) is available in the Resources section of the Cost of Capital Navigator at dpcostofcapital.com (subscription required).

156 Rolf W. Banz, "The Relationship between Return and Market Value of Common Stocks," *Journal of Financial Economics* (March 1981): 3–18. This paper is often cited as the first comprehensive study of the size effect.

large-cap companies. Banz's 1981 study is often cited as the first comprehensive study of the size effect. Banz found that there is a significant (negative) relationship between size and historical equity returns as size decreases, returns tend to increase, and vice versa.

## Possible Explanations for the Greater Returns of Smaller Companies

Some valuation analysts treat small firms as equivalent to scaled-down large firms. This is likely an erroneous assumption.

There are theoretical reasons for the greater returns of smaller companies (i.e., the "size effect"), which might include: (i) small stocks are less liquid (with higher associated transaction costs), (ii) small stocks are riskier and harder to diversify, (iii) small stocks have higher betas which tend to be underestimated, (iv) investors must do more analysis per dollar invested, (v) investment data is less available.

Valuation analysts also cite more practical reasons that small firms have risk characteristics that differ from those of large firms. For example, large firms may have greater ability to enter the market of the small firm and take market share away. Large companies likely have more resources to "weather the storm" in economic downturns. Large firms can generally spend more cash on R&D, advertising, and typically even have greater ability to hire the "best and brightest." Larger firms may have greater access to capital, broader management depth, and less dependency on just a few customers. A larger number of analysts typically follow large firms relative to small firms so there is probably more information available about large firms. Small firms have fewer resources to fend off competition and redirect themselves after changes in the market occur.[157]

Any one of these differences (not an all-encompassing list) would tend to increase investors' required rate of return to induce them to invest in small companies rather than investing in large companies.

The size effect is not without controversy, nor is this controversy something new. Traditionally, small companies are believed to have greater required rates of return than large companies because small companies are inherently riskier. It is not clear, however, whether this is due to size itself, or to other factors closely related to or correlated with size, and thus the qualification that Banz noted in his 1981 article remains pertinent today:[158,159]

*"It is not known whether size [as measured by market capitalization] per se is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size."*

---

157 M. S. Long and J. Zhang, "Growth Options, Unwritten Call Discounts and Valuing Small Firms", *EFA 2004 Maastricht Meetings Paper no. 4057*, March 2004. Available at http://www.ssrn.com/abstract=556203.

158 Even after controlling for size, research suggests that liquidity is still a systematic factor and a predictor of returns. See: Ibbotson, Roger G., and Daniel Y.-J Kim, "Liquidity as an Investment Style: 2018 Update," available at www.zebracapm.com Updated version of: Ibbotson, Roger G., Chen, Zhiwu, Kim, Daniel Y.-J., and Hu, Wendy Y. "Liquidity as an Investment Style," *Financial Analysts Journal*, May/June 2013.

159 "Liquidity" is discussed in detail in Chapter 9, "Liquidity Investing."

## Aspects of the Company Size Effect

The company size phenomenon is remarkable in several ways. First, the greater risk of small-cap stocks does not, in the context of the capital asset pricing model, fully account for their higher returns over the long term. In the capital asset pricing model (CAPM), only systematic, or beta risk, is rewarded; small-cap stock returns have exceeded those implied by their betas. Second, the calendar annual return differences between small- and large-cap companies are serially correlated. This suggests that past annual returns may be of some value in predicting future annual returns. Such serial correlation, or autocorrelation, is practically unknown in the market for large-cap stocks and in most other equity markets but is evident in the size premium series. Third, the size effect is seasonal. For example, small-cap stocks outperformed large-cap stocks in January in a large majority of the years. Such predictability is surprising and suspicious in light of modern capital market theory. These three aspects of the size effect – long-term returns in excess of systematic risk, serial correlation, and seasonality – will be discussed in the following sections.

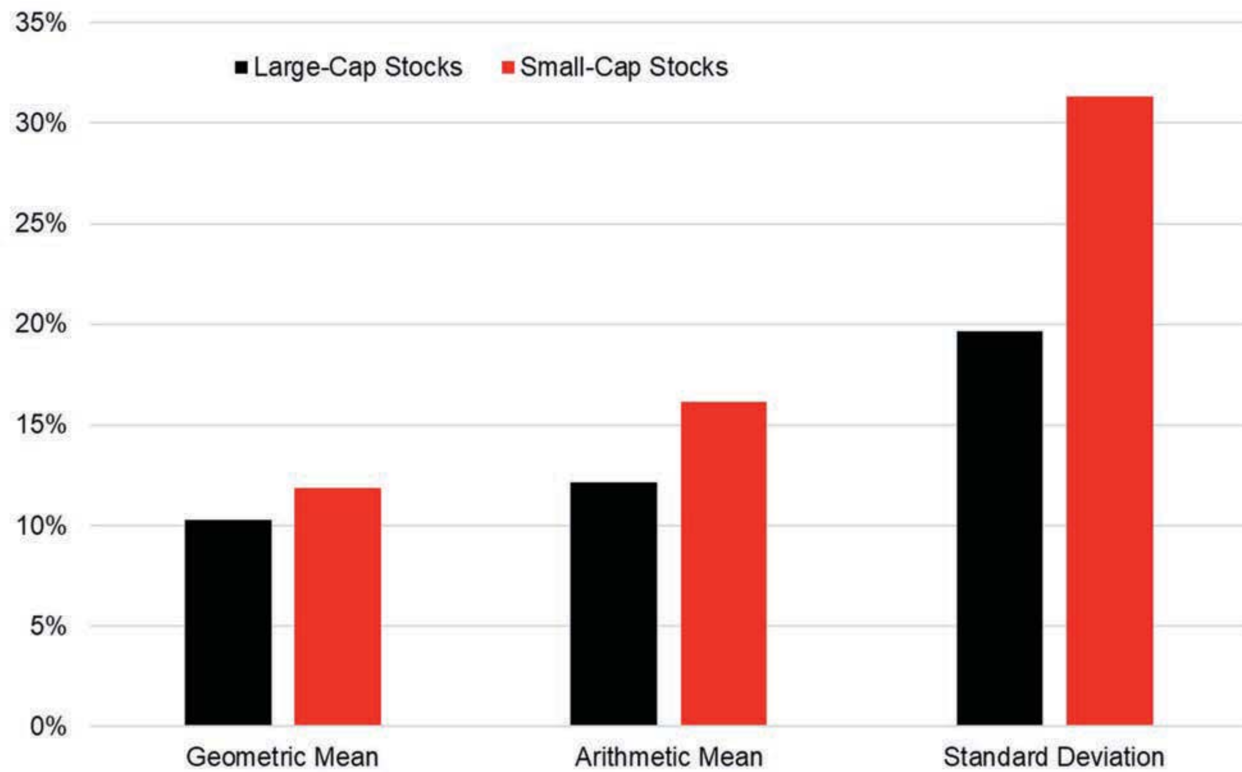
## The Size Effect: Empirical Evidence

Summary statistics of annual total returns for Large-Cap stocks and Small-Cap stocks are illustrated in Exhibit 7.1 over the 1926–2020 period. The differences in return between large-cap stocks and small-cap stocks is apparent.[160] For example, the annual arithmetic mean return of large-cap stocks was just over 12% over the 1926–2020 period, while the annual arithmetic mean return of small-cap stocks was just over 16%.

Note that this increased return comes at a price: risk (as measured by standard deviation) increases from just under 20% for large-cap stocks to just over 31% for small-cap stocks. The relationship between risk and return is a fundamental principle of finance. History tells us that small companies are riskier than large companies. Investors are compensated for taking on this additional risk by the higher returns provided by small companies.

160 Traditionally, researchers have used market value of equity (i.e., market capitalization, or simply "market cap") as a measure of size in conducting historical rate of return studies. However, market cap is not the only measure of size that can be used to predict return, nor is it necessarily the best measure of size to use. In the online D&P/Kroll Cost of Capital Navigator platform, the size effect is examined in relation to *eight* measures of company size (including market cap): (i) market capitalization, (ii) book value of equity, (iii) 5-year average net income, (iv) market value of invested capital (MVIC), (v) total assets, (vi) 5-year average EBITDA, (vii) sales, and (viii) number of employees. The Cost of Capital Navigator guides the Analyst through the process of estimating the cost of equity capital, a key component of any valuation analysis. The Cost of Capital Navigator includes the critical information and data from the 1999–2021 CRSP Deciles Size Study and Risk Premium Report Study, as published in the former *Valuation Handbook – U.S. Guide to Cost of Capital* from 2014 to 2017, and, before that, in the former Ibbotson Associates/ Morningstar *Stocks, Bonds, Bills, and Inflation® (SBBI®) Valuation Yearbook* and *Risk Premium Report*, respectively, from 1999 to 2013. The valuation data and information in the Cost of Capital Navigator is the actual "as published" valuation data from those former publications. The 1999–2013 Ibbotson Associates/Morningstar size premia, industry risk premia, and other valuation data that are available within the Cost of Capital Navigator are used with permission from Morningstar, Inc. CRSP® is a registered trademark and service mark of Center for Research in Security Prices, LLC and has been licensed for use by D&P/Kroll. The D&P/Kroll publications and services are not sponsored, sold or promoted by CRSP®, its affiliates or its parent company. To learn more about CRSP, visit www.crsp.com. To learn more and to purchase the Cost of Capital Navigator, visit dpcostofcapital.com.

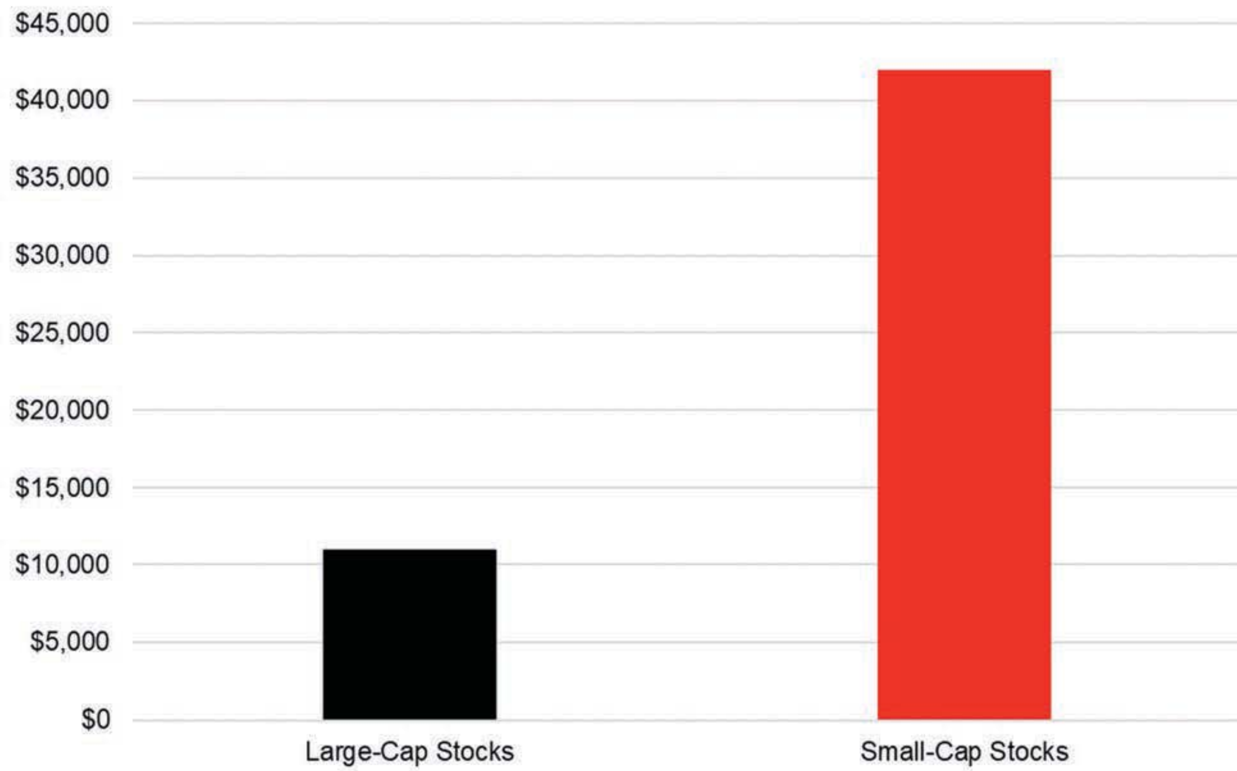
**Exhibit 7.1**: Illustration of Summary Statistics of Large-Cap Stocks and Small-Cap Stocks (%) 1926–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

The differences in the performance of large-cap stocks and small-cap stocks can have important implications for investors. Exhibit 7.2 is a graphical depiction of the value of $1 invested at the end of 1925 in large-cap and small-cap stocks and held through December 31, 2020 (a total of 95 years).

**Exhibit 7.2**: The Value of $1 Invested in Large-Cap and Small-Cap Stocks, 1926–2020 Index (Year-end 1925 = $1.00)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

The geometric (i.e., compound annual) returns of these series (as illustrated in Exhibit 7.1) can be used to calculate terminal index values as follows: [161,162]

*Terminal Index Value = (1 + Geometric Mean Return)*$^n$

Where *n* is the number of periods (in this case, 95 years).

[161] For more information on calculating annual total and income returns, see Chapter 5, "Annual Returns and Indexes".

[162] *Precalculated* annualized monthly returns for each of the six SBBI series plus inflation, for each year over the 1926–2019 time horizon, are presented in table format in the full-version *2020 SBBI® Yearbook* in that book's appendices. For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

### Do Small-Cap Stocks Always Outperform Large-Cap Stocks?

The increased risk faced by investors in small stocks is quite real. It is important to note, however, that the risk/return profile is over the *long-term*. The long-term expected return for any asset class can be quite different from short-term expected returns. Investors in small-cap stocks should expect losses and periods of *underperformance* relative to large-cap stocks. While this might lead some market observers to speculate that there is no size premium, statistical evidence suggests that periods of smaller stocks' underperformance should be expected. The evidence also suggests that the longer small-cap companies are given to "race" against large-cap companies, the greater the chance that small-cap companies outpace their larger counterparts.

In Exhibit 7.3, a detailed summary of the results of various "races" between small-cap companies and large-cap companies is shown, where the holding periods are limited to *exactly* 1 month, 60 months (5 years), 120 months (10 years), 240 months (20 years), and 360 months (30 years). The entire January 1926–December 2020 period is examined, as well as three more recent start date windows: April 1981–December 2020, January 1990–December 2020, and January 2000–December 2020. All three of the three more recent periods are *after* Banz wrote his March 1981 article that identified the size effect, and so they are labeled "Post Banz."[163]

In Exhibit 7.3 the number of periods examined is shown first, followed by the outperformance percentage of the total periods in parentheses.

[163] Banz, Rolf W. "The Relationship between Return and Market Value of Common Stocks". *Journal of Financial Economics* (March 1981): 3–18. Professor Banz's 1981 article is often cited as the first comprehensive study of the size effect.

**Exhibit 7.3**: Small-cap Companies' Performance minus Large-cap Companies' Performance Over Periods of Exactly 1, 60, 120, 240, and 360 Months January 1926–December 2020

| Holding Period | All Dates Jan 1926–Dec 2020 | Post Banz Apr 1981–Dec 2020 | Post Banz Jan 1990–Dec 2020 | Post Banz Jan 2000–Dec 2020 |
|---|---|---|---|---|
| *Exactly 1 month* | | | | |
| Small Stocks Outperform | 568 (50%) | 226 (47%) | 183 (49%) | 130 (52%) |
| Large Stocks Outperform | 572 (50%) | 251 (53%) | 189 (51%) | 122 (48%) |
| *Exactly 60 months (5 years)* | | | | |
| Small Stocks Outperform | 598 (55%) | 184 (44%) | 176 (56%) | 106 (55%) |
| Large Stocks Outperform | 483 (45%) | 234 (56%) | 137 (44%) | 87 (45%) |
| *Exactly 120 months (10 years)* | | | | |
| Small Stocks Outperform | 686 (67%) | 199 (56%) | 199 (79%) | 89 (67%) |
| Large Stocks Outperform | 335 (33%) | 159 (44%) | 54 (21%) | 44 (33%) |
| *Exactly 240 months (20 years)* | | | | |
| Small Stocks Outperform | 791 (88%) | 193 (81%) | 133 (100%) | 13 (100%) |
| Large Stocks Outperform | 110 (12%) | 45 (19%) | 0 (0%) | 0 (0%) |
| *Exactly 360 months (30 years)* | | | | |
| Small Stocks Outperform | 752 (96%) | 113 (96%) | 13 (100%) | – |
| Large Stocks Outperform | 29 (4%) | 5 (4%) | 0 (0%) | – |

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the following Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

In the top row of Exhibit 7.3 (in which the holding period is restricted to a single month), large-cap companies *barely* outperformed small-cap companies in the January 1926–December 2020 period (572 versus 568 months). In the "Post-Banz" April 1981–December 2020 and January 1990–December 2020 time horizons, large-cap stocks outperformed small-cap stocks 53% and 51% of the time, respectively. In the more recent January 2000–December 2020 time horizon small-cap companies outperformed 52% of the time.

As the holding period is increased and the time that small-cap companies and large-cap companies are given to "race" against each other is lengthened, small-cap stocks tend to increasingly outperform large-cap stocks. For example, over the entire range January 1926–December 2020 (see leftmost column of Exhibit 7.3), as the holding period is increased to 60 months (5-years), to 120 months (10-years), to 240 months (20-years) and finally to 360 months (30-years), small stocks increasingly outperform large stocks (55%, 67%, 88%, and 96% of the time, respectively).

This same pattern of increasing outperformance of small stocks as the holding period is increased can also be seen in the three "Post Banz" periods.

## Long-term Returns in Excess of Systematic Risk

The capital asset pricing model, or CAPM, does not fully account for the higher returns of small-cap stocks. The textbook CAPM can be expressed as follows:

$$k_e = R_f + \beta \times (RP_m)$$

Where:

$k_e$ = Cost of equity capital

$R_f$ = Risk-free rate

$\beta$ = Beta

$RP_m$ = Equity risk premium (also referred as ERP)

According to the CAPM, the expected return on a security should consist of the riskless rate plus an additional return to compensate for the systematic risk of the security. The return in excess of the riskless rate is estimated in the context of the CAPM by multiplying the equity risk premium by beta. The equity risk premium is the return that compensates investors for taking on risk equal to the risk of the market as a whole (systematic risk). Beta measures the extent to which a security or portfolio is exposed to systematic risk.

The beta of each decile indicates the degree to which the decile's return moves with that of the overall market. A beta greater than one indicates that the security or portfolio has greater systematic risk than the market; according to the CAPM equation, investors are compensated for taking on this additional risk.

CAPM is an attempt to predict *future* returns. CAPM can be used to see how well it would have done predicting returns that we *already know* (i.e., historical returns; returns that have already occurred). This is called "back-testing." If what "actually happened" is greater than "what CAPM would have predicted," then CAPM fell short of explaining what actually happened.

Smaller companies tend to have returns that are not fully explained by their higher betas, so return in excess of that predicted by CAPM tends to *increase* as one moves from the largest companies to the smallest companies. This size related phenomenon prompted a revision to the CAPM to include a size premium. A size premium (as used in the CAPM equation) is thus a measure of "what actually happened" minus "what textbook CAPM predicted":

*Size Premium = Actual Excess Return – Excess Return Predicted by CAPM*

A size premium is a common adjustment that analysts make to the textbook CAPM when developing cost of equity capital estimates.[164,165] This is sometimes referred to as the "modified" CAPM:

$$k_e = R_f + \beta \times (RP_m) + RP_s$$

Where:

| | | |
|---|---|---|
| $k_e$ | = | Cost of equity capital |
| $R_f$ | = | Risk-free rate |
| $\beta$ | = | Beta |
| $RP_m$ | = | Equity risk premium (also referred as ERP) |
| $RP_s$ | = | Size Premium |

The size effect is not without controversy, nor is this controversy something new. Traditionally, small companies are believed to have greater required rates of return than large companies because small companies are inherently riskier. It is not clear, however, whether this is due to size itself, or to other factors closely related to or correlated with size (e.g., liquidity).[166]

164 The CRSP Deciles Size Study and Risk Premium Report Study, both of which provide size premia and other risk premia based upon data licensed from the Center for Research in Security Prices (CRSP) at the University of Chicago Booth School of Business, are fully available in the D&P/Kroll online Cost of Capital Navigator platform at dpcostofcapital.com (subscription required). CRSP® is a registered trademark and service mark of Center for Research in Security Prices, LLC and has been licensed for use by D&P/Kroll. The D&P/Kroll publications and services are not sponsored, sold or promoted by CRSP®, its affiliates or its parent company. To learn more about CRSP, visit www.crsp.com.

165 A detailed discussion of company size and return (and size premia) based upon CRSP deciles 1-10 and the 10th decile split (10a, 10b, 10w, 10x, 10y, and 10z) is available in the Resources section of the Cost of Capital Navigator at dpcostofcapital.com. (subscription required).

166 For more information, see the Resources section of the Cost of Capital Navigator at dpcostofcapital.com. A comprehensive discussion of the size effect is in the Cost of Capital Navigator "U.S. Cost of Capital Module's" Resources section in Chapter 4, "Basic Building Blocks of the Cost of Equity Capital – Size Premium."

# Chapter 8
# Growth and Value Investing

Investment style can be defined broadly as an overarching description of groups of stocks or portfolios based on shared characteristics. Probably the first discussion and consideration of style concerned large-company versus small-company investing, and even this distinction was not prominent until the 1960s. Styles of investing are now broken down into more detail and used for performance measurement, asset allocation, and other purposes. Mutual funds and other investment portfolios are often measured against broad growth or value benchmarks. In some cases, investment-manager-specific style benchmarks are constructed to separate pure stock-selection ability from style effects.

Most investors agree on the broad definitions of growth and value, but when it comes to specifics, definitions can vary widely. In general, growth stocks have high relative growth rates in regard to earnings, sales, or return on equity. Growth stocks usually have relatively high price to-earnings and price-to-book ratios. Value stocks will generally have lower price-to-earnings and price-to-book values and often have higher dividend yields. Value stocks are often turnaround opportunities, companies that have had disappointing news, or companies with low growth prospects. Value investors generally believe that a value stock has been unfairly beaten down by the market, leading the stock to sell below its "intrinsic" value. Therefore, they buy the stock with the hope that the market will realize the stock's full value and eventually bid the price up to its fair value.

## Fama-French Growth and Value Series

For the analysis in this chapter the Fama-French growth and value "benchmark" portfolios (discontinued) have been replaced by the Fama-French growth and value "research" portfolios.[167]

While *individual* period returns of the Fama-French growth and value research portfolios are significantly different in some cases than individual period returns of the discontinued Fama French growth and value benchmark portfolios, the *summary* statistics of the two are quite similar. Moreover, the *relative performance* of the Large Growth, Large Value, Small Growth, and Small Value remains essentially the same.[168]

---

167 A July 2012 note on Professor Kenneth R. French's data library site states "…we believe the research factors are more useful than the benchmark factors…". The benchmark factors, which the site continued to publish, were employed by Morningstar in the *SBBI Yearbook* through the 2015 edition, and D&P/Kroll continued to using the benchmark series through the 2019 edition of the *SBBI Yearbook* (D&P/Kroll has published the *SBBI Yearbook* since 2016; for more information about the full-version *2021 SBBI® Yearbook*. visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook). In spring of 2019, the benchmark series were discontinued, and the "research" series replaced them replaced them in the *SBBI Yearbook*.

168 The time horizon in previous versions of the *SBBI Yearbook* over which summary statistics for the Fama-French growth and value benchmark portfolios were calculated was 1928–present. The time horizon in this book over which summary statistics for the Fama-French growth and value research portfolios are calculated is 1927–present.

The following commentary and corresponding data make use of the Fama-French growth and value data series.[169,170]

### Fama-French Index Construction Methodology[171]

The Fama-French growth and value portfolios are constructed at the end of each June. The portfolios are the intersections of two portfolios formed on size (market equity) and three portfolios formed on the ratio of book equity to market equity. The size breakpoint for year *t* is the median NYSE market equity at the end of June of year *t*. Book equity to market equity for June of year *t* is the book equity for the last fiscal year end in *t-1* divided by market equity for December of *t-1*. The book equity to market equity breakpoints are the 30th and 70th NYSE percentiles:

| | | |
|---|---|---|
| | Small Value | Large Value |
| 70th Book Equity/Market Equity Percentile | | |
| | Small Neutral | Large Neutral |
| 30th Book Equity/Market Equity Percentile | | |
| | Small Growth | Large Growth |

The portfolios for July of year t to June of *t+1* include all NYSE, AMEX, and NASDAQ stocks for which (i) market equity data for December of *t-1* and June of *t* is available, and (ii) (positive) book equity data for *t-1*.

## Historical Returns of the Fama-French Series

Using the Fama-French series ("F-F"), Exhibit 8.1 depicts the growth of $1.00 invested in F-F small-growth, F-F small-value, F-F large-growth, and F-F large-value stocks from the end of 1926 to the end of 2020. All results assume reinvestment of dividends and exclude transaction costs. The top two performers during this time period were small-value and large-value stocks, followed by small-growth and large-growth stocks. Over the period from 1927 to 2020 (94 years), small-value stocks outperformed all other stock series in the graph. One dollar invested in small-value stocks at the end of 1926 grew to over $270,000 by the end of 2020. Alternatively, one dollar

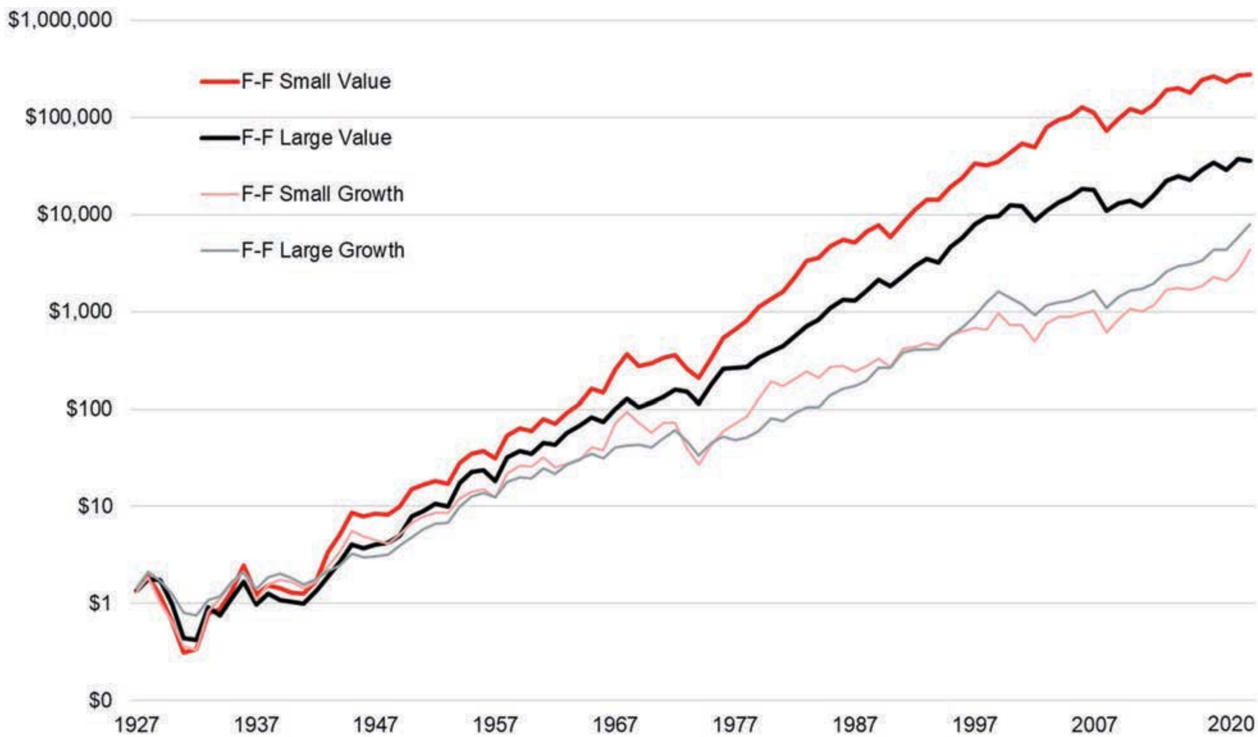
169 Source of Fama-French growth and value return series data used in this chapter: Monthly Historical Research Returns from the Kenneth R. French data library. Returns from the Kenneth R. French data library. Fama-French growth and value "research" portfolios are revised often. To learn more visit: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. All summary calculations performed by D&P/Kroll.

170 Eugene F. Fama is the 2013 Nobel laureate in economic sciences, the Robert R. McCormick Distinguished Service Professor of Finance at the University of Chicago Booth School of Business, and an advisory editor of the *Journal of Financial Economics*. Ken French is the Roth Family Distinguished Professor of Finance at the Tuck School of Business at Dartmouth College. Fama and French's paper "The Cross-Section of Expected Stock Returns" was the winner of the 1992 Smith Breeden Prize for the best paper in *The Journal of Finance*. See Eugene Fama and Kenneth French, "The Cross-Section of Expected Stock Returns," *Journal of Finance* (June 1992): 427–486. Also see Eugene F. Fama and Kenneth R. French, "A five factor asset pricing model," *The Journal of Financial Economics* 116 (2015): 1–22.

171 Source: Kenneth R. French data library at: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

invested in large value, large growth, and small growth stocks at the end of 1926 grew to approximately $35,000, $7,000, and $4,000, respectively, by the end of 2020.[172]

**Exhibit 8.1:** F-F Small-Value Stocks, F-F Small-Growth Stocks, F-F Large-Value Stocks, and F-F Large Growth Stocks Index (Year-End 1926 = $1.00) 1927–2020




**Source of underlying data**: Dr. Kenneth R. French's website at: https://mba.tuck.dartmouth.edu/pages/faculty/ ken.french/data_library.html. Series used: Under "6 Portfolios Formed on Size and Book-to Market (2 x 3)": (i) Small Value, (ii) Small Growth, (iii) Big (i.e., "Large") Value, and (iv) Big (i.e., "Large") Growth.

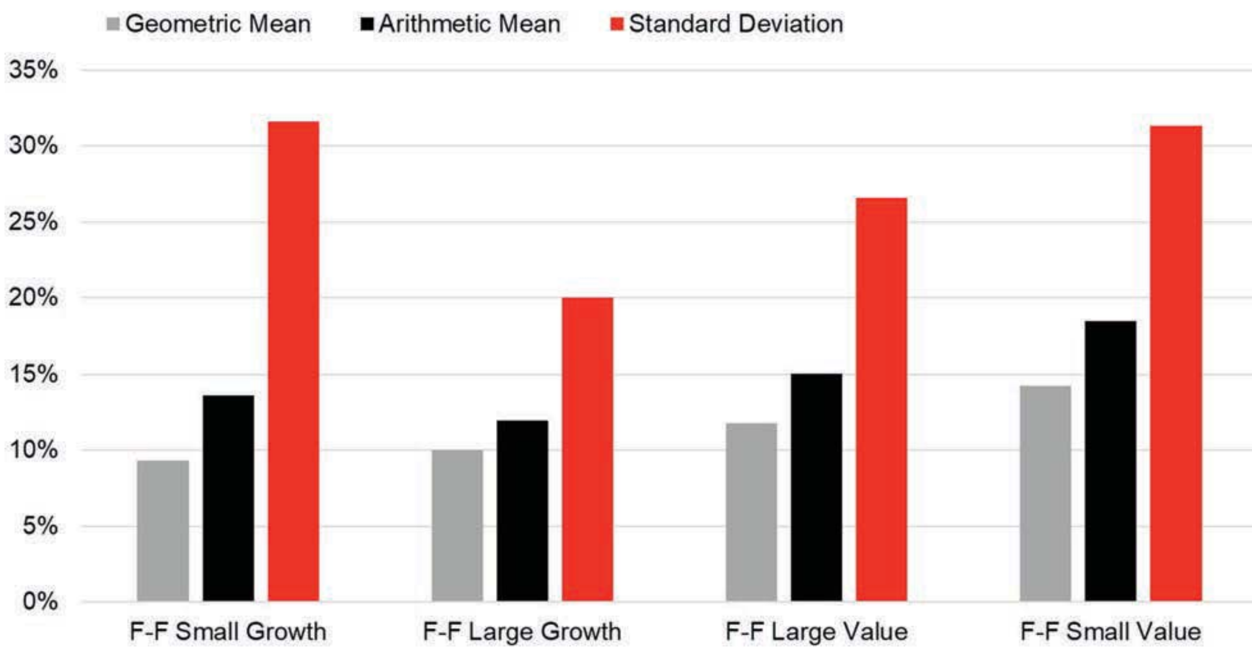
## Summary Statistics for the Fama-French Series

Exhibit 8.2 illustrates summary statistics of annual total returns for the Fama-French growth and value series from 1927 to 2020. The summary statistics are geometric mean, arithmetic mean, and standard deviation. Value significantly outperformed growth across the market capitalization spectrum. In the large-cap arena, the extra return of value over growth was at the expense of increased risk, as the standard deviation of large-value was over 26% versus approximately 20.0% for large-growth. In the small-cap series, small value significantly outperformed small-growth, and did so with the same volatility (approximately 32%).[173]

172 For more information on calculating index values, see Chapter 5, "Annual Returns and Indexes". Precalculated index values from 1927 through December 2020 are presented in the full-version *2021 SBBI® Yearbook* for the F-F growth and value series. For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

173 For more information on calculating annual returns and other summary statistics, see Chapter 5, "Annual Returns and Indexes". Precalculated summary statistics from 1927 through December 2020 are presented in the full-version *2021 SBBI® Yearbook* for the F-F growth and value series. For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

**Exhibit 8.2:** Illustration of Fama-French Growth and Value Series Summary Statistics of Annual Returns (%) 1927–2020




**Source of underlying data:** Dr. Kenneth R. French's website at: https://mba.tuck.dartmouth.edu/pages/faculty/ ken.french/data_library.html. Series used: Under "6 Portfolios Formed on Size and Book-to-Market (2 x 3)": (i) Small Value, (ii) Small Growth, (iii) Big (i.e., "Large") Value, and (iv) Big (i.e., "Large") Growth. Calculations by D&P/Kroll.

## Relative Performance of the Fama-French Growth and Value Series by Decade

Exhibit 8.3 shows the relative performance of the Fama-French growth and value series by decade as measured by compound annual return (best performer at top, worst performer at bottom). Small-value stocks beat small-growth stocks in all decades except the 1930s and the 2010s. It is also interesting to note that small-value stocks were never the worst performing among all four stock series in any decade, with the exception of the 2010s.

In Exhibit 8.3, "Value" (either Large Value or Small Value) was the *best* performer or the *second* best performer 15 times. Alternatively, "Growth" (either Large Growth or Small Growth) was the *best* performer or the *second* best performer 5 times.[174]

[174] For more information on calculating geometric (i.e., compound) returns over periods, see Chapter 5, "Annual Returns and Indexes". Precalculated compound annual returns by decade from 1927 through December 2019 are presented in the full-version *2020 SBBI® Yearbook* for the F-F growth and value series. For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

**Exhibit 8.3:** The Relative Performance of the Fama-French Growth and Value Series by Decade (Best Performer at Top, Worst Performer at Bottom)

| 1920s* | 1930s | 1940s | 1950s |
|---|---|---|---|
| F-F Large Value | F-F Small Growth | F-F Small Value | F-F Large Value |
| F-F Large Growth | F-F Small Value | F-F Large Value | F-F Small Value |
| F-F Small Value | F-F Large Growth | F-F Small Growth | F-F Small Growth |
| F-F Small Growth | F-F Large Value | F-F Large Growth | F-F Large Growth |

| 1960s | 1970s | 1980s |
|---|---|---|
| F-F Small Value | F-F Small Value | F-F Small Value |
| F-F Large Value | F-F Large Value | F-F Large Value |
| F-F Small Growth | F-F Small Growth | F-F Large Growth |
| F-F Large Growth | F-F Large Growth | F-F Small Growth |

| 1990s | 2000s | 2010s |
|---|---|---|
| F-F Large Growth | F-F Small Value | F-F Large Growth |
| F-F Small Value | F-F Large Value | F-F Small Growth |
| F-F Large Value | F-F Large Growth | F-F Large Value |
| F-F Small Growth | F-F Small Growth | F-F Small Value |

* Based on the period 1927–1929.

**Source of underlying data:** Dr. Kenneth R. French's website at: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. Series used: Under "6 Portfolios Formed on Size and Book-to- Market (2 x 3)": (i) Small Value, (ii) Small Growth, (iii) Big (i.e., "Large") Value, and (iv) Big (i.e., "Large") Growth. Calculations by D&P/Kroll.

## Correlation of Fama-French Series

Exhibit 8.4 presents the annual cross-correlations and serial correlations for the Fama-French growth and value series.

**Exhibit 8.4:** Fama-French Growth and Value Series Serial and Cross-Correlations of Historical Annual Returns 1927–2020

| | F-F Large Growth | F-F Large Value | F-F Small Growth | F-F Small Value | U.S.Treasury Bills | Inflation |
|---|---|---|---|---|---|---|
| F-F Large Growth | 1.00 | | | | | |
| F-F Large Value | 0.78 | 1.00 | | | | |
| F-F Small Growth | 0.81 | 0.77 | 1.00 | | | |
| F-F Small Value | 0.70 | 0.90 | 0.84 | 1.00 | | |
| U.S. Treasury Bills | -0.04 | 0.00 | -0.11 | -0.04 | 1.00 | |
| Inflation | -0.06 | 0.05 | 0.00 | 0.04 | 0.42 | 1.00 |
| Serial Correlation | 0.02 | -0.07 | 0.02 | 0.06 | 0.91 | 0.63 |

**Source of underlying data:** Dr. Kenneth R. French's website at: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. Series used: Under "6 Portfolios Formed on Size and Book-to- Market (2 x 3)": (i) Small Value, (ii) Small Growth, (iii) Big (i.e., "Large") Value, and (iv) Big (i.e., "Large") Growth. Calculations by D&P/Kroll.

## Conclusion

What can explain this value effect? Readers of Benjamin Graham and David L. Dodd's "Security Analysis," first published in 1934, would say that the outperformance of value stocks is due to the market coming to realize the full value of a company's securities that were once undervalued.[175]

The Graham and Dodd approach to security analysis is to do an independent valuation of a company using accounting data and common market multiples, then look at the stock price to see if the stock is under- or overvalued. Several academic studies have shown that the market *overreacts* to bad news and *underreacts* to good news. This would lead us to conclude that there is more room for value stocks (which are more likely to have reported bad news) to improve and outperform growth stocks which already have high expectations built into them.

[175] The sixth edition of this book was published in 2008. See also Cottle, S., Murray, R.F., & Block, F.E. 1988. Graham and Dodd's *Security Analysis*, 5th ed. (New York: McGraw-Hill).

# Chapter 9
# Liquidity Investing

## What Is Liquidity?[176]

Liquidity has many different, but similar meanings. In every case it is related to the ease of movement. Even within the context of financial markets, liquidity has several different meanings. In the banking system, liquidity measures the degree to which loans are made. In the securities markets, liquidity is the ease with which transactions can be made. In valuation, this liquidity impacts value, so that the more liquidity an asset has the more value it has, all other things being equal. The absence of liquidity lowers the value of the asset by the amount of a liquidity discount.

In this chapter, we focus on liquidity as the ease of trading securities in general, especially equities. We focus on liquidity's impact on valuation and in particular its impact on security returns. We will demonstrate that less liquid securities have higher expected returns.

## Valuation as Present Value of Cash Flows

In equilibrium, an asset has a value that equals its present value, or the discounted sum of its expected cash flows. These future cash flows are unobservable except for risk-free assets. For stocks, there is great disagreement as to what these expected cash flows might be. This disagreement is the primary reason that stocks are traded. A secondary reason is that they are bought or sold to meet liquidity needs.

The other component of a present value calculation is the discount rate. Similar to the expected cash flows, these discount rates are unobservable. We can usually observe the riskless discount rates from a term structure of riskless bonds which we unravel from U.S. government discount bonds. But there are usually other premiums that we would add to the riskless term structure. The most common one is an equity risk premium which is often modified by a beta in the CAPM framework. We might also add a premium for size and another one for value (or distress). We argue here that another premium should be added for lack of liquidity.

The difference of opinion that investors have about expected cash flows leads to the additional risk of a security. The risk of the security reflects not only the changing economy and company cash-flow expectations, but also the divergence of opinion that changes from moment to moment. This risk reduces the value of a security. Ironically though, this divergence of opinion also leads to most of the trading of a security, thereby making the security more liquid for trades, whether they be active or liquidity traders. The higher liquidity increases the security's value.

---

176 This chapter was written by Roger G. Ibbotson, Professor Roger G. Ibbotson, Professor Emeritus of Finance at the Yale School of Management, Chairman of Zebra Capital LLC, and former Chairman and founder of Ibbotson Associates, now part of Morningstar, Inc.

We do not mean to imply that most investors actually make these present-value calculations. Instead, investors rely on simple metrics, such as the price/earnings ratio, or PE ratio, trying to buy stocks with relatively high but unspecified cash flow projections, at relatively low PE ratios. Or they may simply feel that a stock's price is too low or high relative to its estimated value, leading them to buy or sell a security.

## The Liquidity Premium

Most conventional present-value calculations ignore the liquidity premium. These calculations usually implicitly assume that securities are perfectly liquid. If they are somewhat liquid, a liquidity discount is often made to the present value, at the end of the calculation. Thus, a liquid stock is priced at the present value of the expected cash flows, discounted by the riskless rate and various other risk premiums, such as a beta-adjusted equity risk premium, a size premium, and a value premium. The final present value is then reduced by some percentage due to its lack of liquidity.

The other way to calculate a present value is to add a liquidity premium into the discount rate. Less-liquid securities would then have their cash flows discounted at higher rates. The benefit of this approach is that this liquidity premium can be thought of as causing a higher discount rate. These discount rates are equivalent, under certain conditions, to the expected return that an investor receives for investing in less-liquid securities.

The liquidity premium is the extra return an investor would demand to hold a security that cannot costlessly be traded. This premium is not exactly a risk premium since it more reflects a transaction cost. We can think of the premium as related to risk, however, because it is the risk of having to buy or sell a security quickly. The less liquid and more hurried the transaction, the higher the cost.

The liquidity premium is potentially interesting to investors who can afford to hold a security over time, instead of continuously trading it. For investors with longer-term horizons, the trading costs become trivial because they happen so infrequently. The liquidity premium is a benefit to the longer-term investor. It means that the less liquid securities will have higher returns and these higher returns are not likely to be affected by trading costs.

It is sometimes argued that part of the expected return that is demanded from real estate, private equity, or venture capital comes from their relative liquidity.[177] In addition to any of their return for other risk characteristics, investors want an extra return for holding an illiquid asset. Thus, investors would want to invest in alternative illiquid assets only if they thought they would receive extra compensation for their lack of liquidity.

The liquidity premium also is substantial within publicly traded securities. There is a difference in the return of the more highly traded securities versus the less traded securities, even though most all public securities can be readily traded. We now examine the relative impact of liquidity across

[177] Ibbotson, Roger, Siegel, Laurence B., and Diermeier, Jeffrey, "The Demand for Capital Market Returns," *Financial Analysts Journal*, January/February 1984.

publicly traded stocks on the New York Stock Exchange (NYSE), the NYSE MKT (formerly the NYSE Amex), and the NASDAQ Stock Market.

## Liquidity and Stock Returns

In the U.S. stock market, liquidity has substantial impact on stock returns. We examine the monthly data for the largest 3,500 U.S. stocks by capitalization over the period 1972 through 2020. These stocks are traded on either the NYSE, the NYSE MKT, or the NASDAQ. All are publicly traded and relatively liquid, but of course some are more liquid than others.

We separate the stocks into four quartiles separated from the prior year by the turnover rate. The turnover rate is the number of shares traded during the year divided by the number of shares outstanding for the stock. The stocks with the highest turnover rates are the most liquid, and the stocks with the lowest turnover rates the least liquid. The return, share volume, and capitalization data are from the Center for Research in Security Prices at the University of Chicago Booth School of Business.

Exhibit 9.1 summarizes the results for the four liquidity quartiles. The exhibit illustrates the historical magnitude of the liquidity premium over the period from 1972–2020. Note that there is a substantial difference in the returns of the least-liquid quartile versus the most-liquid quartile, as well as a continual progression of higher returns as we move to less liquid quartiles. The less-liquid stocks are not necessarily more risky. Measured by the standard deviation, risk seems to increase with liquidity.

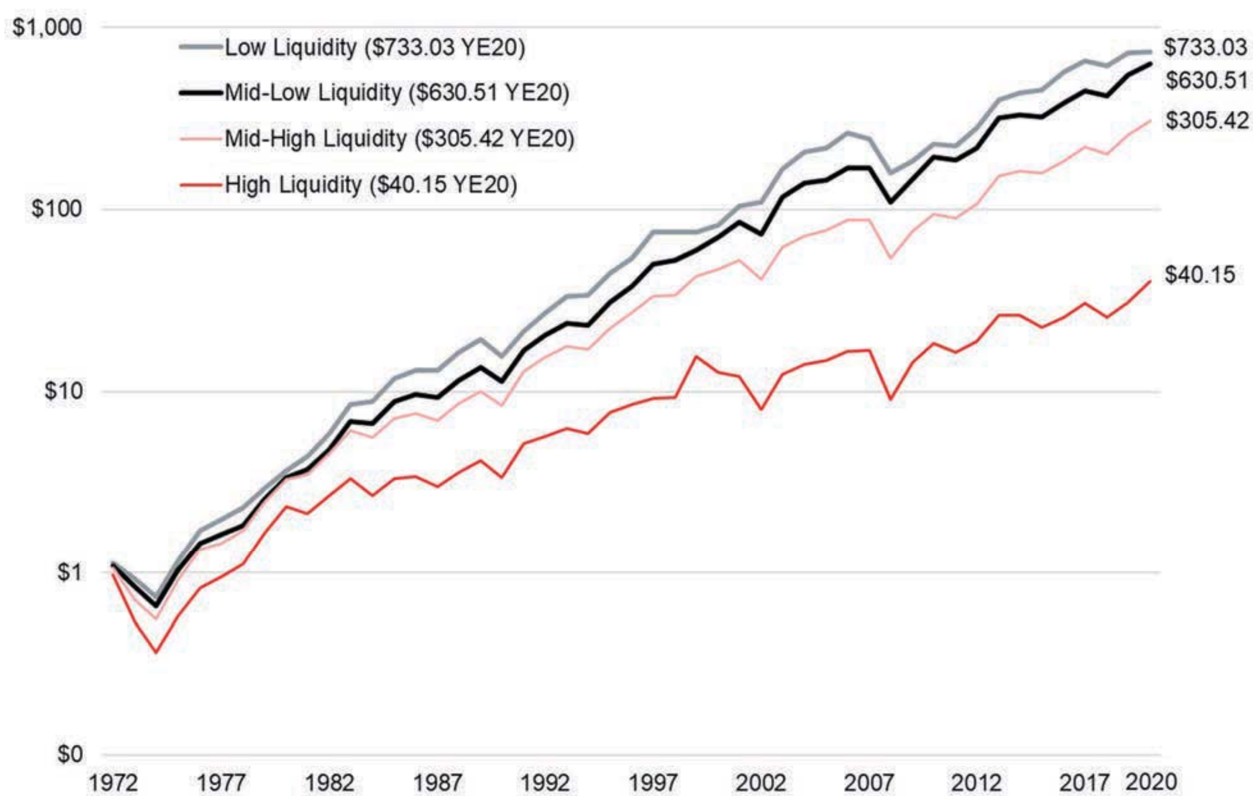
**Exhibit 9.1:** Liquidity Quartiles of the NYSE/NYSE MKT/NASDAQ, Annualized Returns (%) 1972–2020

| Quartile | Geometric Mean | Arithmetic Mean | Standard Deviation |
|---|---|---|---|
| 1-Less Liquid | 14.41 | 16.11 | 19.39 |
| 2 | 14.06 | 15.94 | 20.49 |
| 3 | 12.39 | 14.58 | 21.86 |
| 4-More Liquid | 7.83 | 11.20 | 26.77 |

Calculated by Zebra Capital Management at www.zebracapital.com. This is an update to the research published in Ibbotson, Roger G., and Daniel Y.-J Kim, "Liquidity as an Investment Style: 2018 Update," available at www.zebracapital.com, which itself is an updated version of the original paper: Ibbotson, Roger G., Chen, Zhiwu, Kim, Daniel Y.-J., and Hu, Wendy Y. "Liquidity as an Investment Style," *Financial Analysts Journal*, May/June 2013.

Exhibit 9.2 shows the same four quartiles of liquidity, but here presented as indexes of cumulative wealth. The quartiles consist of equally weighted portfolios with all dividends reinvested. The least-liquid quartile of stocks is at the top of the graph, and $1.00 invested at the end of 1971 grows to $733.03 by the end of 2020. One dollar invested in the second-least-liquid quartile grows to $630.51 over the period. One dollar invested in the third-least-liquid quartile (the second-most-liquid- quartile) grows to $305.42 over the period. One dollar invested at year-end 1971 into the most-liquid quartile grows to only $40.15 over the period. These large differences in terminal wealth reflect investments at different share turnover rates but include most types of companies in each liquidity quartile.

**Exhibit 9.2:** Wealth Indices of Investments in Low to High Quartiles of Liquidity in NYSE/NYSE MKT/ NASDAQ Stocks, Cumulative Total Returns: Index (Year-End 1971 = $1.00) 1972–2020




**Source:** Zebra Capital Management at www.zebracapital.com.

## Liquidity as an Investment Style[178]

Similar to small-versus-large or value versus-growth, liquid-versus-illiquid can be viewed as an investment style. Returns are on average higher for small, value, or illiquid stocks. In this way, liquidity can be thought of as another risk factor, with a risk premium. There are some years in which each style outperforms, as well as some years of underperformance, buteach style has a *long-run* positive payoff for investing in it.

Returns on stocks typically are greater than the returns on riskless (or default-free) bonds. This extra expected return is called the equity risk premium. The styles of investing can also add or

[178] Ibbotson, Roger G., and Daniel Y.-J Kim, "Liquidity as an Investment Style: 2018 Update," available at www.zebracapital.com. Updated version of: Ibbotson, Roger G., Chen, Zhiwu, Kim, Daniel Y.-J., and Hu, Wendy Y. "Liquidity as an Investment Style," *Financial Analysts Journal*, May/June 2013.

detract from the investor's return. In fact, styles explain about half of the cross-sectional variation in equity mutual funds, with stock selection, market timing, and fees explaining the other half. Styles seem to explain more of the variation in mutual fund portfolio returns than do industry sectors.[179]

The premiums in the equity market are as follows:

- **Equity Risk Premium:** The excess return of stocks relative to risk-free (default-free) government bonds. This premium can be measured over various bond horizons, and the bonds may themselves contain a horizon premium.

- **Size Premium:** The excess return on small stocks versus the return on larger stocks.

- **Value Premium:** The excess return on value stocks versus growth stocks.

- **Liquidity Premium:** The excess return on less-liquid stocks versus more-liquid stocks.

### Liquidity Versus Size

It is natural to think that liquidity and size would be related. The total number of shares of a company that are traded in a given period (say a year) are the number of shares outstanding times the turnover rate. Turnover is a measure of liquidity, adjusted for the number of shares outstanding.

Exhibit 9.3 breaks the universe of stocks into four turnover quartiles and four size-capitalization quartiles, each independently sorted. The numbers in the exhibit are the compound annual (geometric mean) rate of returns for each category. Note that small stocks tend to outperform large stocks in general, but not for the most-liquid stocks. In fact, for the most-liquid stocks shown in column four, the pattern is reversed. The poorest performing category is the highly liquid stocks that are the smallest in size (i.e., that upper-right quartile with a return of 0.33% per year).

The best-performing category is column one which represents the least-liquid stocks. The worst-performing category is column four, the most-liquid stocks. There is a clear pattern of generally decreasing returns as the liquidity of the stocks increase. The best-performing categories are the small, relatively less liquid stocks in the upper left corner of Exhibit 9.3. The Micro-Cap mid-low liquidity and Micro-Cap low liquidity categories performed the best over the 1972–2020 time horizon (15.57% and 15.13%, respectively). In previous SBBI® Yearbooks, the Micro-Cap low liquidity category performed the best, but the pattern is unchanged: small, less-liquid stocks tend to perform better than their more liquid counterparts, across all size categories.

179 Xiong, James X., Roger G. Ibbotson, Thomas M. Idzorek, and Peng Chen., "The Equal Importance of Asset Allocation and Active Management," *Financial Analysts Journal,* March/April 2010.

As shown in the low-minus-high liquidity column (i.e., “Liquidity Effect (%)”), the impact of liquidity is strongest for the smallest companies and weakest for the largest companies. However, the impact of liquidity is strong and consistent across all categories. Liquidity appears to be a much better predictor of returns than is size. Note the mixed results for size shown in the bottom small-minus large row (i.e.,”Size Effect (%)”).

**Exhibit 9.3:** Size and Liquidity Quartile Portfolios, Independently Sorted Each Year Compound Annual Returns (%) 1972–20**20**

| | Low Liquidity | Mid-Low Liquidity | Mid-High Liquidity | High Liquidity | Liquidity Effect (%) |
|---|---|---|---|---|---|
| **Micro-Cap** | | | | | |
| Geometric Mean (%) | 15.13 | 15.57 | 9.72 | 0.33 | **14.79*** |
| **Small-Cap** | | | | | |
| Geometric Mean (%) | 14.83 | 14.21 | 12.08 | 6.07 | **8.76** |
| **Mid-Cap** | | | | | |
| Geometric Mean (%) | 13.55 | 13.53 | 12.96 | 8.46 | **5.08*** |
| **Large-Cap** | | | | | |
| Geometric Mean (%) | 11.47 | 12.42 | 11.85 | 9.32 | **2.15** |
| **Size Effect (%)** | **3.66** | **3.15** | **-2.13** | **-8.98*** | |

*Difference due to rounding.

**Source:** Compound annual returns (%) from 1972–2020. Calculated by Zebra Capital Management at www.zebracapital.com. This is an update to the research published in Ibbotson, Roger G., and Daniel Y.-J Kim, “Liquidity as an Investment Style: 2018 Update,” available at www.zebracapital.com, which itself is an updated version of the original paper: Ibbotson, Roger G., Chen, Zhiwu, Kim, Daniel Y.-J., and Hu, Wendy Y. “Liquidity as an Investment Style,” *Financial Analysts Journal*, May/June 2013.

## Liquidity Versus Value/Growth

As noted in Chapter 8, value tends to outperform growth over time. In this chapter, less-liquid stocks are shown to outperform more liquid stocks. In this section, we examine how liquidity and value/growth interact.

The stocks are ranked by turnover rates and separated into quartiles. Similarly, the stocks are ranked by the earnings-to-price ratios and separated into quartiles. The high-earnings-to-price companies are considered value companies, while the low earnings-to-price companies are growth companies. The inverse, of course, is the PE ratio, with the growth companies having high PE ratios, and the value companies having low PE ratios.

The earnings used are the trailing reported earnings. The earnings data is from *Compustat*, owned by Standard & Poor’s. The portfolios are rebalanced once per year with the earnings lagged by two months to reflect delays in compiling the accounting earnings.

Exhibit 9.4 presents the quartile results for the different levels of liquidity and value/growth. Note that both liquidity and value/growth have a strong impact on stock market returns across all categories. The results appear to be additive. There is an excess return for investing in either low-liquidity or value stocks, and the best return of all was earned by investing in the upper-left category: high-value, low liquidity stocks, which have a realized return of 18.33%. The worst category is the lower-right corner, high liquidity growth stocks, which have a return of 2.84%.

**Exhibit 9.4:** Summary Statistics of Value vs. Growth and Liquidity Quartile Portfolios, Independently Sorted Each Year; Compound Annual Returns (%) 1972–2020

| | **Low Liquidity** | **Mid-Low Liquidity** | **Mid-High Liquidity** | **High Liquidity** | **Liquidity Effect (%)** |
|---|---|---|---|---|---|
| **High-Value** | | | | | |
| Geometric Mean (%) | 17.60 | 15.99 | 15.46 | 9.57 | **8.02*** |
| **Mid-Value** | | | | | |
| Geometric Mean (%) | 14.56 | 14.24 | 12.75 | 11.66 | **2.90** |
| **Mid-Growth** | | | | | |
| Geometric Mean (%) | 12.77 | 12.79 | 10.99 | 7.55 | **5.22** |
| **High-Growth** | | | | | |
| Geometric Mean (%) | 10.46 | 12.73 | 9.81 | 3.78 | **6.68** |
| **Size Effect (%)** | **7.14** | **3.27*** | **5.66*** | **5.80*** | |

**Source:** Compound annual returns (%) from 1972–2020. Calculated by Zebra Capital Management at www.zebracapital.com. This is an update to the research published in Ibbotson, Roger G., and Daniel Y.-J Kim, "Liquidity as an Investment Style: 2018 Update," available at www.zebracapital.com, which itself is an updated version of the original paper: Ibbotson, Roger G., Chen, Zhiwu, Kim, Daniel Y.-J., and Hu, Wendy Y. "Liquidity as an Investment Style," *Financial Analysts Journal*, May/June 2013.

## Conclusion

The results confirm that liquidity impacts returns across styles and locations. Investing in less liquid securities generates higher returns. Liquidity seems to be an investment style that is different from size or value. This result seems to hold up in almost any equity market subset and in any location.

*The following section is an excerpt from a CFA Institute Research Foundation monograph entitled,* Popularity: A Bridge Between Classical and Behavioral Finance *by Roger G. Ibbotson and colleagues Thomas M. Idzorek, CFA, Paul D. Kaplan, CFA, and James X. Xiong, CFA.*[180]

## What's Next?

For many years, academics have sought to explain and understand asset prices, with a strong emphasis on market premiums and market anomalies. These premiums and anomalies can be explained by social or behavioral phenomenon in many settings. In a 2014 article, Roger Ibbotson and Tom Idzorek said, "Most of the best-known market premiums and anomalies can be explained by an intuitive and naturally occurring (social or behavioral) phenomenon observed in countless settings: popularity.[181]

## Popularity

The existence of various market premiums and anomalies is well established in the finance literature. To date, however, no single agreed-upon explanation for them has emerged. Investment finance is largely divided into two camps, classical and behavioral. Classical finance is based mainly on the idea that investors are risk averse, so market premiums are generally interpreted as risk premiums. In behavioral finance, premiums are considered to be the result of either cognitive errors that investors systematically make or preferences for company or security characteristics that might not be related to risks. We believe that most of the best-known market premiums and anomalies can be explained by an intuitive and naturally occurring (social or behavioral) phenomenon observed in countless settings: popularity.

## What Is Popularity?

Popularity is the condition of being admired, sought after, well-known, and/or accepted. A wide range of possible categories – people, food, fashion, music, places to live, types of pet, vacation destinations, television shows, and so on – contain an implicit popularity spectrum or rank. Each of the categories has various criteria for estimating popularity.

For our purposes, the quality of the ranking criteria is not important; what is important is that any given category comprises a natural ordering in which some constituents are more popular than others. Such relative popularity evolves over time. Some aspects of popularity are systematic, or more or less permanent (for example, modern society seems to prefer thin to fat, tall to short). Other aspects of popularity may be transitory or exist only as fads (for example, necktie width, high-waisted jeans, men wearing wigs). Whether the result of systematic trends or idiosyncratic evolution, these rankings are in flux. Some popular items become relatively less popular, and

---

180 Available for download at: https://www.cfainstitute.org/en/research/foundation/2018/popularity-bridge-between-classical-and-behavioral-finance, or go to the CFA Institute website at cfainstitute.org and search for "popularity". Copyright 2018, CFA Institute Research Foundation. Reproduced from *Popularity: A Bridge between Classical and Behavioral Finance* with permission from CFA Institute Research Foundation. All rights reserved.

181 Ibbotson, R.G., Idzorek, T.H. "Dimensions of Popularity," *Journal of Portfolio Management*, Vol. 40 No. 5, (Special 40th Anniversary Issue 2014), P. 68–74.

some of the unpopular items become relatively more popular. While unsustainable, some popular items will temporarily become even more popular. For example, liquidity is permanently popular, but on a relative basis during times of market distress, it is especially sought after. Society places a greater relative value (monetary or otherwise) on the more popular items.

In *Popularity: A Bridge Between Classical and Behavioral Finance*, popularity refers to investor preferences – that is, how much an asset is liked or disliked. Of course, the primary preference for investors is to seek returns. Investors do not know what the returns will be, but they can distinguish one asset from another in terms of their observable characteristics, for which they may have clearly defined preferences. Thus, even with the same set of expected cash flows, investors may have more demand for one asset over another, which gives the preferred asset a higher current price and a lower expected return. An asset could be liked (or disliked) for *rational* or *irrational* reasons.[182]

In this way, popularity spans ideas from both classical and behavioral finance, thus providing a bridge between the two camps.

In classical finance, the primary preference, beyond maximizing expected return, is to take less risk. This fact has given rise to various models that usually assume no other preferences. In the most well-known model, the capital asset pricing model (CAPM), the only "priced" characteristic is exposure to undiversifiable market risk. We consider a broader set of preferences that lead to other priced characteristics, which might include the rational preferences to reduce catastrophic losses, increase liquidity, be tax efficient, and so on. We also consider preferences that might be more in line with what the literature considers "behavioral," such as desiring to hold companies with strong brands, investments with strong past price increases, or companies that have strong ESG (environmental, social, and governance) characteristics.

The popularity framework presented in *Popularity: A Bridge Between Classical and Behavioral Finance,* includes a generalization of a wide range of characteristics in classical finance and behavioral finance that influence how investors value securities. We can classify these characteristics into two broad categories with two subcategories each as follows:

## Classical Finance

- **Risks.** In classical finance, risk usually refers to fluctuations in asset values, but risk can be interpreted more broadly as any risks to which a rational investor, who assumes away any real-world frictions in the holding and trading of securities, would be averse. Thus,

[182] Throughout *Popularity: A Bridge Between Classical and Behavioral Finance*, we describe preferences, or the reasons for preferences, as being either rational or irrational. Rational reasons for preferences are those considered in classical finance, broadly defined. The reasons include expected returns, risk, liquidity, taxes, and trading costs. Generally, rational preferences are pecuniary. Irrational reasons for preferences generally are those identified in behavioral finance and result from the various biases and heuristics identified in that literature. Irrational preferences are generally nonpecuniary. Although Ibbotson, Diermeier, and Siegel (1984) acknowledged the possibility of nonpecuniary security characteristics playing a role in asset pricing (such as in the art market), their focus was on pecuniary characteristics that we consider to be subject to rational preferences. Our popularity framework extends their idea to irrational preferences.

risks may be multidimensional, including various types of stock or bond risks, or may arise from catastrophic events.

- **Frictional**. These characteristics are often assumed away in classical finance, but a rational investor would consider them. Examples include taxes, trading costs, and asset divisibility.

## Behavioral Finance

- **Psychological**. Investors consider these characteristics because of their psychological impact. For example, buying a company with a small carbon footprint might make an investor feel good.

- **Cognitive**. Investors consider these factors or fail to accurately interpret such factors because of systematic cognitive errors. For example, investors may overvalue the importance of a company's brand when evaluating its stock because they do not realize that the value of the brand is already embedded in the market price of the stock.

Our fourfold classification of security characteristics partially overlaps with the threefold classification in Statman (2017), in which investors are described as holding securities for utilitarian, expressive, and emotional reasons. Utilitarian reasons correspond risk and frictional characteristics, and expressive and emotional reasons correspond to psychological characteristics.

In *Popularity: A Bridge Between Classical and Behavioral Finance*, we focus primarily on the stock market, although we believe the concepts can be applied to fixed-income securities, real estate, and numerous other real assets. Periodically, as necessary, we attempt to distinguish between characteristics of a company and characteristics of the security in question – both of which can have attributes that are more or less popular among investors. Assets are priced not only by their expected cash flows but also by the popularity of the other characteristics associated with the company or security. The less popular stocks have lower prices (relative to the expected discounted value of their cash flows), thus higher expected returns. The more popular stocks have higher prices and, therefore, lower expected returns. Popularity can be related to risk (an unpopular characteristic), and it can also be related to other rational preferences. But popularity can also be related to behavioral concepts. For instance, investors may want to brag about their past winners (or purchase recent winners – for example, in the practice called "window dressing") or hold recognizable securities that are consistent with their social values. Any aspect that can affect the popularity of a stock will affect its demand and thus its price.[183]

Popularity is a bridge between classical finance and behavioral finance because both types of finance rely on preferences. Popularity is an expression of these preferences, whether they are\

[183] By demand, we mean the sum of the demand of all market participants.

rational, irrational, or somewhere in between.[184] Popularity does not make a value judgment but, instead, takes preferences as a given and recognizes that preferences can change over time. *Popularity: A Bridge Between Classical and Behavioral Finance* is presented in an equilibrium framework, so asset prices and expected returns reflect the aggregate impact of investor preferences.

[184] The same preference may be rational for one investor and irrational for another investor. For example, it is rational for a taxable investor to consider tax efficiency and irrational for nontaxable investor to seek out tax efficient investments.

# Chapter 10
# Using Historical Data in Wealth Forecasting and Portfolio Optimization

When forecasting the return on an asset or a portfolio, investors are (or should be) interested in the entire probability distribution of future outcomes, not just the mean or "point estimate." An example of a point estimate forecast is "large-cap stocks will have a return of 12% in 2021." It is more helpful to know the uncertainty surrounding this point estimate than to know the point estimate itself. One measure of uncertainty is standard deviation. The large cap stock return forecast can be expressed as 12% representing the mean and 20% representing the standard deviation.[185]

If the returns on large-cap stocks are normally distributed, the mean (expected return) and the standard deviation provide enough information to forecast the likelihood of any return. Suppose one wants to ascertain the likelihood that large-cap stocks will have a return of -25% or lower in 2021. Given the above example, a return of -25% is [12 - (-25)] / 20 = 1.9 standard deviations below the mean. The likelihood of an observation of 1.9 or more standard deviations below the mean is 2.9%. This can be looked up in any statistics textbook in the table showing values of the cumulative probability function for a normal distribution. Thus, a likelihood that the stock market will fall by 25% or more in 2020 is 2.9%. This is valuable information, both to the investor who believes that stocks are a sure thing and to the investor who is certain that they will crash tomorrow.

However, historical stock returns are not exactly normally distributed, and a slightly different method needs to be used to make accurate probabilistic forecasts. A description of the model used to forecast the distribution of stock returns appears later in this chapter.

## Probabilistic Forecasts

Probabilistic forecasts might seem to be too wide to be useful – the most widely quoted forecasters, after all, sometimes make very specific predictions. However, the forecast of a probability distribution actually reveals much more than the point estimate. The point estimate reflects what statisticians call an "expected value," but the actual return will likely be higher or lower than the point estimate. By knowing the extent to which actual returns are likely to deviate

[185] *Precalculated* summary statistics of annual returns (1926–2020) are presented in table format in the full-version *2021 SBBI® Yearbook* for the following Ibbotson Associates(IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: Large-Cap Stocks (total return, income return, and capital appreciation return), Small-Cap Stocks (total return), Long-term Corporate (i.e., 20-year) Bonds (total return), Long-term (i.e., 20-year) Government Bonds (total return, income return, and capital appreciation return), Intermediate-term (5-year) Government Bonds (total return, income return, and capital appreciation return), (30-day) U.S. Treasury Bills (total return), and Inflation. For more information, visit dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

from the point estimate, the investor can assess the risk of every asset, and thus compare investment opportunities in terms of their risks as well as their expected returns. As Harry Markowitz showed nearly a half century ago in his Nobel Prize-winning work on portfolio theory, investors care about avoiding risk as well as seeking return. Probabilistic forecasts enable investors to quantify these concepts.

### The Lognormal Distribution

In the lognormal model, the natural logarithms of asset return relatives are assumed to be normally distributed. A return relative is one plus the return. That is, if an asset has a return of 15% in a given period, its return relative is 1.15 (1 + 0.15).

The lognormal distribution is skewed to the right. This means that the expected value, or mean, is greater than the median. Furthermore, if return relatives are lognormally distributed, returns cannot fall below negative 100%. These properties of the lognormal distribution make it a more accurate characterization of the behavior of market returns than does the normal distribution.

In all normal distributions, moreover, the probability of an observation falling one standard deviation below the mean equals the probability of falling one standard deviation above the mean; each has a probability of about 34%. In a lognormal distribution, these probabilities differ and depend on the parameters of the distribution.

### Forecasting Wealth Values and Rates of Return

Using the lognormal model, it is fairly simple to form probabilistic forecasts of both compound rates of return and ending period wealth values. Wealth at time *n* (assuming reinvestment of all income and no taxes) is:

$$W_n = W_0(1+r_1)(1+r_2)...(1+r_n)$$

Where:

| | | |
|---|---|---|
| $W_n$ | = | The wealth value at time *n* |
| $W_0$ | = | The initial investment at time 0 |
| $r_1$, $r_2$, etc. | = | The total returns on the portfolio for the rebalancing ending at times 1, 2, and so forth |

The compound rate of return or geometric mean return over the same period, $r_G$, is:

$$r_G = \left(\frac{W_n}{W_0}\right)^{\frac{1}{n}} - 1$$

Where:

$r_G$ = The geometric mean return

$W_n$ = The ending period wealth value at time $n$

$W_0$ = The initial wealth value at time 0

$n$ = The inclusive number of periods

By assuming that all $(1+r_n)$ values are lognormally distributed with the same expected value and standard deviation and are all statistically independent of each other, it follows that $W_n$ and $(1+r_G)$ are lognormally distributed. In fact, even if the $(1+r_n)$ values are not themselves lognormally distributed but are independent and identically distributed, $W_n$ and $(1+r_G)$ are approximately lognormal for large enough values of $n$. This "central-limit theorem" means that the lognormal model can be useful in long-term forecasting even if short term returns are not well described by a lognormal distribution.

**Calculating Parameters of the Lognormal Model**

To use the lognormal model, we must first calculate the expected value and standard deviation of the natural logarithm of the return relative of the portfolio. These parameters, denoted $m$ and $s$ respectively, can be calculated from the expected return ($m$) and standard deviation ($s$) of the portfolio as follows:

$$m = \ln(1+\mu) - \left(\frac{s^2}{2}\right)$$

$$s = \sqrt{\ln\left[1+\left(\frac{\sigma}{1+\mu}\right)^2\right]}$$

Where:

ln = The natural logarithm function

To calculate a particular percentile of wealth or return for a given time horizon, the only remaining parameter needed is the z score of the percentile. The z-score of a percentile ranking is that percentile ranking expressed as the number of standard deviations that it is above or below the mean of a normal distribution. For example, the z-score of the 95th percentile is 1.645 because in a normal distribution, the 95th percentile is 1.645 standard deviations above the 50th percentile or median, which is also the mean. Z-scores can be obtained from a table of cumulative values of the standard normal distribution or from software that produces such values.

Given the logarithmic parameters of a portfolio (*m* and *s*), a time horizon (*n*), and the z-score of a percentile (*z*), the percentile in question in terms of cumulative wealth at the end of the time horizon ($W_n$) is:

$$e^{\left(mn+zs\sqrt{n}\right)}$$

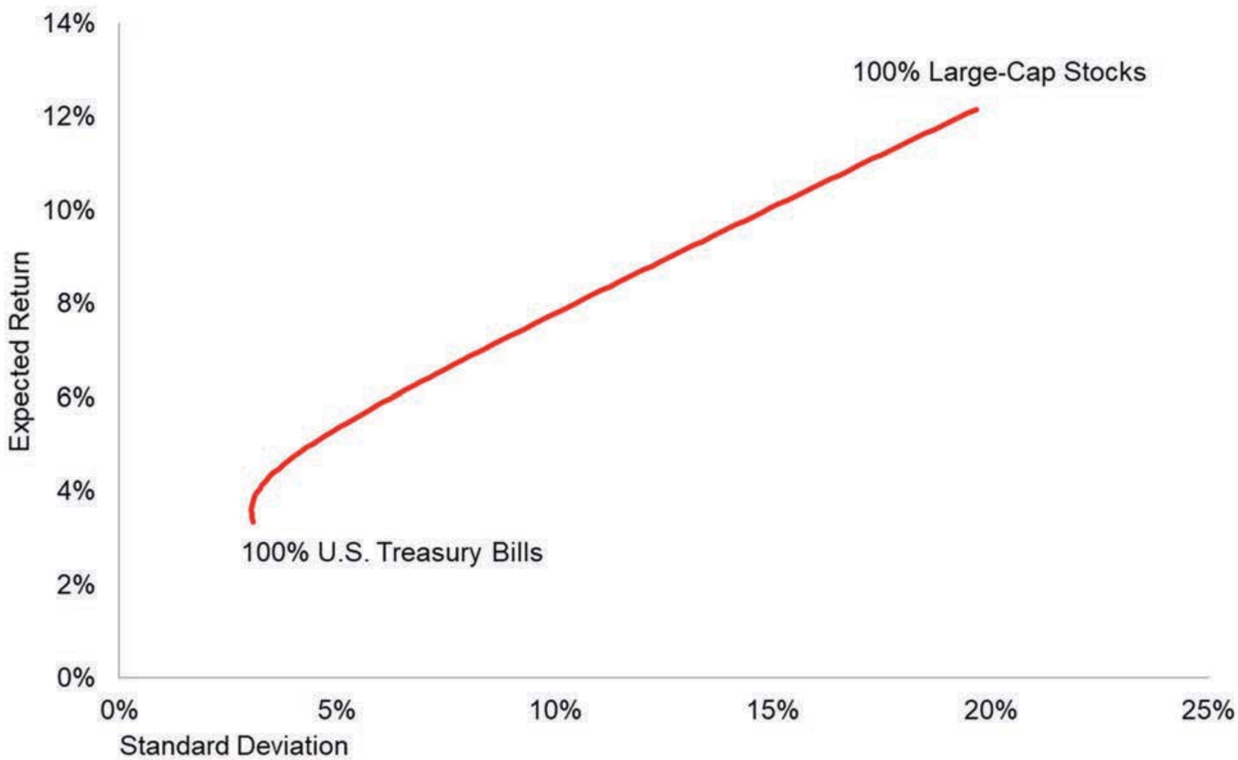
Similarly, the percentile in question in terms of the compound rate of return for the period ($r_G$) is:

$$e^{\left(m+z\frac{s}{\sqrt{n}}\right)}-1$$

## Mean-Variance Optimization

One important application of the probability forecasts of asset returns is mean variance optimization. Optimization is the process of identifying portfolios that have the highest possible expected return for a given level of risk or the lowest possible risk for a given expected return. Such a portfolio is considered "efficient," and the locus of all efficient portfolios is called the efficient frontier. A simple two-asset efficient frontier constructed from large-cap stocks and U.S. Treasury Bills is shown in Exhibit 10.1. All investors should hold portfolios that are efficient with respect to the assets in their opportunity set.

**Exhibit 10.1:** Efficient Frontier; Large-cap Stocks and U.S. Treasury Bills 1926–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, and (ii) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

The most widely accepted framework for optimization is Markowitz, or mean-variance, optimization (MVO), which makes the following assumptions: (i) The forecast mean, or expected return, describes the attribute that investors consider to be desirable about an asset; (ii) The risk of the asset is measured by its expected standard deviation of returns; and (iii) The interaction between one asset and another is captured by the expected correlation coefficient of the two assets' returns. MVO thus requires forecasts of the return and standard deviation of each asset and the correlation of each asset with every other asset.[186]

In the 1950s, Harry Markowitz developed both the concept of the efficient frontier and the mathematical means of constructing it (mean-variance optimization).[187]

186 The standard deviation is the square root of the variance; hence the term "mean-variance" in describing this form of the optimization problem.

187 Markowitz, H.M. 1959. *Portfolio Selection: Efficient Diversification of Investments* (New York: John Wiley & Sons).

## Estimating Returns, Risks, and Correlations

To simulate future probability distributions of asset and portfolio returns, one typically estimates parameters of the historical return data. The parameters that are required to simulate returns on an asset are its mean and standard deviation. To simulate returns on portfolios of assets, one must also estimate the correlation of each asset in the portfolio with every other asset. Thus, the parameters required to conduct a simulation are the same as those required as inputs into a mean-variance optimization.[188] The techniques used to estimate these parameters are described below.

### Means, or Expected Returns

The mean return (forecast mean or expected return) on an asset is the probability-weighted average of all possible returns on the asset over a future period. Estimates of expected returns are based on models of asset returns. While many models of asset returns incorporate estimates of gross national product, the money supply, and other macroeconomic variables, the model employed in this chapter does not. This is because we assume (for the present purpose) asset markets are informationally efficient, with all relevant and available information fully incorporated in asset prices. If this assumption holds, investor expectations (forecasts) can be discerned from market-observable data. Such forecasts are not attempts to outguess, or beat, the market. They are attempts to discern the market's expectations, i.e., to read what the market itself is forecasting.

For some assets, expected returns can be estimated using current market data alone. For example, the yield on a riskless bond is an estimate of its expected return. For other assets, current data are not sufficient. Stocks, for example, have no exact analogue to the yield on a bond. In such cases, we use the statistical time series properties of historical data in forming the estimates.

To know which data to use when estimating expected returns, we need to know the rebalancing frequency of the portfolios and the investment horizon. In our example we will assume an annual rebalancing frequency and a 20-year planning horizon. The rebalancing frequency gives the time units in which returns are measured.

With a 20-year horizon, the relevant riskless rate is the yield on a 20-year coupon bond. This riskless rate is the baseline from which the expected return on every other asset class is derived by adding or subtracting risk premiums.

[188] It is also possible to conduct a simulation using entire data sets, that is, without estimating the statistical parameters of the data sets. Typically, in such a nonparametric simulation, the frequency of an event occurring in the simulated history is equal to the frequency of the event occurring in the actual history used to construct the data set.

## Large-Cap Stocks

The expected return on large-cap stocks is the riskless rate, plus the expected risk premium of large-cap stocks over bonds that are riskless over the investment horizon.

## Bonds and Bills

For default-free bonds with a maturity equal to the planning horizon, the expected return is the yield on the bond. For bonds with other maturities, the expected bond horizon premium should be added to the riskless rate (for longer maturities) or subtracted from the riskless rate (for shorter maturities). Because expected capital gains on a bond are zero, the expected horizon premium is estimated by the historical average difference of the income returns on the bonds.[189]

For U.S. Treasury Bills, the expected return over a given time horizon equals the expected return on a Treasury bond of a similar horizon, less the expected horizon premium of bonds over bills. The long-term horizon premium is estimated by the historical average of the difference of the income return on bonds and the return on bills This forecast typically differs from the current yield on a U.S. Treasury Bill because a portfolio of U.S. Treasury Bills is rolled over (the proceeds of maturing bills are invested in new bills, at yields not yet known) during the time horizon described.

## Standard Deviations

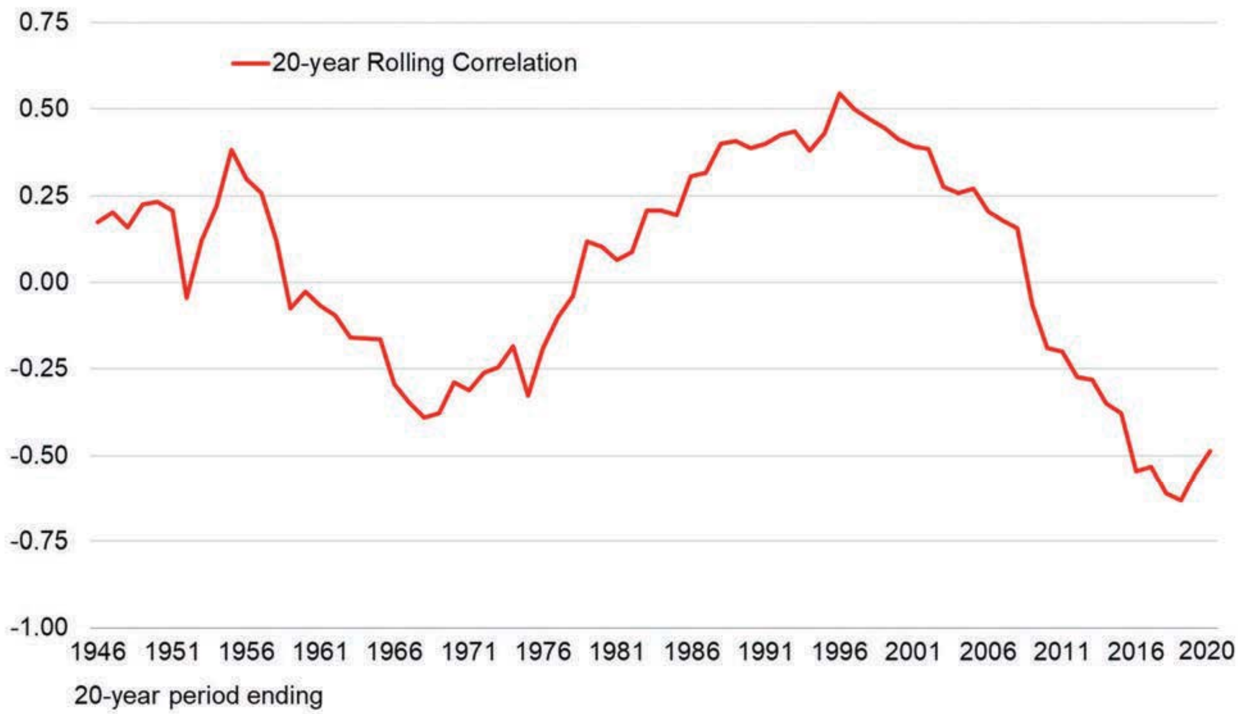
Standard deviations can be estimated from historical data as described in Chapter 6. There is no evidence of a major change in the variability of returns on large-cap stocks, so the entire period 1926 to 2020 can be used to estimate the standard deviation of these asset classes. For long-term government bonds and Treasury bills, the period 1970 to 2020 can be used to estimate these inputs. This is because the departure from the Bretton Woods fixed-rate currency exchange agreement in the early 1970s caused a structural shift in the U.S. interest-rate environment. As a result, bond volatility spiked and has remained well above levels experienced before the regime shift, rendering pre-1970 risk comparisons inappropriate.

## Correlations

Correlations between the asset classes are estimated from historical data as described in Chapter 6. Correlations between major asset classes change over time. Exhibit 10.2 shows the historical correlation of annual returns on large-cap stocks and long-term government bonds over 20-year rolling periods from 1926–1945 through 2000–2020.

[189] The expected capital gain on a par bond is self-evidently zero. For a zero coupon (or other discount) bond, investors expect the price to rise as the bond ages, but the expected portion of this price increase should not be considered a capital gain. It is a form of income return.

**Exhibit 10.2:** 20-Year Rolling-Period Correlations of Annual Returns of Large-Cap Stocks and Long-term Government Bonds 1926–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, and (ii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

## Using Inputs to Form Other Portfolios

In Exhibit 10.3, inputs are provided that can be used in forming portfolios.[190]

[190] *Precalculated* "Building Blocks for Expected Return Construction" are presented in table format in the full-version *2020 SBBI® Yearbook* as of December 31, 2020 for the following: (i) **Yields** (Long-term (20-year) U.S. Treasury Coupon Bond Yield; Intermediate-term (5-year) U.S. Treasury Coupon Note Yield; Short-term (30-day) U.S. Treasury Bill Yield), (ii) **Fixed Income Risk Premiums** (Expected default premium; Expected long-term horizon premium; Expected intermediate-term horizon premium, (iii) **Equity Risk Premiums** (Long-horizon expected equity risk premium; Intermediate-horizon expected equity risk premium; Short-horizon expected equity risk premium; Small-cap premium). For more information, visit: dpcostofcapital.com/stocks-bonds-bills-inflation-sbbi-yearbook.

**Exhibit 10.3:** Optimization Inputs: Year-end 2020 Large-Cap Stocks, Long-term Government Bonds, and U.S. Treasury Bills (%)

| | | | Correlation | | |
|---|---|---|---|---|---|
| | Expected Return (%) | Standard Deviation (%) | Stocks | Bonds | Bills |
| Stocks | 8.6 | 19.7 | 1.00 | | |
| Bonds | 1.4 | 11.7 | 0.01 | 1.00 | |
| Bills | 0.1 | 3.4 | -0.02 | 0.17 | 1.00 |

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, and (iii) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

Given a complete set of inputs, the expected return and standard deviation of any portfolio (efficient or other) of the asset classes can be calculated. The expected return of a portfolio is the weighted average of the expected returns of the asset classes:

$$r_p = \sum_{i=1}^{n} x_i r_i$$

Where:

$r_p$ = The expected return of the portflio $p$

$n$ = The number of asset classes

$x_i$ = The portfolio weight of asset class $i$, scaled such that:

$$\sum_{i=1}^{n} x_i = 1$$

Where:

$r_i$ = The expected return of asset class $i$

For example, referring to the inputs in Exhibit 10.3, a portfolio comprised of large-cap stocks only would have an expected return of 9.4% and a standard deviation of 19.8%. If the portfolio mix is changed to, say, 60.0% large-cap stocks, 35.0% long-term government bonds, and 5.0% U.S. Treasury Bills, the expected return of this new portfolio mix can be calculated by applying the above formula (again, using the inputs in Exhibit 10.3):

5.7% = (60.0% x 8.6%) + (35.0% x 1.4%) + (5.0% x 0.1%)

The standard deviation of the portfolio depends not only on the standard deviations of the asset classes, but also on all of the correlations. It is given by:

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \sum_{i=1}^{n} x_i x_j \sigma_i \sigma_j p_{ij}}$$

Where:

| | | |
|---|---|---|
| $\sigma_p$ | = | The standard deviation of the portfolio |
| $x_i$ and $x_j$ | = | The portfolio weights of asset classes *i* and *j* |
| $\sigma_i$ and $\sigma_j$ | = | The standard deviations of returns on asset classes *i* and *j* |
| $p_{ij}$ | = | The correlation between returns on asset classes *i* and *j* |
| | | *(note that $r_{ij}$ equals one and that $r_{ij}$ is equal to $r_{ji}$).* |

The standard deviation of the new portfolio (60.0% large-cap stocks, 35.0% long-term government bonds, and 5.0% U.S. Treasury Bills) can be calculated using the inputs from Exhibit 10.3 as shown in Exhibit 10.4:

**Exhibit 10.4:** Calculation of Example Portfolio Comprised of 60.0% Large-cap stocks, 35.0% Long-term Government Bonds, and 5.0% U.S. Treasury Bills

| **Stocks (asset class 1)** | **Bonds (asset class 2)** | **Bills (asset class 3)** |
|---|---|---|
| Stocks & Stocks | Stocks & Bonds | Stocks & Bills |
| $x_1^2\sigma_1^2 p_{1,1} =$ $0.60^2 \times 0.197^2 \times 1.00 =$ | $x_1 x_2 \sigma_1 \sigma_2 p_{1,2} =$ $0.60 \times 0.35 \times 0.197 \times 0.117 \times 0.015 =$ | $x_1 x_3 \sigma_1 \sigma_3 p_{1,3} =$ $0.60 \times 0.05 \times 0.197 \times 0.034 \times -0.024 =$ |
| =0.013923 | =0.000070 | =-0.000005 |
| Bonds & Stocks | Bonds & Bonds | Bonds & Bills |
| $x_1 x_2 \sigma_1 \sigma_2 p_{1,2} =$ $0.35 \times 0.60 \times 0.117 \times 0.197 \times 0.015 =$ | $x_2^2\sigma_2^2 p_{2,2} =$ $0.35^2 \times 0.117^2 \times 1.00 =$ | $x_2 x_3 \sigma_2 \sigma_3 p_{2,3} =$ $0.35 \times 0.05 \times 0.117 \times 0.034 \times 0.168 =$ |
| =0.000070 | =0.001689 | =0.000012 |
| Bills & Stocks | Bills & Bonds | Bills & Bills |
| $x_1 x_3 \sigma_1 \sigma_3 p_{1,3} =$ $0.05 \times 0.60 \times 0.034 \times 0.197 \times -0.024 =$ | $x_2 x_3 \sigma_2 \sigma_3 p_{2,3} =$ $0.05 \times 0.35 \times 0.034 \times 0.117 \times 0.168 =$ | $x_3^2\sigma_3^2 p_{3,3} =$ $0.05^2 \times 0.034^2 \times 1.00 =$ |
| =-0.000005 | =0.000012 | =0.000003 |

By summing these terms and taking the square root of the total, the result is a standard deviation of 12.6%.

$$\sqrt{\begin{matrix} 0.013923 + 0.000070 + -0.000005 + \\ 0.000070 + 0.001689 + 0.000012 + \\ -0.000005 + 0.000012 + 0.000003 \end{matrix}} = 12.6\%$$

## Enhancements to Mean-Variance Optimization

Ibbotson Associates was an early adopter of mean-variance optimization to develop asset class model guidelines and continues to assist the industry in the development of enhancements to the traditional mean-variance approach as well as the state-of-the-art techniques described later in the chapter. Over the last half century, the Markowitz mean-variance optimization (MVO) framework has become the textbook approach for creating these optimal asset allocations, but the approach has several shortcomings.

### Shortcomings of Traditional Optimization Techniques

One notable shortcoming is that the output (optimal asset allocation weights) is very sensitive to the inputs (expected returns, standard deviations, and correlations). Input sensitivity often leads to highly concentrated allocations in only a small number of the available asset classes. For example, if a typical optimization starts with an opportunity set of about 10 asset classes, just a few of these asset choices might end up in the resulting optimal allocation with the remaining asset choices not even getting a mention.

Mean-variance optimization is a powerful tool, but it needs to be used with caution. For instance, basing mean-variance optimization inputs on shorter periods can contribute to extreme results. Basing the mean-variance optimization inputs on longer periods, such as those presented elsewhere in this book, can help mitigate the extreme asset allocations mixes. Also, there is usually a more consistent ratio of return to risk amongst the different asset classes when using longer periods.

Placing maximum and minimum allocation constraints on each asset is the most common solution to the problem of highly concentrated asset allocations. For instance, we could specify a minimum allocation of 5% and a maximum allocation of 15% for each of the nine asset choices. This would ensure that each asset gets represented in the final allocation and that no single asset completely dominates in the final allocation mix. Unfortunately, these artificial minimums and maximums are arbitrary and usually end up limiting the ability of the optimizer to properly act on the information contained in the inputs.

### Black-Litterman and Resampling Techniques

Two popular enhancements to traditional optimization techniques have emerged in recent years that can help overcome these difficulties. While both of these methods can help develop well diversified asset allocations, they approach the problem in very different ways. The first of these, the Black-Litterman model, attempts to create better inputs. The second, resampled mean variance optimization, attempts to build a better optimizer.

The Black-Litterman model was created by Fischer Black and Robert Litterman in the late 1980s. The Black-Litterman model combines investors' views regarding expected returns and the expected returns predicted by the capital asset pricing model to form a single blended estimate of expected returns. When this new combined estimate is used as an input within a traditional mean-variance optimization framework, it produces well-diversified portfolios that include not only market-based asset allocations but also allocations in assets that received favorable views.

The second approach, resampled mean-variance optimization, grew out of the work of a number of authors, but it is most closely associated with the work of Richard Michaud. While traditional mean-variance optimization treats the capital market assumptions as if they were known with complete certainty, resampled mean-variance optimization recognizes that the capital market assumptions are forecasts and are therefore not known with complete certainty.

Conceptually, resampled mean-variance optimization is a combination of Monte Carlo simulation and the more traditional Markowitz mean-variance optimization approach.[191] The simulation randomly resamples possible returns from a forecasted return distribution or randomly resamples possible returns from a historical distribution. The simulated returns lead to a simulated set of capital market assumptions that are used in a traditional mean-variance optimizer, and the asset allocations are recorded. After combining the asset allocations from the numerous intermediate optimizations, the resulting asset allocations are those that, on average, are predicted to perform best over the range of potential outcomes implied by the capital market assumptions. Research has shown that asset allocations selected from a resampled efficient frontier may outperform those from a traditional efficient frontier.[192]

In addition to the problem of getting results that are highly concentrated in just a few of the assets available, there are two more criticisms of the traditional mean-variance optimization framework.

First, the traditional approach focuses on a subset of the total portfolio. Traditionally, the focus is on finding a mix of asset classes that maximizes the expected return, subject to a risk constraint. However, because the purpose of most asset portfolios is to fund a specified future cash-flow

---

191 Monte Carlo simulation is a problem-solving technique utilized to approximate the probability of certain outcomes by performing multiple trial runs, called simulations, using random variables. The probability distribution of the results is calculated and analyzed in order to infer which outcomes are most likely to be produced.

192 See Markowitz, H. & Usmen, N. 2003. "Resampled Frontiers vs. Diffuse Bayes: An Experiment." *Journal of Investment Management*, Vol. 1, No. 4.

stream – a liability – the true risk for the portfolio is not the standard deviation of the assets or the performance of the assets relative to that of peers, but not being able to fund the future liability. An asset allocation approach that takes the future liability into account is called liability-relative optimization (or surplus optimization). The usual method employed to accomplish this is to constrain the optimizer to hold short an asset class representing the liability.

Second, the traditional mean-variance optimization framework assumes that the returns of the assets in the optimization are normally distributed. As illustrated in Exhibit 2.3, the return distributions of different asset classes do not always follow a standard, symmetrical bell-shaped curve. Some assets have distributions that are skewed to the left or right, while others have distributions that are skinnier or fatter than others. These more complicated characteristics are called skewness and kurtosis, respectively. The next wave of enhancements to the traditional mean-variance optimization are frameworks that incorporate these additional types of abnormalities into the optimization.

## Markowitz 1.0

In 1952, Harry Markowitz, invented portfolio optimization. His genius was based on three principles: risk, reward and the correlation of assets in a portfolio. Over the years, technologies advanced, markets crashed, but the portfolio optimization models used by many investors did not evolve to compensate. This is surprising in light of the fact that Markowitz was a pioneer of technological advancement in the field of computational computer science. Furthermore, he did not stand by idly in the area of portfolio modeling but continued to make improvements in his own models and to influence the models of others. Few of these improvements, however, were picked up broadly in practice.

Because Markowitz's first effort was so simple and powerful, it attracted a great number of followers. The greater the following became, the fewer questioners debated its merits. Markowitz's original work is synonymous with modern portfolio theory and has been taught in business schools for generations and, not surprisingly, is still widely used today.

Then came the crash of 2008, and people started to ask questions. The confluence of the economic trauma and the technological advances of recent decades made the post-crash environment the perfect moment to upgrade to a new model built around Markowitz's fundamental principles of risk, reward and correlation. We dub our updated model "Markowitz 2.0." This section is an adaptation of a 2009 article, "The New Efficient Frontier," by Paul D. Kaplan, Morningstar Canada's director of research, and Sam L. Savage, consulting professor at Stanford University.

## Markowitz 2.0

### The Flaw of Averages

The 1952 mean-variance model of Harry Markowitz was the first systematic attempt to cure what Savage (2009) called the "flaw of averages." In general, the flaw of averages is a set of systematic errors that occur when people use single numbers (usually averages) to describe uncertain future quantities. For example, if you plan to rob a bank of $10 million and have one chance in 100 of getting away with it, your average take is $100,000. If you described your activity beforehand as "making $100,000" you would be correct on average. But this is a terrible characterization of a bank heist. Yet as Savage discussed, this very "flaw of averages" is made all the time in business practice and helps explain why everything is behind schedule, beyond budget, and below projection. This phenomenon was an accessory to the global financial crisis that culminated in 2008.

Markowitz's mean-variance model distinguished between different investments that had the same average (expected) return but different risks, measured as variance or its square root (standard deviation). This breakthrough systematic attempt to cure the flaw of averages ultimately garnered a Nobel Prize for its inventor. However, the use of standard deviation and covariance introduces a higher order version of the flaw of averages in that these concepts are themselves a version of averages.

### Making a Great Idea Better

By taking advantage of the very latest in economic thought and computer technology, we can, in effect, add more thrust to the original framework of the Markowitz portfolio optimization model. The result is a dramatically more powerful model that is more aligned with 21st century investor concerns, markets, and financial instruments, such as options.

Our discussion here will focus on five practical enhancements to traditional portfolio optimization that can be made with current technology:

1. First, we use a scenario-based approach to allow for fat-tailed distributions. "Fat-tailed" return distributions are not possible within the context of traditional mean-variance optimization where return distributions are assumed to be adequately described by mean and variance.

2. Second, we replace the single-period expected return with the long-term forward-looking geometric mean as this takes into account accumulation of wealth.

3. Third, we substitute conditional value at risk, or CVaR, which focuses on tail risk for standard deviation, which looks at average variation.

4. Fourth, the original Markowitz model used a covariance matrix to model the distribution of returns on asset classes; we replace this with a scenario-based model that can be generated with Monte Carlo simulation and can incorporate any number of distributions.

5. Finally, we exploit new statistical technologies pioneered by Savage in the field of probability management. Savage invented an astonishing new technology called the Distribution String, or DIST, which encapsulates thousands of trials as a single data element or spreadsheet cell, thus eliminating the main disadvantage of the scenario-based approach – the need to store and process large amounts of data.

**The Scenario Approach vs. Lognormal Distributions**

One of the limitations of the traditional mean-variance optimization framework assumes the distribution of returns of the assets in the optimization can be adequately described simply by mean and variance alone. The most common depiction of this assumption is to draw the distribution of each asset class as a symmetrical bell-shaped curve, but asset class returns do not always fall into normal distributions.

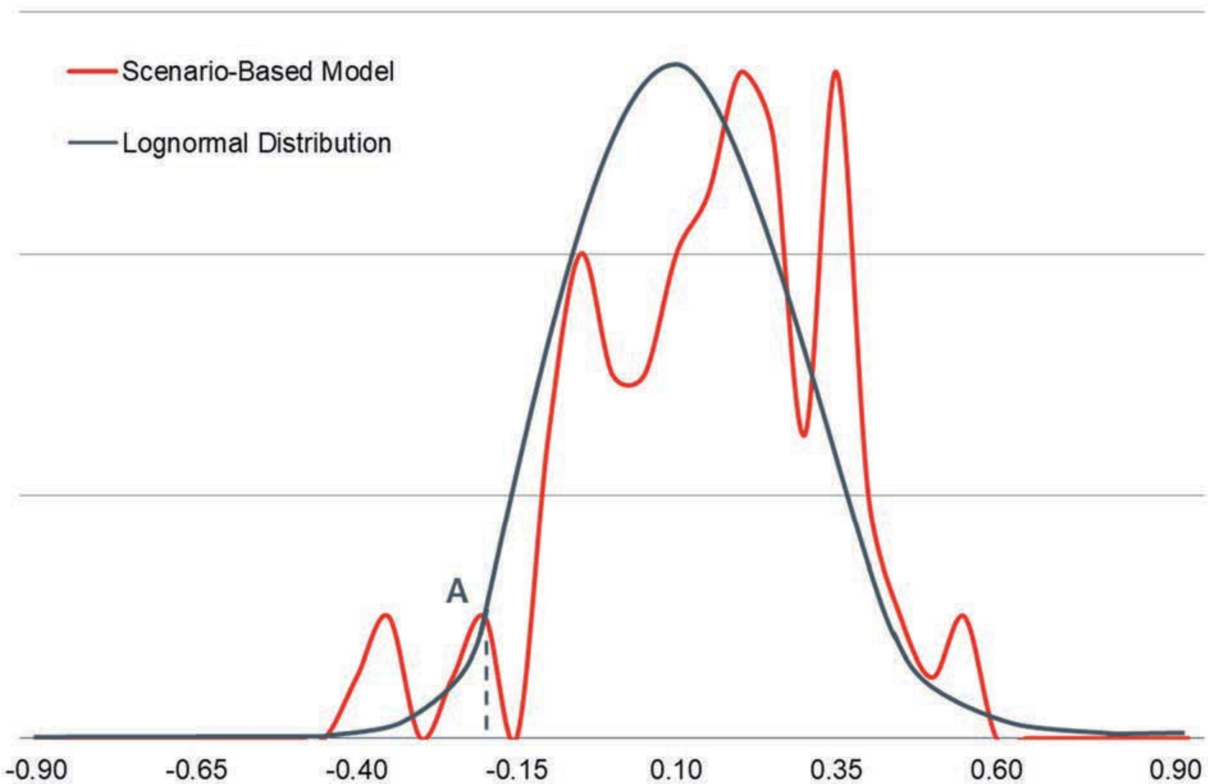
Over the years, various alternatives have been put forth to replace mean-variance optimization with an optimization framework that takes into account the non-normal features of return distributions. Some researchers have proposed using distribution curves that exhibit skewness and kurtosis (i.e., have fat tails) while others have proposed using large numbers of scenarios based on historical data, or Monte Carlo simulation.

The scenario-based approach has two main advantages over a distribution curve approach: (i) It is highly flexible. For example, nonlinear instruments such as options can be modeled in a straightforward manner; and (ii) it is mathematically manageable. For example, portfolio returns under the scenarios are simply weighted averages of asset class returns within the scenarios. In this way, the distribution of a portfolio can be derived from the distributions of the assets classes without working complicated equations that might lack analytical solutions; only straightforward portfolio arithmetic is needed.

In standard scenario analysis there is no precise graphical representation of return distributions. Histograms serve as approximations, such as those shown in Exhibit 2.3. We augment the scenario approach by employing a smoothing technique so that smooth curves represent return distributions. For example, Exhibit 10.5 shows the distribution curve of annual returns of large-cap stocks under our scenario-based approach. Comparing Exhibit 10.5 with the large-cap stock histogram in Exhibit 2.3, we can see that the smooth distribution curve retains the properties of the historical distribution making it more esthetically pleasing and precise. Further, our model can bring all of the power of continuous mathematics to the scenario approach. This was previously enjoyed only by models based on continuous distributions.

In Exhibit 10.5, the solid gray line represents the distribution of annual returns of large-cap stocks when our smoothed scenario-based approach is used, and the red line represents the distribution curve of annual returns of large-cap stocks when traditional mean-variance analysis is used and we assume that returns follow a lognormal distribution.

**Exhibit 10.5:** Distribution of Annual Returns: Large-cap Stocks (%) Lognormal Distribution vs. Scenario-Based Model




If we extend a vertical line from Point A down to the x-axis, the area to the left (and underneath) each of the curves represents the occurrences of annual returns equal to or less than, in this case, negative 26%. Because these are cumulative distributions, we can calculate the probability that the annual returns of large cap stocks will be less than or equal to negative 26% by dividing the area underneath each of the smaller curves (to the left of Point A) by the total area underneath each of the entire curves.

For example, looking to the scenario-based model, the area to the left of the vertical line under the scenario-based distribution represents 5% of the total area underneath this entire distribution line. This implies that the probability of large cap stocks having a loss of 26% or more is 5%. Correspondingly, the area to the left of the vertical line for the lognormal distribution represents 1.6% of the total area under the entire lognormal distribution line. This implies that the probability of large-cap stocks returning negative 26% or less using the traditional mean-variance model is 1.6%.

As Kaplan et al. (2009) discuss, “tail events” have occurred often throughout the history of capital markets all over the world, but the probabilities associated with them may be systematically underestimated within the context of traditional mean-variance analysis where return distributions are assumed to be lognormal. The scenario-based model proposed by Kaplan and Savage is a real step forward as it better models the nontrivial probabilities associated with tail events.

For a more detailed discussion of tail events and their nontriviality, see Chapter 11, where Kaplan introduces a set of monthly real stock market total returns going back a full 150 years. Using these new returns, we demonstrate that the severity of the financial crisis of 2008 was not unique but was merely the latest chapter in a long history of market meltdowns.

### Geometric Mean vs. Single-Period Expected Return

In mean-variance optimization, reward is measured by expected return, which is a forecast of arithmetic mean. However, over long periods, investors are not concerned with simple averages of return rather they are concerned with the accumulation of wealth. We use forecasted long-term geometric mean as the measure of reward because investors who plan on repeatedly reinvesting in the same strategy over an indefinite period would seek the highest rate of growth for the portfolios as measured by geometric mean.[193]

### Conditional Value at Risk vs. Standard Deviation

As for risk, much has been written about how investors are not concerned merely with the degree of dispersion of returns (as measured by standard deviation), but rather with how much wealth they could lose. A number of downside risk measures, including value at risk, conditional value at risk, and maximum drawdown, have been proposed to replace standard deviation as the measure of risk in strategic asset allocation. While any one of these could be used, our preference is to use conditional value at risk.

CVaR is related to value at risk. VaR describes the left tail in terms of how much capital can be lost over a given period of time. For example, a 5% VaR answers a question of the form: Having invested $10,000 there is a 5% chance of losing $X or more in 12 months. (The “or more” implications of VaR are sometimes overlooked by investors with serious implications.) Applying this idea to returns, the 5% VaR is the negative of the 5th percentile of the return distribution. CVaR is the expected or average loss of capital should VaR be breached. Therefore CVaR is always greater than VaR.

### Scenarios vs. Correlation

In mean-variance analysis, the covariation of the returns of each pair of asset classes is represented by a single number, the correlation coefficient. This is mathematically equivalent to assuming that a simple linear regression model is an adequate description of how the returns on

[193] Ranking investment strategies by forecasted geometric mean is sometimes described as applying the Kelly Criterion, an idea promoted by William Poundstone in his 2005 book, *Fortune’s Formula*.

the two asset classes are related. In fact, the R-squared statistic of a simple linear regression model for two series of returns is equal to the square of the correlation coefficient.

However, for many pairs of asset classes, a linear model misses the most important features of the relationship. For example, during normal times, non-U.S. equities are considered to be good diversifiers for U.S. equity investors. But during global crises, all major equity markets move down together.

Furthermore, suppose that the returns on two asset class indices were highly correlated, but instead of including direct exposures to both in the model, one was replaced with an option on itself. Instead of having a linear relationship, we now have a nonlinear relationship which cannot be captured by a correlation coefficient. Fortunately, these sorts of nonlinear relationships between returns on different investments can be handled in a scenario-based model. For example, in scenarios that represent normal times, returns on different equity markets could be modeled as moving somewhat apart from each other, while scenarios that represent global crises could model the markets as moving downward together.

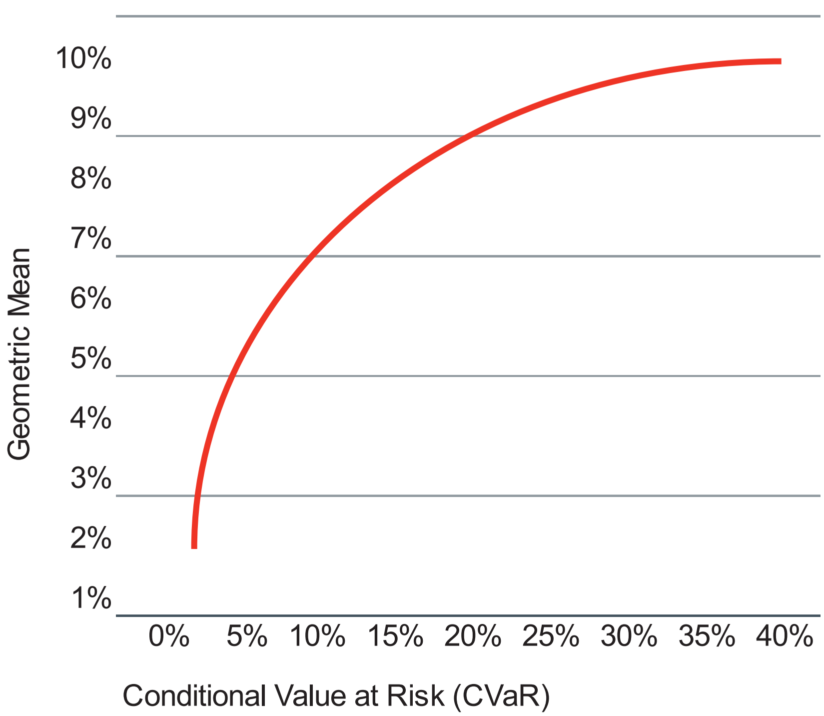
**Probability Management Enables Scenario Analysis**

Because it may take thousands of scenarios to adequately model return distributions, until recently, a disadvantage of the scenario-based approach has been that it requires large amounts of data to be stored and processed. Even with the advances in computer hardware, the conventional approach of representing scenarios with large tables of explicit numbers remained problematic. The phenomenal speed of computers has given rise to the field of probability management, an extension of data management to probability distributions, rather than numbers. The key component of probability management is the Distribution String, or DIST, that can encapsulate thousands of trials as a single data element. The use of DISTs greatly saves on storage and speeds up processing time so that a Monte Carlo simulation consisting of thousands of trials can be performed on a personal computer in an instant. Monte Carlo simulations that use DISTs are also very adaptable, allowing for almost any return distribution or underlying probability model rather than being contained by parameters. While not all asset management organizations are prepared to create the DISTs needed to drive geometric mean-CVaR optimization, some outside vendors, such as Ibbotson Associates/Morningstar, can fulfill this role. Another facet of probability management is interactive simulation technology, which can run thousands of scenarios through a model before the sound of your finger leaving the <Enter> key reaches your ear. These supersonic models allow much deeper intuition into the sensitivities of portfolios and encourage the user to interactively explore different portfolios, distributional assumptions, and potential black swans. For more information visit: http://www.ProbabilityManagement.org.

**Finale: The New Efficient Frontier**

Putting it all together, we form an efficient frontier of forecasted geometric mean and conditional value at risk as shown in Exhibit 10.6,[194] incorporating our scenario approach to covariance and new statistical technology. We believe that this efficient frontier is more relevant to investors than the traditional expected return versus standard deviation frontier of MVO because it shows the trade-off between reward and risk that is meaningful to investors, namely, long-term potential growth versus short-term potential loss.

**Exhibit 10.6:** Geometric Mean – Conditional Value at Risk Efficient Frontier (%)




## Approaches to Calculating the Equity Risk Premium

Researchers have estimated the expected outperformance of stocks over risk-free bonds – the equity risk premium – using many approaches. Such studies can be categorized into four groups based on the approaches they have taken, using:

- Historical returns between stocks and bonds
- Fundamental information such as earnings, dividends, or overall productivity (supply-side models)

[194] Other researchers have also proposed using GM and CVaR as the measures or reward and risk in an efficient frontier. See, for example: Sheikh, A.Z. & Qiao, H. 2009. "Non-Normality of Market Returns: A Framework for Asset Allocation Decision Making". Whitepaper, J.P. Morgan Asset Management.

- Payoffs demanded by equity investors for bearing the additional risk (demand-side models)

- Broad surveys of opinions of financial professionals.

The rest of this chapter will focus on the historical and supply-side methods.

## The Historical Equity Risk Premium

The expected equity risk premium (ERP) can be defined as the additional return an investor expects to receive to compensate for the additional risk associated with investing in equities as opposed to investing in riskless assets.

Unfortunately, the expected equity risk premium is unobservable in the market and therefore must be estimated. Typically, this estimation is arrived at using historical data. The historical equity risk premium can be calculated by subtracting the long-term average of the income return on the riskless asset (Treasuries) from the long-term average stock market return (measured over the same period as that of the riskless asset).

In using a historical measure of the equity risk premium, one assumes what has happened in the past is representative of what might be expected in the future. In other words, the assumption one makes when using historical data to measure the expected equity risk premium is the relationship between the returns of the risky asset (equities) and the riskless asset (Treasuries) is stable.

### The Stock Market Benchmark

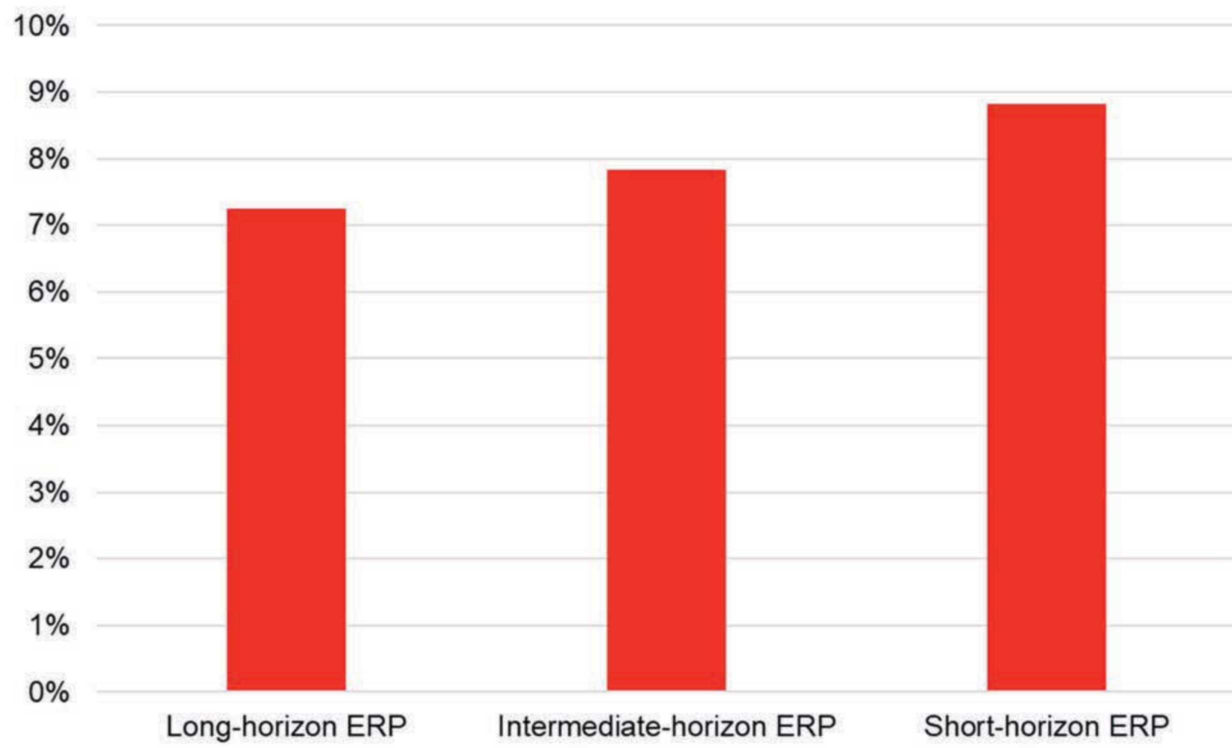
The stock market benchmark chosen should be a broad index that reflects the behavior of the market as a whole. Commonly used indexes include the S&P 500 and the Russell 3000. Although the Dow Jones Industrial Average is a popular index, it would be inappropriate for calculating the equity risk premium because it is too narrow.

We use the total return of our large-cap stock index (currently represented by the S&P 500) as our market benchmark when calculating the equity risk premium.[195] The S&P 500 was selected as the appropriate market benchmark because it is representative of a large sample of companies across a large number of industries. The S&P 500 is also one of the most widely accepted market benchmarks and is a good measure of the equity market as a whole.

Exhibit 10.7 illustrates the equity risk premium calculated using the S&P 500 and the income return on three government bonds of *different* horizons.

[195] The SBBI Large-cap Stocks total return series is essentially the S&P 500 Index.

**Exhibit 10.7**: Equity Risk Premia Calculated Using the S&P 500 and the Income Return on Three Government Bonds of Different Horizons (%) 1926–2020




**Source of underlying data**: Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Long-Term (i.e. 20-year) Government Bonds income return series: IA SBBI® US LT Govt IR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds income return series: IA SBBI® US IT Govt IR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD (for U.S. Treasury Bills, the income return and total return are the same). For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

Note that the long-horizon ERP is *lower* than the intermediate-horizon ERP, which in turn is *lower* than the short-horizon ERP. This is because the equity risk premium is calculated by subtracting the arithmetic mean of the government bond *income* return from the arithmetic mean of the stock market total return. When calculating the ERPs in these examples:

- The average income return of a *long*-term (20-year) government bond is used when calculating the *long*-horizon ERP. The average annual income return of 20-year government bonds will be *greater* than the average income return of a 5-year government bond.

- The average income return of an *intermediate*-term (5-year) government bond is used when calculating the *intermediate*-horizon ERP. The average annual income return of 5-year government bonds will be *greater* than the average income return of a 30-day Treasury bill.

- The average income return of a *30-day* Treasury bill is used when calculating the *short-*horizon ERP.

Because the ERPs in these examples are calculated as:

ERP = (Avg. Annual Total Return of the S&P 500) – (Avg. Annual Income Return of Risk-Free Security)

It follows that the ERP would *increase* as the value subtracted from the average annual total return of the stock market benchmark *decreases*.

**The Market Benchmark and Firm Size**

Although not restricted to the 500 largest companies, the S&P 500 is considered a large-cap index. The returns of the S&P 500 are market cap-weighted. The larger companies in the index therefore receive the majority of the weight. If using a large-cap index to calculate the equity risk premium, an adjustment is usually needed to account for the different risk and return characteristics of small stocks, as discussed in Chapter 7.

**The Risk-Free Asset**

The equity risk premium can be calculated for a variety of time horizons when given the choice of risk-free asset to be used in the calculation. The long-horizon, intermediate-horizon, and short-horizon equity risk premia calculated in Exhibit 10.7 use the income return from (i) a 20-year Treasury bond, (ii) a 5-year Treasury bond, and (iii) a 30 day Treasury bill, respectively.[196]

**20-Year vs. 30-Year Treasuries**

The U.S. Treasury periodically changes the maturities it issues. For example, in April 1986 the U.S. Treasury stopped issuing 20-year Treasuries, and from October 2001 through January 2006 the U.S. Treasury did not issue 30-year bonds (it resumed issuing 30-year Treasury bonds in February 2006), making the 10-year bond the longest-term Treasury security issued over the October 2001 January 2006 period. Most recently, on January 16, 2020 the U.S. Department of the Treasury announced it plans to issue a 20-year nominal coupon bond in the first half of calendar year 2020, the first time a 20-year maturity will be offered since March 1986.[197,198]

Our methodology for estimating the long-horizon equity risk premium makes use of the income return on a 20-year Treasury bond. While a 30-year bond is theoretically more correct when

---

196 For U.S. Treasury Bills, the income return and total return are the same.

197 To learn more, visit the U.S. Department of the Treasury website at: https://home.treasury.gov/news/press-releases/sm878.

198 See Kate Davidson, "Treasury to Issue New 20-Year Bond in First Half of 2020", *The Wall Street Journal*, January 16, 2020 at: https://www.wsj.com/articles/treasury-to-issue-new-20-year-bond-in-first-half-of-2020-11579217450

dealing with the long-term nature of business valuation,[199] 30-year Treasury securities have an issuance history that is on-again-off-again. Ibbotson Associates creates a series of returns using bonds on the market with approximately 20 years to maturity because Treasury bonds of this maturity are available over a long history, while Treasury bonds of 30-years are not.

**Income Return** Another point to keep in mind when calculating the equity risk premium is the income return on the appropriate-horizon Treasury security, rather than the total return, is used in the calculation.

The total return comprises three return components: the income return, the capital appreciation return, and the reinvestment return. The income return is defined as the portion of the total return that results from a periodic cash flow or, in this case, the bond coupon payment. The capital appreciation return results from the price change of a bond over a specific period. Bond prices generally change in reaction to unexpected fluctuations in yields. Reinvestment return is the return on a given month's investment income when reinvested into the same asset class in the subsequent months of the year. The income return is thus used in the estimation of the equity risk premium because it represents the truly riskless portion of the return.

### Arithmetic vs. Geometric Mean

The equity risk premium data presented in this book are arithmetic average risk premiums as opposed to geometric average risk premiums. The arithmetic average equity risk premium can be demonstrated to be most appropriate when discounting future cash flows. For use as the expected equity risk premium in either the CAPM or the building-block approach, the arithmetic mean or the simple difference of the arithmetic means of stock market returns and riskless rates is the relevant number.

This is because both the CAPM and the building-block approach are additive models, in which the cost of capital is the sum of its parts. The geometric average is more appropriate for reporting past performance because it represents the compound average return.

### Appropriate Historical Period

The equity risk premium can be estimated using any historical time period. For the U.S., market data exist at least as far back as the late 1800s. Therefore, it is possible to estimate the equity risk premium using data that covers roughly the past 125 years.

---

[199] An equity risk premium is an input in developing cost of capital estimates (i.e., "expected return", "required return", or "discount rate") for use in a discounted cash flow model. **Note**: The D&P/Kroll "Cost of Capital Navigator" guides financial professionals through the process of estimating the cost of capital, a key component of any valuation analysis. The Cost of Capital Navigator can be used to estimate country-level cost of equity capital globally, for approximately 180 countries, from the perspective of investors based in any one of up to 56 countries. For more information, visit dpcostofcapital.com.

Our equity risk premium covers 1926 to the present. The original data source for the time series comprising the equity risk premium is the Center for Research in Security Prices (CRSP).[200] CRSP chose to begin its analysis of market returns with 1926 for two main reasons. CRSP determined that 1926 was approximately when quality financial data became available. They also made a conscious effort to include the period of extreme market volatility from the late 1920s and early 1930s; 1926 was chosen because it includes one full business cycle of data before the market crash of 1929.

Implicit in using history to forecast the future is the assumption that investors' expectations for future outcomes conform to past results. This method assumes that the price of taking on risk changes only slowly, if at all, over time. This "future equals the past" assumption is most applicable to a random time-series variable. A time-series variable is random if its value in one period is independent of its value in other periods.

### Choosing an Appropriate Historical Period

The estimate of the equity risk premium depends on the length of the data series studied. A proper estimate of the equity risk premium requires a data series long enough to give a reliable average without being unduly influenced by very good and very poor short-term returns. When calculated using a long data series, the historical equity risk premium is relatively stable. Furthermore, because an average of the realized equity risk premium is quite volatile when calculated using a short history, using a long series makes it less likely that the analyst can justify any number he or she wants. The magnitude of how shorter periods can affect the result will be explored later in this chapter.

Some analysts estimate the expected equity risk premium using a shorter, more recent period on the basis that recent events are more likely to be repeated in the near future; furthermore, they believe that the 1920s, 1930s, and 1940s contain too many unusual events. This view is suspect because all periods contain unusual events. Some of the most unusual events of the last 100 years took place quite recently, including the inflation of the late 1970s and early 1980s, the October 1987 stock market crash, the collapse of the high-yield bond market, the major contraction and consolidation of the thrift industry, the collapse of the Soviet Union, the development of the European Economic Community, the attacks of Sept. 11, 2001, the global financial crisis of 2008– 2009, and most recently, the market crash in the first quarter of 2020 that was precipitated by the spread of the COVID-19 virus.

It is even more difficult for economists to predict the economic environment of the future. For example, if one were analyzing the stock market in 1987 before the crash, it would be statistically improbable to predict the impending short-term volatility without considering the stock market crash and market volatility of the 1929-1931 period.

200 CRSP® is a registered trademark and service mark of Center for Research in Security Prices, LLC and has been licensed for use by D&P/Kroll. The D&P/Kroll publications and services are not sponsored, sold or promoted by CRSP®, its affiliates or its parent company. To learn more about CRSP, visit www.crsp.com.

Without an appreciation of the 1920s and 1930s, no one would believe that such events could happen. The 95-year period starting with 1926 represents what can happen: It includes high and low returns, volatile and quiet markets, war and peace, inflation and deflation, and prosperity and depression. Restricting attention to a shorter historical period underestimates the amount of change that could occur in a long future period. Finally, because historical event-types (not specific events) tend to repeat themselves, long-run capital market return studies can reveal a great deal about the future. Investors probably expect unusual events to occur from time to time, and their return expectations reflect this.

### A Look at the Historical Results

It is interesting to look at the realized returns and realized equity risk premium in the context of the above discussion, since a longer historical period provides a more stable estimate of the equity risk premium. The reason is that any unique period will not be weighted heavily in an average covering a longer historical period. It better represents the probability of these unique events occurring over a long period of time.

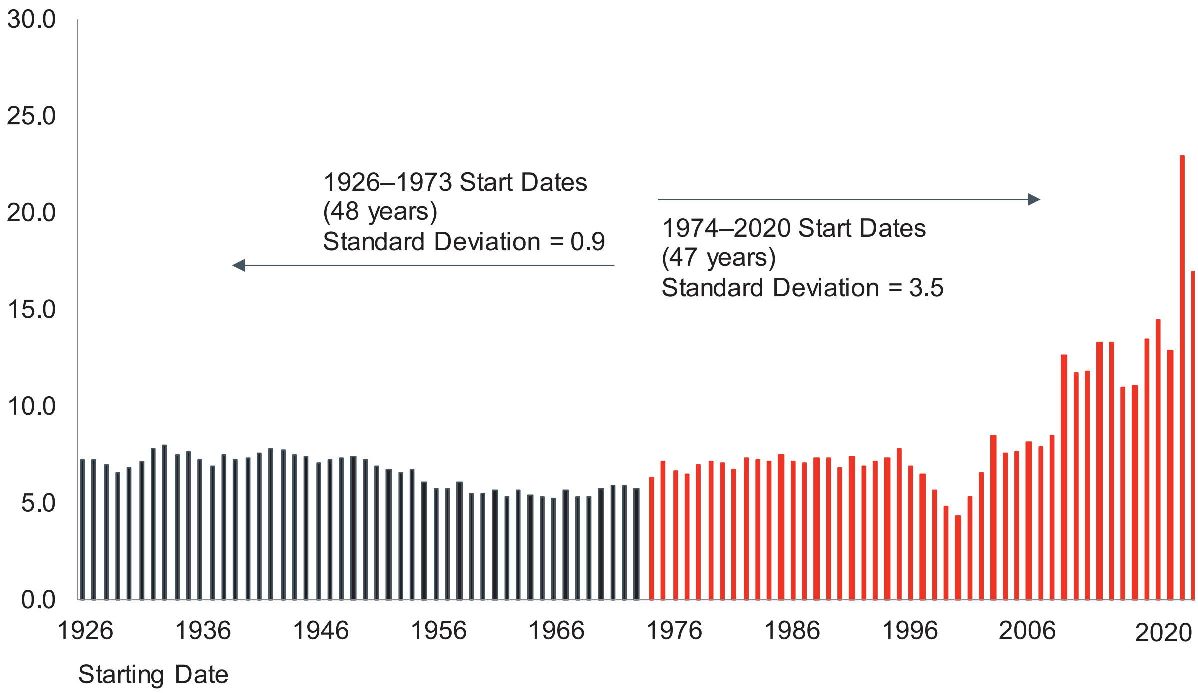
Exhibit 10.8 helps to clarify this point. Exhibit 10.8 shows the realized equity risk premium for a series of periods through 2020, starting with 1926. In other words, the first value on the graph represents the average realized equity risk premium over the period 1926–2020. The next value on the graph represents the average realized equity risk premium over the period 1927–2020, and so on, with the rightmost value representing the average for a single year, 2020.

Concentrating on the left side of Exhibit 10.8, one notices that the realized equity risk premium when measured over *longer* periods is relatively stable and has a standard deviation of 0.9.

Alternatively, the realized equity premia on the right side of Exhibit 10.8 are measured over *shorter* periods are less stable and have a standard deviation of 3.5.[201]

[201] If the unusually large realized equity risk premia measured over the years 2019–2020 (23.0) and the single year 2020 (17.0) are excluded (the rightmost two bars in Exhibit 10.8), the standard deviation of the realized equity risk premia measured with starting dates 1973–2018 drops to 2.5. This is still more than twice the standard deviation of the realized equity risk premia measured with starting dates 1926–1972 (the left side of Exhibit 10.8) of 0.9.

**Exhibit 10.8:** Average Long-Horizon Equity Risk Premium Calculated Using Variable Start Dates (1926–2020), and Fixed End Date (2020) (%)




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, and (ii) Long-Term (i.e. 20-year) Government Bonds income return series: IA SBBI® US LT Govt IR USD. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission.

Some practitioners argue for a shorter historical period, such as 30 years, as a basis for the equity risk premium estimation. The logic for the use of a shorter period is that historical events and economic scenarios present before this time are unlikely to be repeated. However, the impact of adding one additional year of data to a historical average is *lessened* the *greater* the initial period of measurement. As is demonstrated in Exhibit 10.8, shorter-term averages can be affected considerably by one or more unique observations, while longer term averages tend to produce more stable results.

A dramatic example of this is the second rightmost point in Exhibit 10.8, which is the "average" ERP as measured over a single year (2019). In 2019 large-cap stocks (represented by the S&P 500) produced a total return of over 31.49% and the income return of long-term government bonds was 2.55%, implying an "average" ERP of 28.94% (31.49% - 2.55%). Using an estimate of the ERP developed over such a short time horizon is logical only to the extent that one believes that stocks will outperform the risk-free instrument by nearly 29% per year, in perpetuity.

Having said that, the effect of "adding one additional year" when using historical data to estimate the ERP can still lead to counterintuitive conclusions, even when the average is taken over *longer* periods. A very recent example of a result that was "counterintuitive" occurred in the December 2008–2009 Financial Crisis. The historical ERP at the end of 2007 (as calculated over the time period 1926–2007) was over 7%. A year later at the end of 2008, at the height of the financial crisis and risks were likely at an all-time *high*, the historical ERP (as calculated over the time period 1926– 2008) *declined* to less than 7%, implying that risks were actually *lower* than they were a year earlier.

What happened? In 2008 the S&P 500 declined nearly 37%, an unusually large decline for a single year. This single period's unusually large decline caused the average annual return of the S&P 500 to fall from *over* 12% (as calculated over the 1926–2007 time period) to *less* than 12% (as calculated over the 1926–2008 time period). The historical ERP is calculated as the average annual equity return minus the average annual risk-free rate, so a decline in the average equity return causes a 1 for 1 decline in the ERP, all other things held the same. Such large moves in a single year can produce a "tail wagging the dog" effect.

## The Supply-Side Equity Risk Premium

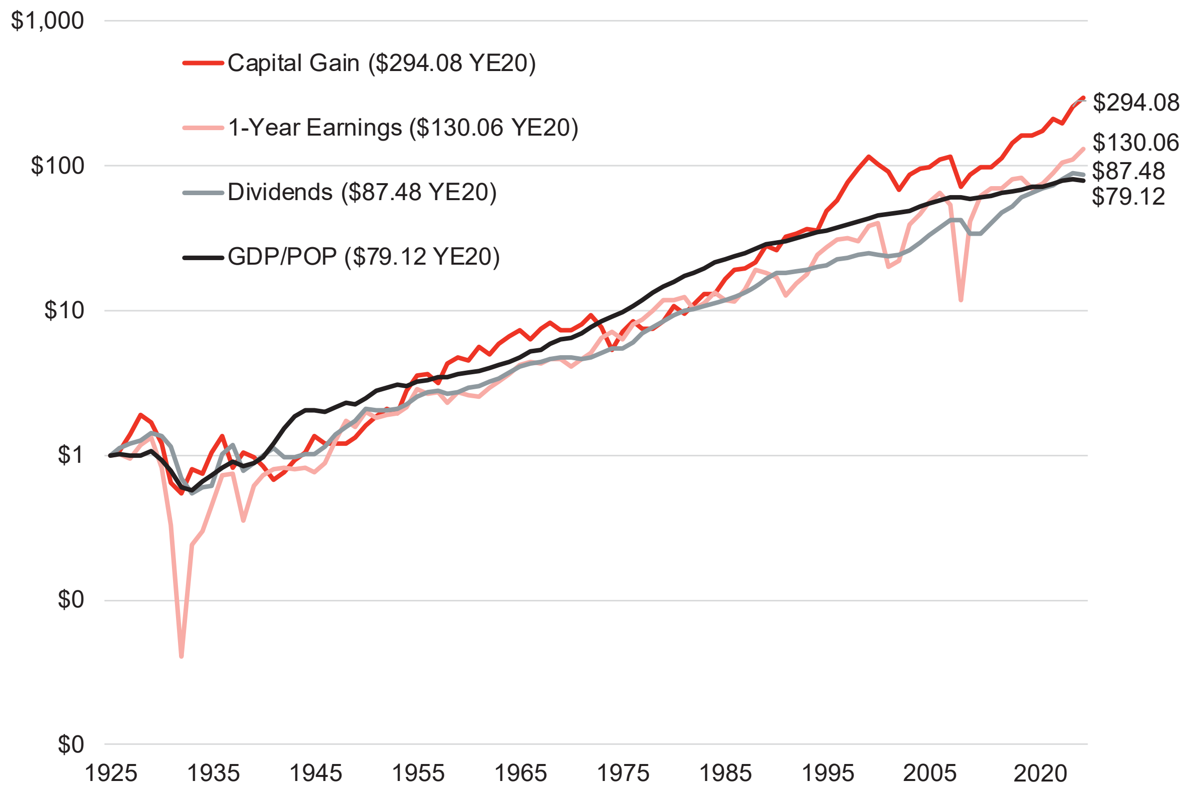
This section is based on the work by Roger G. Ibbotson and Peng Chen, who combined the first and second approaches to arrive at their forecast of the equity risk premium.[202] By proposing a new supply-side methodology, the Ibbotson-Chen study challenges current arguments that future returns on stocks over bonds will be negative or close to zero. The results affirm the relationship between the stock market and the overall economy.

Long-term expected equity returns can be forecasted by the use of supply-side models. The supply of stock market returns is generated by the productivity of the corporations in the real economy. Investors should not expect a much higher or lower return than that produced by the companies in the real economy. Thus, over the long run, equity returns should be close to the long-run supply estimate.

Earnings, dividends, and capital gains are supplied by corporate productivity. Exhibit 10.9 illustrates that earnings and dividends have historically grown in tandem with the overall economy (GDP per capita). However, GDP per capita did not outpace the stock market. This is primarily because the 3-year average P/E ratio increased 2.7 times during the same period. So, assuming the economy will continue to grow, all three should continue to grow as well.

[202] Ibbotson, R.G., & Chen, P. 2003. "Long-Run Stock Returns: Participating in the Real Economy". *Financial Analysts Journal*, Vol. 59, No. 1, P. 88.

**Exhibit 10.9:** Capital Gains, GDP Per Capita, Earnings, and Dividends Index (Year-end 1925 = $1.00) 1926–2020

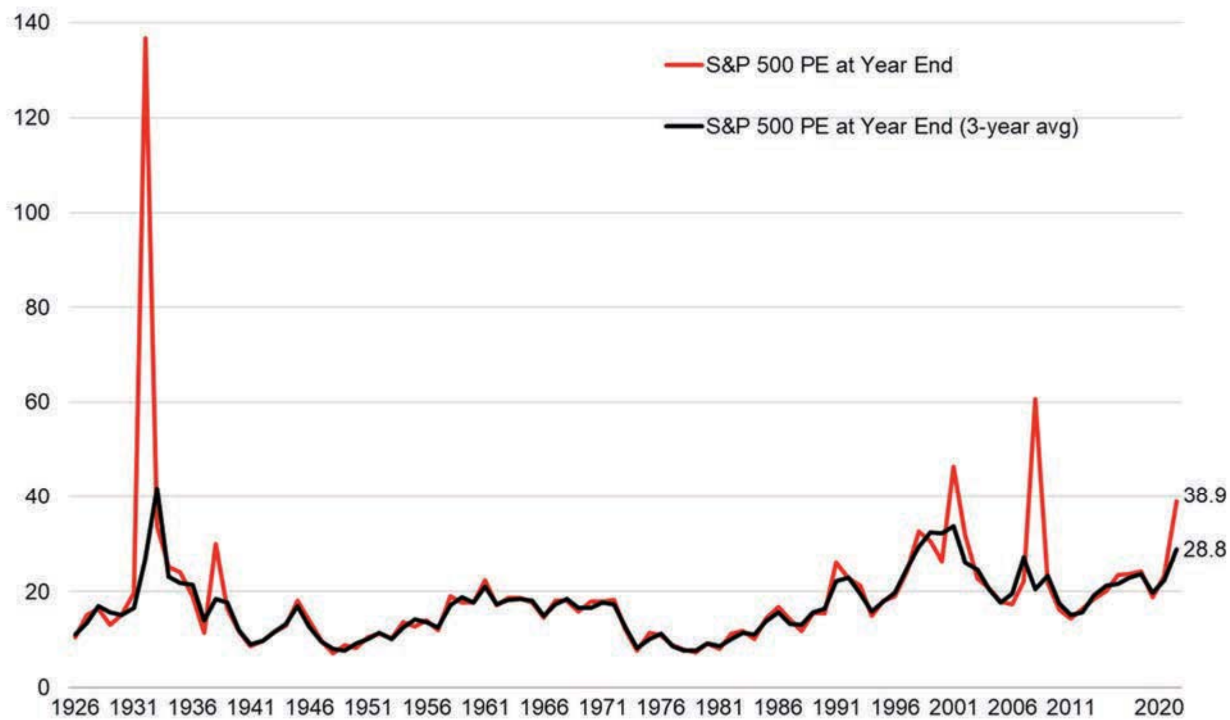



## Forward-Looking Earnings Model

Ibbotson and Chen forecast the equity risk premium through a supply-side model using historical data. They used an earnings model as the basis for their supply-side estimate. The earnings model breaks the historical equity return into four pieces, with only three historically being supplied by companies: inflation, income return, and growth in real earnings per share. The growth in the P/E ratio, the fourth piece, reflects investors' changing prediction of future earnings growth. The past supply of corporate growth is forecasted to continue; however, a change in investors' predictions is not. P/E rose dramatically from 1980 through 2001 because people believed that corporate earnings were going to grow faster in the future. This growth in P/E drove a small portion of the rise in equity returns over the same period.

Exhibit 10.10 illustrates the price-to-earnings ratio from 1926 to 2020. The P/E ratio, using one-year average earnings, was 10.23 at the beginning of 1926 and ended the year 2020 at estimated 38.94, an average increase of 1.40% per year. The highest P/E was 136.69 recorded in 1932, while the lowest was 7.08 recorded in 1948. Ibbotson Associates revised the calculation of the P/E ratio from a one-year to three-year average earnings for use in equity forecasting.

**Exhibit 10.10:** Large-cap Stocks P/E Ratio 1926–2020




This is because reported earnings are affected not only by the long-term productivity, but also by one-time items that do not necessarily have the same consistent impact year after year. The three-year average is more reflective of the long-term trend than the year-by-year numbers. The P/E ratio calculated using the three-year average of earnings had an increase of 0.96% per year.

The historical P/E growth factor, using three-year earnings, of 0.96% per year is subtracted from the equity forecast because it is not believed that P/E will continue to increase in the future. The market serves as the cue. The current P/E ratio is the market's best guess for the future of corporate earnings and there is no reason to believe, at this time, that the market will change its mind. Using this top-down approach, the geometric supply-side equity risk premium is slightly more than 4% which equates to an arithmetic supply-side equity risk premium of approximately 6%.

Another approach in calculating the premium would be to add up the components that constitute the supply of equity return, excluding the P/E component. Thus, the supply of equity return only includes inflation, the growth in real earnings per share, and income return:

$$SR = \left[ \left(1 + CPI\right) \times \left(1 + g_{REPS}\right) + Inc + Rinv \right]$$

Where:

$SR$ = The supply of the equity return

$CPI$ = Consumer Price Index (inflation)

$g_{REPS}$ = The growth in real earning per share

$Inc$ = The income return

$Rinv$ = The reinvestment return

The equity risk premium, based on the supply-side earnings model, is calculated on a geometric basis as follows:

$$SERP = \frac{(1+SR)}{(1+CPI) \times (1+RRf)} - 1$$

Where:

$SERP$ = The supply-side equity risk premium

$SR$ = The supply of the equity return

$CPI$ = Consumer Price Index (inflation)

$RRf$ = The real risk-free rate

The geometric estimate can be converted into an arithmetic estimate as follows:[203]

[203] The 1926–present supply-side equity risk premia estimate is calculated by D&P/Kroll for the full-version *2021 SBBI® Yearbook* using (i) the same methodologies and (ii) the same data sources as were used in previous editions of this book, based upon the work by Roger G. Ibbotson and Peng Chen; see: Ibbotson, R.G., & Chen, P. 2003. "Long-Run Stock Returns: Participating in the Real Economy". *Financial Analysts Journal*, Vol. 59, No. 1, P. 88. An update of this work has been published that considers stock buybacks in addition to dividends; see: Philip U. Straehl and Roger G. Ibbotson, "The Long-Run Drivers of Stock Returns: Total Payouts and the Real Economy", *Financial Analysts Journal*, Third Quarter 2017, Volume 73 Number 3. The *Financial Analysts Journal* is a publication of CFA Institute. For more information, visit www.cfainstitute.org.

$$R_A = R_G + \frac{\sigma^2}{2}$$

Where:

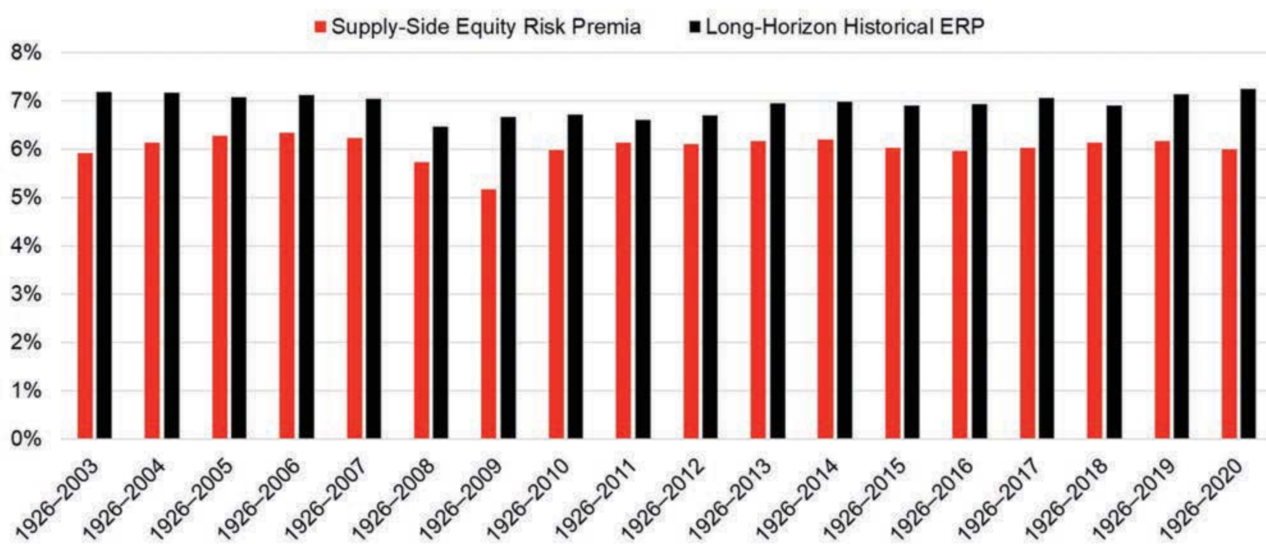
$R_A$ = The arithmetic average

$R_G$ = The geometric average

$\sigma$ = The standard deviation of equity returns

Exhibit 10.11 presents an illustration of the supply-side equity risk premium, on an arithmetic basis, beginning in 1926 and ending in each of the years from 2003 through 2020.[204]

**Exhibit 10.11:** Supply-Side Equity Risk Premia and Long-Horizon Historical Equity Risk Premia over Time.




**Source of underlying data**: Morningstar, Inc. and S&P Global Market Intelligence. All rights reserved. Used with permission. Calculations by D&P/Kroll.

In every year since 2003 the supply-side ERP has been less than the long-term historical ERP. The difference has varied between approximately 0.5% and 1.5% over the course of the 18 observations in Exhibit 10.11.

[204] As published in (i) the *2004–2013 SBBI® Valuation Yearbooks,* (ii) the 2014–2017 *Valuation Handbook – U.S. Guide to Cost of Capital,* and (iii) the Cost of Capital Navigator at dpcostofcapital.com beginning in 2018.

### Long-Term Market Predictions

As of December 31, 2020, the supply-side model estimates that stocks will continue to provide significant returns over the long run, averaging more than 9% per year, assuming historical inflation rates. The equity risk premium, based on the top-down supply-side earnings model, is calculated to be just over 4% on a geometric basis and approximately 6% on an arithmetic basis.

Ibbotson and Chen predict future increased earnings growth that will offset lower dividend yields. The fact that earnings will grow as dividend payouts shrink is in line with the Modigliani-Miller theorem which here refers to the irrelevance over whether a firm pays a dividend or reinvests its returns.

The forecasts for the market are in line with both the historical supply measures of public corporations (i.e., earnings) and overall economic productivity (GDP per capita).

## Stock Buybacks and Return

**Note:** This section is updated through December 2018.

In recent decades a new source of stock market supply has emerged as companies increasingly use share buybacks instead of dividends to return cash to shareholders. The impact of buybacks on stock returns has been largely ignored in practice because many practitioners continue to rely on traditional supply models that use dividends as the sole source of corporate payout.

In a 2017 article, Philip U. Straehl and Roger G. Ibbotson developed three total payout models of stock returns showing that US stock returns between 1871 and 2014 can be attributed almost entirely to the supply of both dividends and buybacks.[205,206,207]

Although Straehl and Ibbotson introduced buybacks into the supply-side model, they did not dispute that there are many supply-side approaches that can be taken. Rather they updated and back dated the Ibbotson and Chen 2003[208] paper to cover the period 1871-2014, decomposing historical returns by six different methods each containing an inflation component:

1. Building Blocks: risk-free rate and equity risk premium

2. Capital Gains and Income

---

205 Philip U. Straehl is head of capital markets and asset allocation at Morningstar Investment Management LLC, Chicago. Roger G. Ibbotson is Professor in the Practice Emeritus of Finance at the Yale School of Management, New Haven, Connecticut, and chairman and chief investment officer at Zebra Capital Management LLC, Stamford, Connecticut.

206 Philip U. Straehl and Roger G. Ibbotson, "The Long-Run Drivers of Stock Returns: Total Payouts and the Real Economy", *Financial Analysts Journal* (a publication of CFA Institute), Third Quarter 2017, pages 32–52.

207 This section is a summary of Philip U. Straehl and Roger G. Ibbotson, "The Long-Run Drivers of Stock Returns: Total Payouts and the Real Economy", *Financial Analysts Journal* (a publication of CFA Institute), Third Quarter 2017, pages 32–52. The original article was through 2014; Straehl and Ibbotson updated the commentary herein through 2018.

208 Ibbotson, Roger G., and Peng Chen. 2003, "Long-Run Stock Returns: Participating in the Real Economy." *Financial Analysts Journal*, vol. 59, no. 1 (January/February).

3. Earnings growth, PE ratio, rate of change, and income

4. Dividends growth, payout ratio rate of change, PE ratio rate of change, and income

5. Book Equity growth, growth in ROE, PE ratio rate of change, and income

6. GDP per capita growth, increase in equity factor share of economy

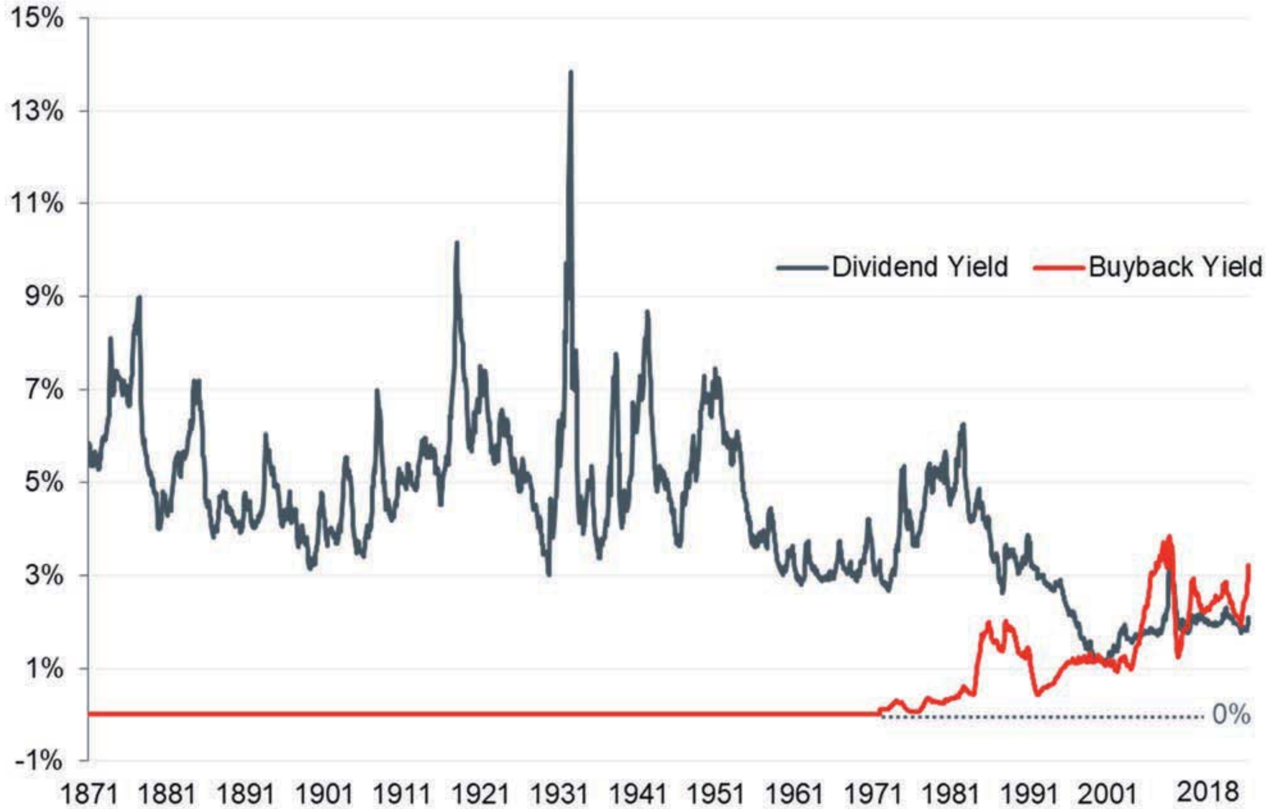
Straehl and Ibbotson focus in particular on method #4, which concentrates on dividend growth, and method #6 which links the stock market to the overall economy. But they make a major departure from Ibbotson and Chen because they include not only dividends but buybacks into the analysis.

## The Rise of Buybacks

A primary objective of Straehl and Ibbotson's study was to shed light on the impact of buybacks on the return generation process. They started by documenting the rise of buybacks as a form of corporate payout relative to dividends.

In 1982, SEC Rate 106-18 provided a safe harbor for firms to conduct share buybacks without the suspicion of share price manipulation. Here we update Exhibit 10.12 from Straehl and Ibbotson to include data through 2018. As can be seen in Exhibit 10.12, there is a major increase in buyback activity starting in the early 1980s. Prior to 1970, buyback activity was so low that it was not included in the study.

**Exhibit 10.12:** Dividend Yield and Buyback Yield, 1871– 2018




In recent decades, companies increasingly prefer to make payouts in the form of buybacks instead of dividend payments. There are several reasons for this. Most important, the amount of buybacks is completely flexible. A company can aggressively buyback one year and skip the next year without major signaling effects, unlike dividend policy. Also, if a company does not have a good use for its cash, buybacks can increase earnings per share by decreasing the number of shares outstanding. Furthermore, companies can buyback shares when they believe the price is attractive, both potentially boosting the price and benefitting the holding shareholders. Finally, through much of history, the tax treatment of buybacks was more favorable than it was for dividend payouts.

Dividend payout models are typically wrongly applied in the era of buybacks. They often estimate the future returns to be the sum of the current dividend yield plus historical dividend growth. This is wrong for two reasons: the current yield is artificially too low since it only includes one source of income, and the historical growth rate is too low because it ignores the shift in payouts away from dividends.

Thus, the advent of buybacks has created a need for models that can explicitly take into account buybacks. Although buybacks are similar to dividends in that they are a way of paying out cash, buybacks have a different impact on the return generating process than dividends do. For example, the buy and hold investor receives dividends in the form of income, while investors receive buybacks as a price increase because the buy and hold investor's share of the company is increased. Prior studies, including Ibbotson and Chen's 2003 paper, disregarded the fact that return components are sensitive to a company's payout method (i.e., dividends versus buybacks).

## Three Total Payout Models of Stock Returns

Miller and Modigliani[209] proved that in a perfect capital market the total return of stocks should be independent of the payout method. The Dividend Per Share Model as typically applied is not independent since higher buybacks make for less historical growth and lower current dividend yields. This is not to say the dividend model is incorrect because it is the future growth that becomes higher as the number of shares diminishes. However, by taking the buybacks explicitly into account, past payout growth can once again be an indicator of future growth, and current yields can reflect the full payouts.

Straehl and Ibbotson present three payout models, all of which include inflation:

1. The Dividend Per Share Model, where the investor gets a dividend yield plus a growth in total payouts, plus the change in payout per share, plus the change in price to total payout. Here, the buy and hold investor gets higher future growth to offset the lower dividend yield.

2. Dividend and Cash Buyback Model, where the investor gets the total yield (dividend plus buyback), plus growth in payout per share adjusted for share decrease, plus change in price to total payout. Here, the buy and hold investor gets the full payouts.

3. Dividend Less Net Issuance Model, where the investor gets the net total yield (dividend plus buyback but diluted by issuance), plus aggregate payout growth, plus rate of change in total payout. Here, the investor gets diluted by issuance but increased ownership by the buybacks.

In all three cases, the historical return is the sum of the components are all equal no matter which of the three methods are used. In the Straehl and Ibbotson historical samples, the total return from 1871-2018 was 9.02%, the total return from 1901-2018 was 9.58%, and the total return from 1970-2018 was 10.20%.

In all three supply-side models the realized return was the same, and most of the real return came from the payouts and the payout growth. However, the nature of the payouts explains what portion of the return comes from the payouts versus the payout growth.

209 Miller, Merton H., and Franco Modigliani 1961, "Dividend Policy, Growth, and the Valuation of Shares" Journal of Business, Vol.34, no.4 (October): 411-433.

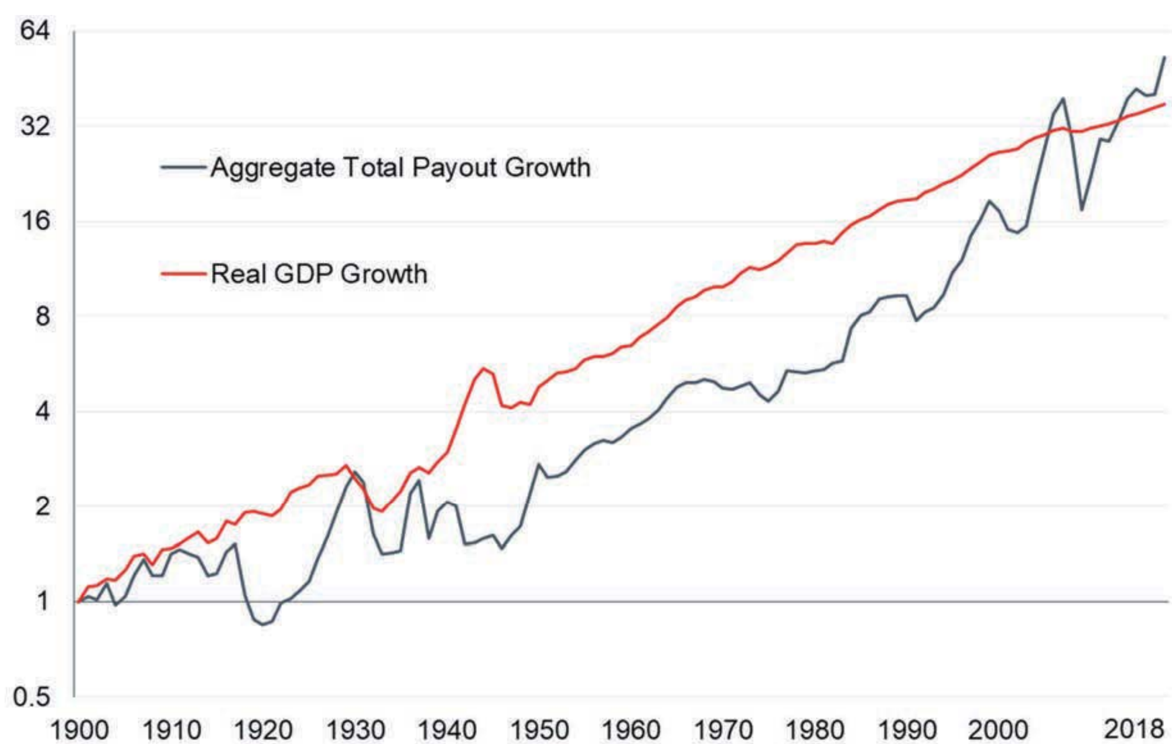
## Total Payouts and the Real Economy

The stock market should link to the real economy over the long run. More generally, Diermeier, Ibbotson, and Siegel (1984)[210] measured the full scope of financial assets and stressed the importance of capital markets being "macro consistent" with the real economy. In the long run, financial assets cannot continually outgrow the real economy, or financial assets would eventually become the whole economy. On the other hand, financial assets cannot continually underperform, or they would become a smaller and smaller part of the economy. For example, Ibbotson and Chen (2003) showed that earning per share growth for the U.S. stocks were comparable to U.S. GDP per capita growth.

In attempting to link the U.S. stock market to the U.S. real economy, Straehl and Ibbotson focus on growth rates rather than the payouts themselves. In particular, the long-term growth rate of aggregate stock payouts should link to the aggregate GOP growth, and the long-term growth rate of per share payout growth rates should link to the GDP per capital growth rate.

Exhibit 10.13 is an updated chart from Straehl and Ibbotson. It shows that the growth in aggregate stock market (as measured by the S&P Composite Index) roughly matches the aggregate real GDP growth. This link up is better than dividend growth by itself, which would have underestimated payout growth.

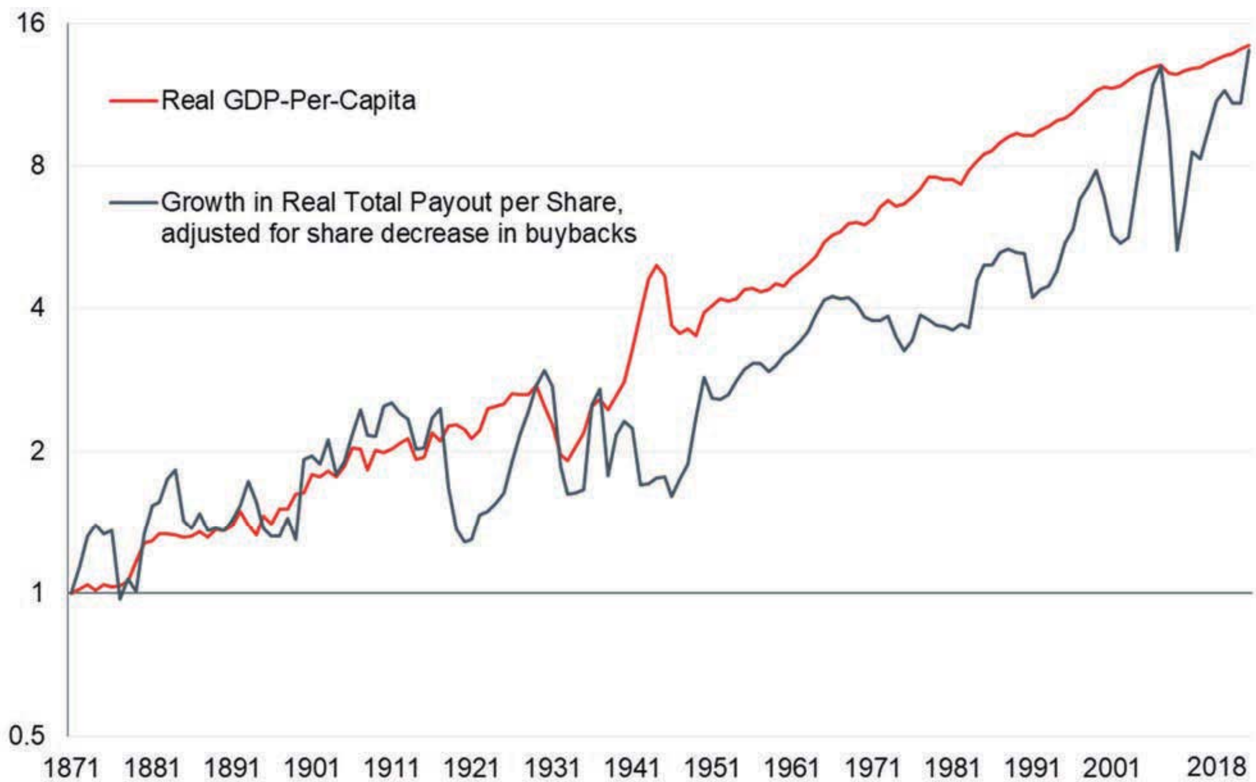
**Exhibit 10.13:** Growth in Aggregate Total Payout vs. GDP Growth 1901–2018




[210] Diermeier, Ibbotson, and Siegel, 1984 "The Supply of Capital Market Returns" Financial Analysts Journal, Vol.40 no.2 (March/April).

Exhibit 10.14 similarly shows real GDP per capital growth is linked to real total payout per share growth. In this case the data goes all the way back to 1871, and again is an updated version of an exhibit in the Straehl and Ibbotson article.

**Exhibit 10.14:** Growth in Total Payout per Share vs. Growth in GDP per Capita 1872–2018




Overall, these results indicate that total payout growth in the stock market is roughly equal to GDP growth over the long run. Of course, this is not necessarily true over shorter intervals, but still total payouts are a useful tool in explaining long run stock returns and their interaction with the overall economy. We now look specifically at forecasting the stock market with supply-side models.

## Forecasting Equity Returns

This chapter of the Yearbook is concerned with using historical data to forecast returns which can potentially be used for long-term planning and optimizing portfolios. The long-term historical data that Straehl and Ibbotson provide can be useful in forecasting.

The payout expected real return models presented here have two components: Payout Yield and Real Payout Growth. It is also necessary to add a small interaction term to match up with the historical rates.

Taking the full historical 1871-2018 period that Straehl and Ibbotson studied, we can compare the annual dividend yield growth (1.59%) to the total payout growth (1.79%). We can also compare a recent current dividend yield (1.83%) to a recent payout yield (4.21%). Adding the historical interaction term to both series (0.25% and 0.23% respectively) gives us the two forecasts. The expected real return using the Dividend Yield method is 3.67%. The expected real return using the Total Payout method (which includes buybacks) is 6.23%.

**Exhibit 10.15:** Long-Run Expected Returns Based On the Current Payout Yield and Historical Growth Over 1871–2018

| | **Recent Yield** | **Historical Growth** | **Interaction** | **Expected Real Return** |
|---|---|---|---|---|
| Dividend Yield Method | 1.83% | 1.59% | 0.25% | 3.67% |
| Total Payout Method | 4.21% | 1.79% | 0.23% | 6.23% |

The dividend yield model (the dividend discount model, DDM) is often incorrectly applied, using the current yield with historical growth rates. As can be seen above, both the current payout and the historical growth are too low. The DDM is a theoretically correct model, but a proper interpretation would be to increase the future growth rate caused by today's artificially low yields. The total payout method makes it clear what is going on. Payouts are switching from dividend yields to buybacks, but the overall total payout yields are relatively constant over time. The switch, however, leads to low historical dividend yield growth and low current dividend yields.

## Conclusion

The total payout models presented here are useful in forecasting equity returns.

1. Changing payout policies should not change expected returns, as shown by Miller and Modigliani.

2. Payout growth should be linked long-term to the real economy.

3. Using the dividend discount model, although theoretically correct, is often incorrectly implemented with low current yields and low historical dividend growth rates.

4. Total Payout Models give more reasonable supply-side forecasts of returns.

# Chapter 11
# Stock Market Returns From 1815–2020

Studies on the long-horizon predictability of stock returns, by necessity, require a database of return information that dates as far back as possible. Since the late 1970s, Ibbotson Associates has produced a broad set of historical returns on asset classes dating back to 1926. Researchers interested in the dynamics of the U.S. capital markets prior to 1926 had to rely on indexes of uneven quality. In 2000, Roger G. Ibbotson and William N. Goetzmann, professors of finance, and Liang Peng, then a Ph.D. candidate in finance, all at Yale School of Management, assembled a New York Stock Exchange database of annual returns for the periods prior to 1926. The first part of this chapter covers the sources and construction of this annual return database extending back to 1815.

The second part of this chapter introduces a new set of monthly real stock market total returns developed by Paul Kaplan, now director of research at Morningstar Canada. Kaplan used these new returns to demonstrate that the severity of the financial crisis of 2008 was not unique but was merely the latest chapter in a long history of market meltdowns.

While we firmly believe that a 1926 starting date was approximately when quality financial data came into existence, our hope is that the continuing development of these data sets will allow modern researchers of pre-1926 stock returns, along with future researchers, to test a broad range of hypotheses about the U.S. capital markets as well as open up new areas for more accurate analysis.

## 1815–1925 Data Series Sources and Collection Methods

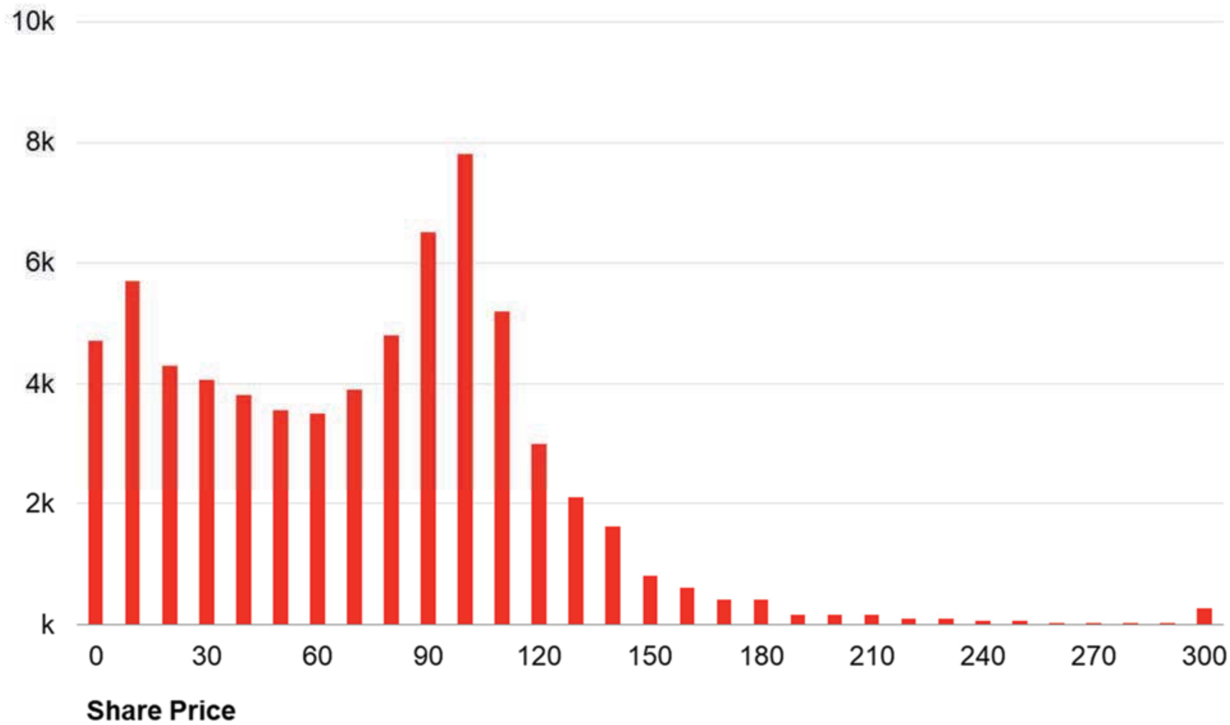
### Share Price Collection

End-of-month equity prices for companies listed on the New York Stock Exchange were hand collected from three sources published January 1815 to December 1870. For the period 1871 through 1925, end-of-month NYSE stock prices were collected from the major New York newspapers. *The New York Shipping List,* later called *The New York Shipping and Commercial,* served as the "official" source for NYSE share price collection up until the early 1850s. In the mid-1850s, *The New York Shipping List* reported prices for fewer and fewer stocks. This led to the collection of price quotes from *The New York Herald* and *The New York Times.* While neither claimed to be the official list for the NYSE, the number of securities quoted by each far exceeded the number quoted by *The New York Shipping List.*

It is important to note that in instances where no transaction took place in December, the latest bid and ask prices were averaged to obtain a year end price. In total, at least two prices from 664 companies were collected. From a low number of eight firms in 1815, the number of firms in the index reached a high point in May of 1883 with 114 listed firms.

Share prices for much of the period of analysis remained around $100. Exhibit 11.1 illustrates this point. The graph shows that the most common price of a share of stock was around $100. The distribution of stock prices is significantly skewed to the left with only a few trading above $200. Such a distribution suggests that management maintained a ceiling on stock prices by paying out most earnings as dividends. No reports of stock splits over the period of data were discovered.

**Exhibit 11.1:** Distribution of Raw Stock Prices 1815–1925




**Source of underlying data:** Morningstar, Inc. Used with permission.

## Dividend Collection

Dividend data was collected for the period 1825 to 1870 by identifying the semiannual dividend announcements for equity securities as reported in *The New York Shipping and Commercial*, *The Banker's Magazine*, *The New York Times*, and *The New York Herald*. From 1871 to 1925, aggregate dividend data from the Alfred Cowles[211] series was used. Whether the above publications reported dividends for all NYSE stocks is unknown; as a result, there is no way of knowing whether missing dividends meant that they were not paid or possibly not reported. Dividend records were collected for more than 500 stocks in the sample and most stocks paid dividends semiannually.

Two approaches were used to estimate the income return for each year. The first approach, the low-dividend return estimate, consisted of the summation of all the dividends paid in a given year

211 Cowles, A. 1939. *Common Stock Indices* (Bloomington, Ind.: Principia Press).

by firms whose prices were observed in the preceding year. This number is then divided by the sum of the last available preceding year prices for those firms. The second approach, the high-dividend return estimate, focused solely on firms that paid regular dividends and for which price data was collected. The sample is restricted to firms that have two years of dividend payments (four semiannual dividends) and for which there was a price observation. Using the second approach, dividend yields tend to be quite high by modern standards. It is important to note that when both a high- and a low-income return series were present, the average was computed. This holds true for the summary statistics table in this chapter as well as the graphs/tables presented throughout. Also, due to missing income return data for 1868, an average of the previous 43 years was computed.

## Price Index Estimation

### Index Calculation Concerns

When attempting to construct an index without having market capitalization data readily available, one is left with one of two options: an equal-weighted index or a price-weighted index. One key concern with an equal weighted index is the effect of a bid-ask bounce. Take for example an illiquid stock that trades at either $1.00 or $2.00 per share. When it rises in price from $1.00 to $2.00, it goes up by 100%. When it decreases in price from $2.00 to $1.00, it drops by 50%. Equally weighting these returns can produce a substantial upward bias. This led us to the construction of a price-weighted index.

### Calculation of the Price-Weighted Index

The procedure used for calculating the price-weighted index is rather simple. For each month, returns are calculated for all stocks that trade in two consecutive periods. These returns are weighted by the price at the beginning of the two periods. The return of the price-weighted index closely approximates the return to a “buy-and-hold” portfolio over the period. Buy-and-hold portfolios are not sensitive to bid-ask bounce bias. We believe that the price-weighted index does a fairly good job of avoiding such an upward bias. Companies were rather concentrated into specific industries. In 1815, the index was about evenly split between banks and insurance companies. Banks, transportation firms (primarily canals and railroads), and insurance companies made up the index by the 1850s. By the end of the sample period, the index was dominated by transport companies and other industrials.

### A Look at the Historical Results

It is important to note that there are a few missing months of data that create gaps in the analysis. The NYSE was closed from July 1914 to December 1914 due to World War I. This is obviously an institutional gap. There are additional gaps; we are missing returns for 1822, part of 1848 and 1849, parts of 1866, all of 1867 and January 1868. We do not know whether the records missing from the late 1860s are due to the Civil War, but the NYSE was certainly open at that time –

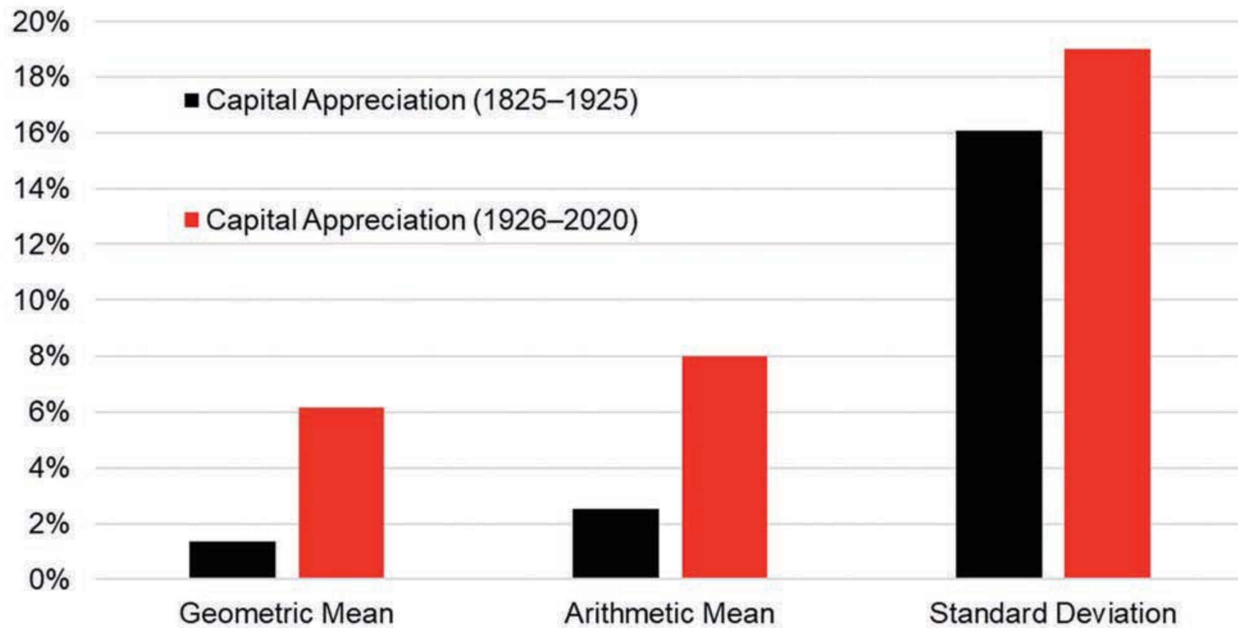
among other things, it was the era of heated speculation and stock price manipulation by legendary financiers Gould, Fisk, and Drew.

The number of available security records was quite lower after 1871. A change in the range of coverage by the financial press is the likely culprit for this. Further data collection efforts hopefully will allow these missing records to be filled in.

### Price Return

Exhibit 11.2 illustrates the annual geometric mean, arithmetic mean, and standard deviation of large-cap stock capital appreciation (i.e., price) returns as measured over two different time horizons: 1825–1925, and 1926–2020. It is interesting to note that large-cap stocks had an annual geometric capital appreciation return from 1825 through 1925 of slightly more than 1%. This number is significantly *lower* when compared to the annual geometric capital appreciation return experienced by large-cap stocks from 1926 to 2020 (slightly more than 6%). This once again alludes to the suggestion that dividend policies have evolved over the past two centuries, and that managers of old companies most likely paid out earnings to keep their stock prices lower. In today's financial world, capital appreciation is accepted as a substitute for dividend payments.

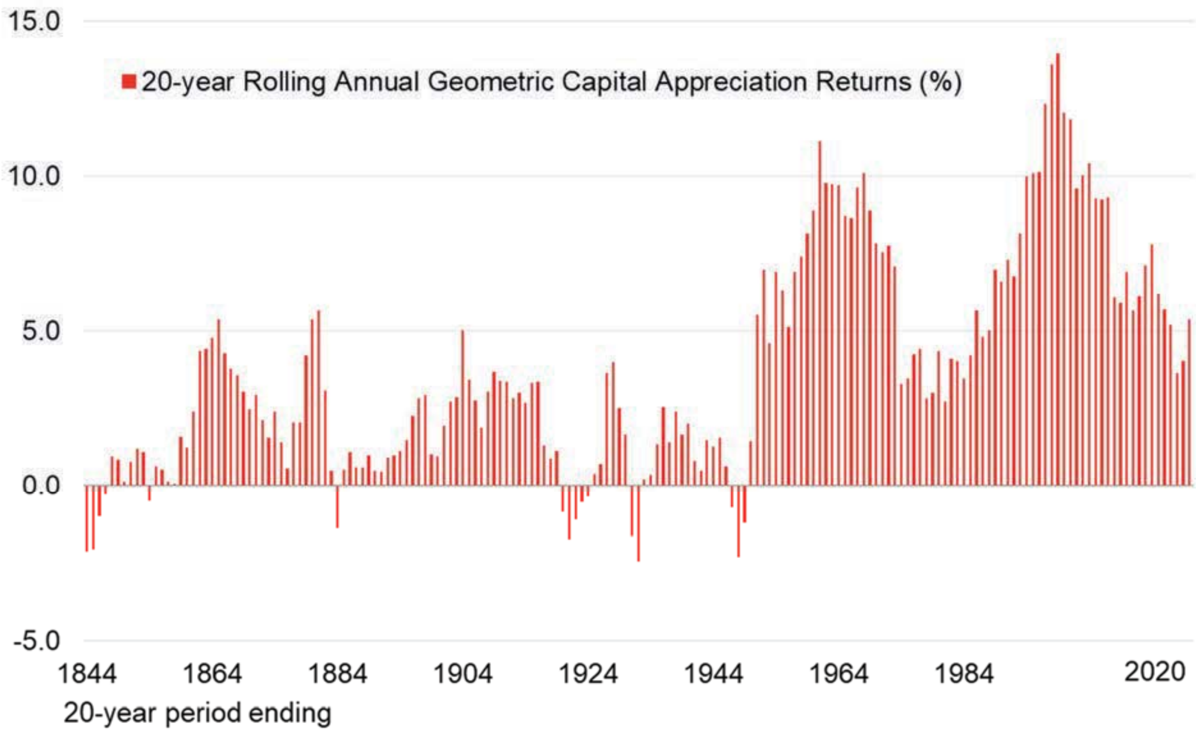
**Exhibit 11.2:** Large-Cap Stocks Capital Appreciation (i.e., "Price") Returns; Annual Geometric Mean, Geometric Mean, and Standard Deviation (%)
1825–1925 and 1926–2020




**Source of underlying data**: Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll.

The rise in capital appreciation returns over the years is more evident when viewing returns on a 20-year rolling period basis as Exhibit 11.3 demonstrates. In Exhibit 11.3, the annual geometric (i.e., compound) capital appreciation return is calculated for all 20-year periods ending 1844 through 2020. For example, the leftmost bar in Exhibit 11.3 represents the annual compound rate of return over the period 1825–1844, and the rightmost bar in Exhibit 11.3 represents the annual compound rate of return over the period 2001–2020.

**Exhibit 11.3:** Large-Cap Stocks: 20-year Rolling Annual Geometric Capital Appreciation Returns (%) 1825–2020

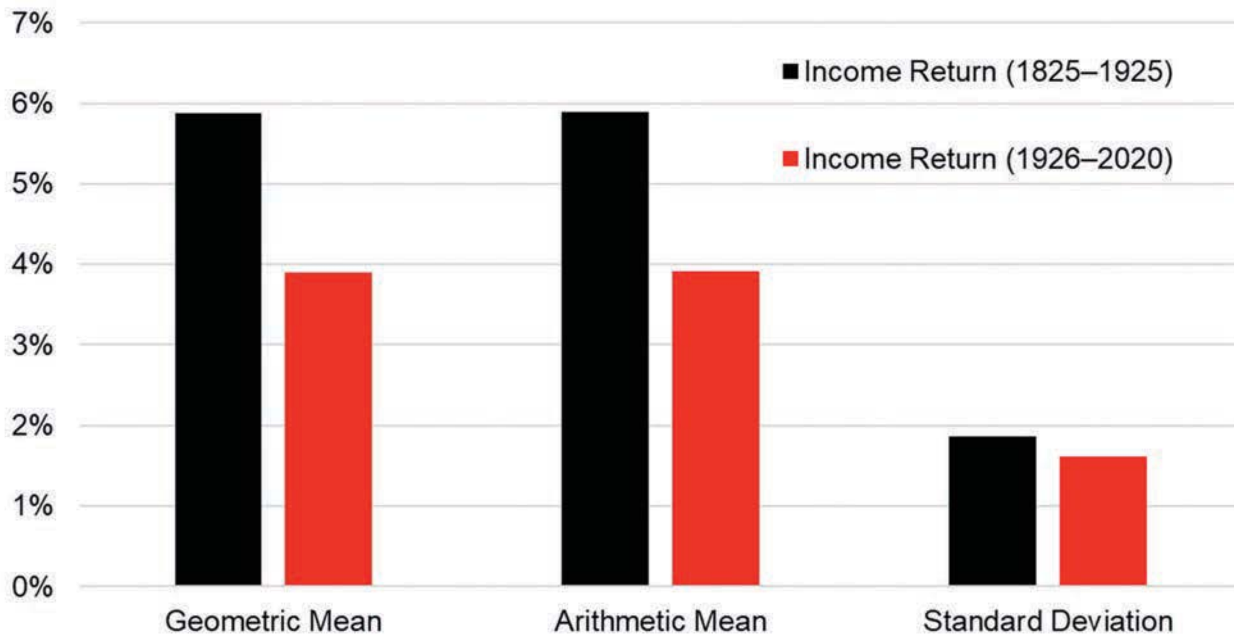



**Source of underlying data**: Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll.

### Income Return

Exhibit 11.4 illustrates the annual geometric mean, arithmetic mean, and standard deviation of large-cap stock income returns as measured over two different time horizons: 1825–1925, and 1926–2020. The *higher* income return of nearly 6% in the earlier period (1825–1925) compared to the *lower* income return in the later period (1926–2020) of less than 4%, and the fact the many stocks traded near par, once again suggest that most companies paid out a large share of their profits rather than retaining them.

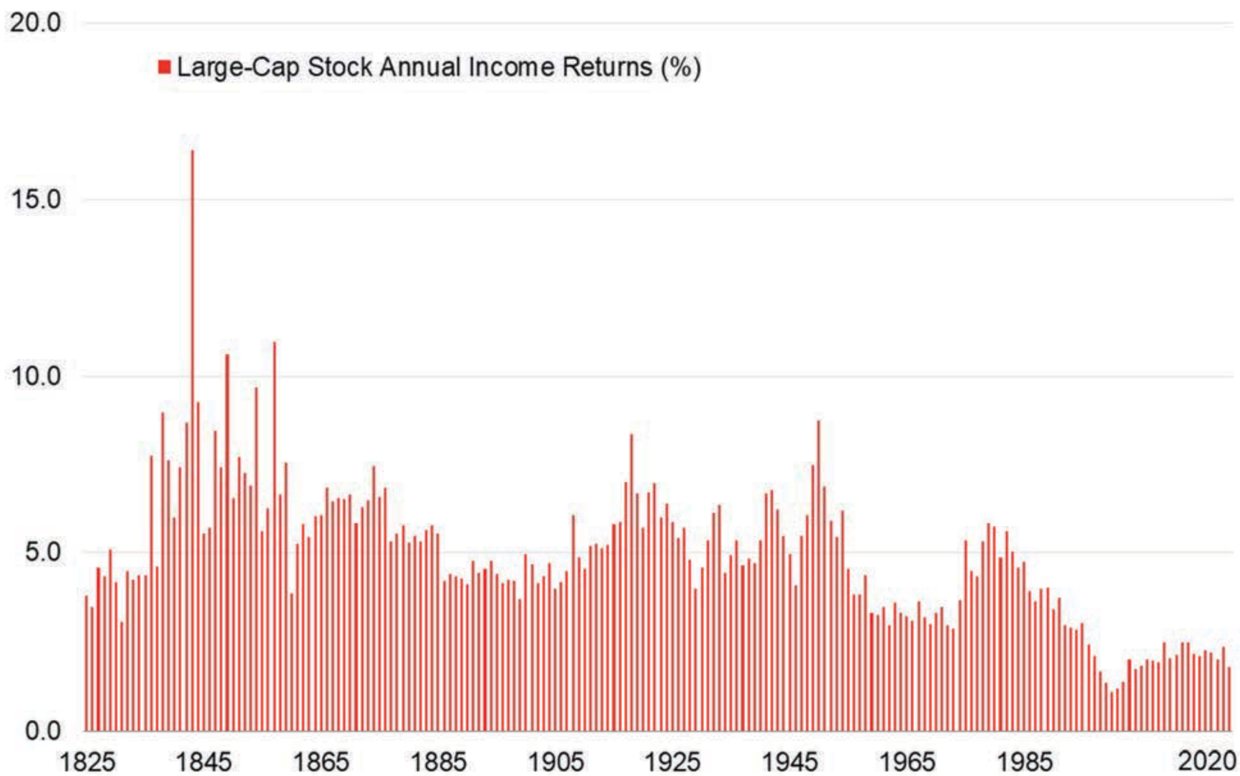
**Exhibit 11.4:** Large-Cap Stocks Income Returns; Annual Geometric Mean, Geometric Mean, and Standard Deviation (%)
1825–1925 and 1926–2020




**Source of underlying data**: Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll.

Exhibit 11.5 shows large-cap stock annual income returns for 1825 to 2020. In fact, when looking at the time distribution of dividend changes over the new period, dividend decreases were only slightly less common than increases, suggesting that managers may have been less averse to cutting dividends than they are today. Perhaps in the pre-income tax environment of the 19th century, investors preferred income return as opposed to capital appreciation.

**Exhibit 11.5:** Large-Cap Stocks Annual Income Returns (%) 1825–2020



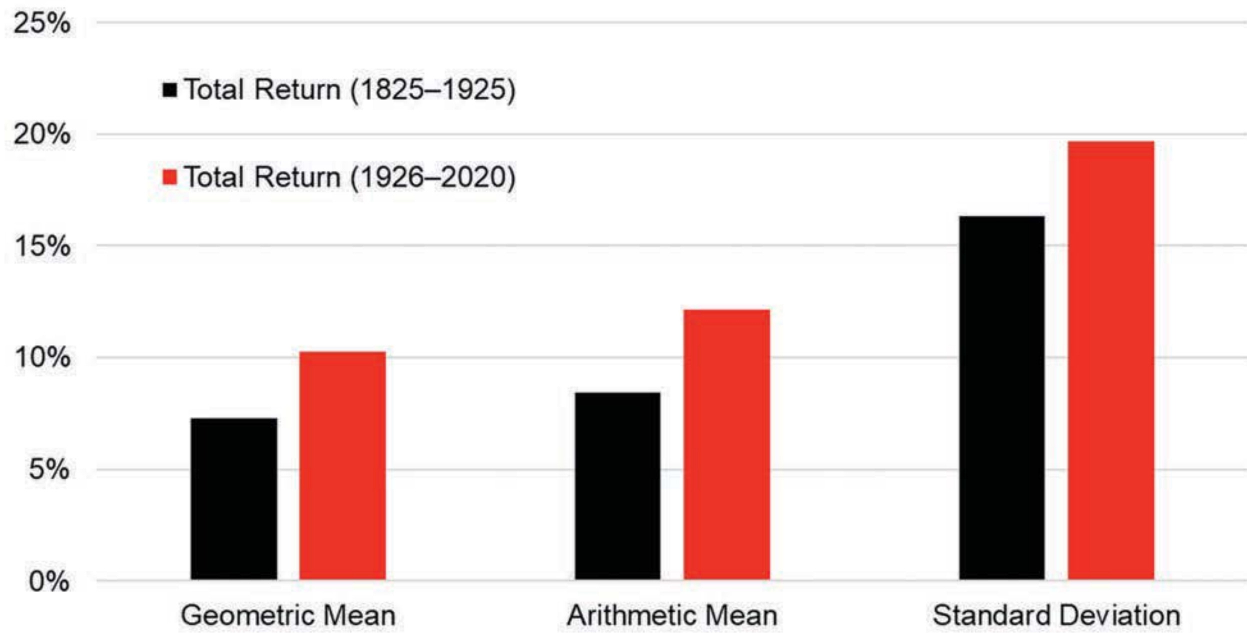

**Source of underlying data**: Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll.

## Total Return

Exhibit 11.6 illustrates the annual geometric mean, arithmetic mean, and standard deviation of large-cap stock total returns as measured over two different time horizons: 1825–1925, and 1926–2020.

It is interesting to notice that the annual geometric total return for large-cap stocks from 1825 to 1925 was a little over 7%. This is quite low when compared to the annual geometric total return of the commonly used 1926 to 2020 period (a little over 10%).

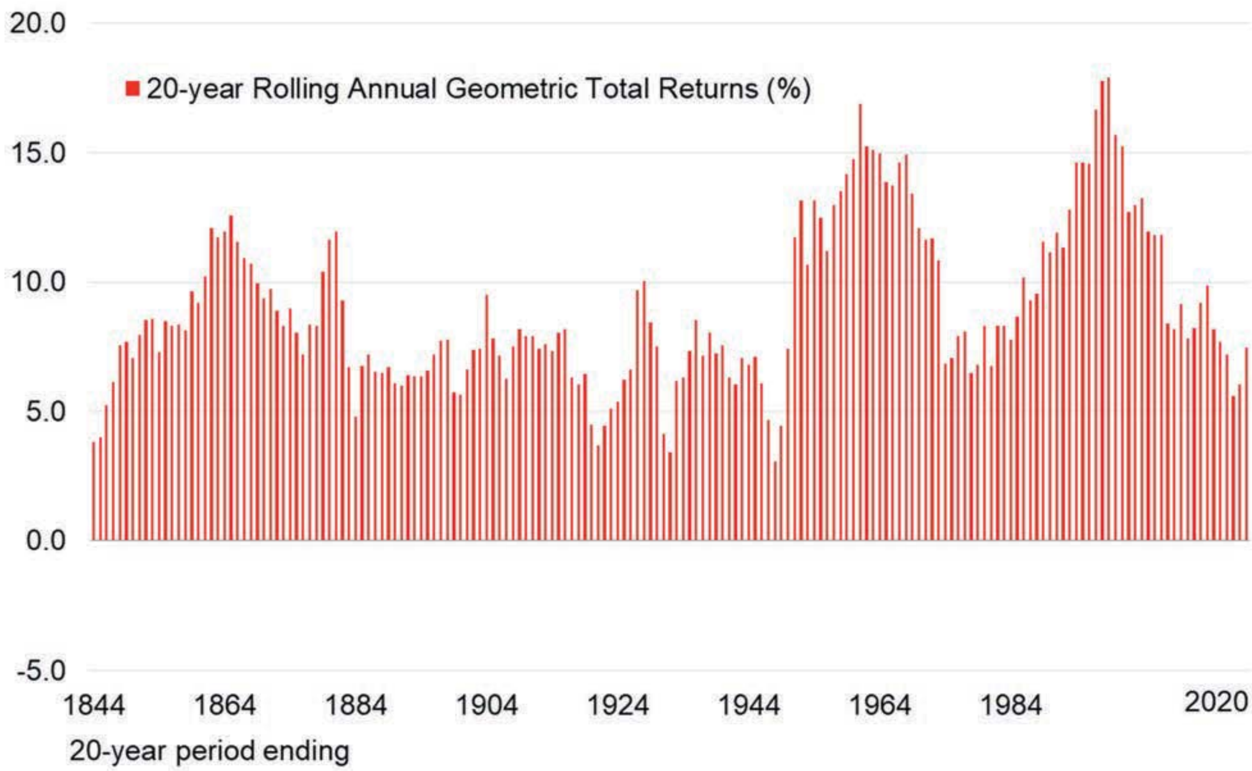
**Exhibit 11.6:** Large-Cap Stocks Total Returns; Annual Geometric Mean, Geometric Mean, and Standard Deviation (%)
1825–1925 and 1926–2020




**Source of underlying data**: Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll.

In Exhibit 11.7, the annual geometric (i.e., compound) total return is calculated for all 20-year periods ending 1844 through 2020. For example, the leftmost bar in Exhibit 11.7 represents the annual compound rate of return over the period 1825–1844, and the rightmost bar in Exhibit 11.7 represents the annual compound rate of return over the period 2001–2020.

**Exhibit 11.7:** Large-Cap Stocks: 20-year Rolling Annual Geometric Total Returns (%) 1825–2020




**Source of underlying data**: Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll.

## 150 Years of Stock Market Drawdowns

Those familiar with the history of U.S. capital markets as documented in this book may have found former Federal Reserve Chairman Alan Greenspan's characterization of the financial crisis of 2008 as a "once-in-a-century credit tsunami" quite surprising. A more appropriate statement may have been the one made by Leslie Rahl (founder of Capital Market Risk Advisors) more than a year before the crisis when she said, "We seem to have a once-in-a-lifetime crisis every three or four years."[212] Ms. Rahl was prescient – another "once in a lifetime" crisis occurred just 12 years later with the market crash in the first quarter of 2020 that was precipitated by the spread of the COVID-19 virus.

The contrast between Mr. Greenspan and Ms. Rahl's perspectives was the inspiration for an article in Morningstar magazine on the history of market meltdowns titled, "Déja Vu All Over Again."[213,214] In that article, Paul Kaplan, CFA, PhD (Director of Research, Morningstar Canada) illustrated the frequency and severity of the major drawdowns for various countries using time

---

212 Wright, C. 2007. "Tail Tales." *CFA Institute Magazine,* March/April.

213 Morningstar magazine is a publication for financial advisors and institutional investors. For more information about Morningstar magazine, call 312 384-4000 or visit us online: global.morningstar.com/MorningstarMagazine.

214 Kaplan, P.D. 2009. "Déjà Vu All Over Again." *Morningstar Advisor magazine,* February/March, P. 28.

series of stock market total returns. For the U.S., Kaplan naturally used the SBBI® large-cap stock index (the SBBI® large-cap stock index is essentially the S&P 500 index). The results of the study clearly demonstrate the severity of the financial crisis of 2008 was not unique but was merely the latest chapter in a long history of market meltdowns.

In 2009, a team of researchers at Morningstar expanded the analysis into a complete study on global equity market history as a contribution to the CFA Institute book on the global history of market crashes.[215] In this study, the research team used monthly *real* total returns that go back into history as far as was possible with reasonably reliable data.[216] The benefit of using real returns is to make meaningful return comparisons as our study spans such a long period. The benefit of going further back in history is, of course, to give a longer-term and more robust historical perspective on market crashes in terms of frequency, length, and magnitude.

To complete the study, the research team needed to find monthly data from before 1925 on both stock returns and inflation and calculate real returns. Because there was no such return series in existence, they had to create one out of readable available data.

Robert J. Shiller, 2013 Nobel laureate in economic sciences and the Sterling professor of economics at Yale University, posts monthly U.S. stock market returns and inflation data on his website that go back to 1871. Unfortunately, Shiller's stock data is based on monthly average prices rather than month-end prices. So, the research team could use his inflation data, but not his stock market data. Separately, Roger Ibbotson and some colleagues created an annual price and total return series for the NYSE that goes back to 1815 (as previously discussed in this chapter).[217] However, annual returns are at too low a frequency to measure the largest drawdowns of the period, such as the large drop in the stock market during the panic of 1907. Fortunately, there is a book that contains daily price data on the Dow Jones Averages going back to 1885.[218] The team estimated the monthly price returns in the broader NYSE price index from the monthly price returns on the Dow Jones Averages and then interpolated the total returns by assuming that the dividend levels remained constant during each year.

The Morningstar team produced a time series of U.S. stock market real total returns from 1871 to 2020. The first 15 years of this history (1871–1885) is *annual* real total returns, and the remaining 135 years (1886–2020) is *monthly* total real returns, for a total of 150 years.

### Truth in Numbers

The significance of this data is in the lessons that we can learn from it. Over the entire 150-year period, the Real U.S. Stock Market Index grew from $1.00 to $22,214.26in 1870 dollars. This is a compound annual real total return of 6.9%, almost the same as the post-1925 compound annual

---

215 Kaplan, P.D., Idzorek, T., Gambera, M., et al. 2009. "The History and Economics of Stock Market Crashes." In *Insights into the Global Financial Crisis*. Edited by Laurence B. Siegel (Charlottesville, Va.: *CFA Institute*).

216 That is, returns that include the reinvestment of dividends and are adjusted for inflation.

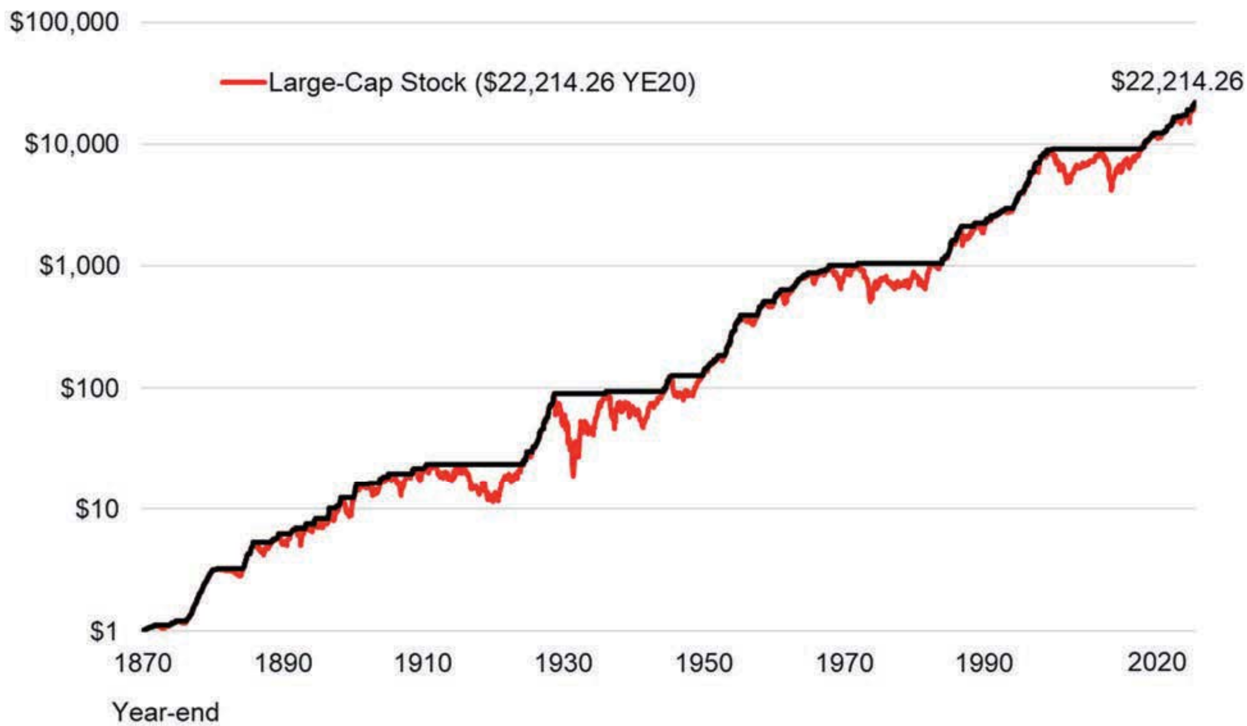
217 Goetzmann, W.N., Ibbotson, R.G., & Peng, L. 2000. "A New Historical Database for the NYSE 1815 to 1925: Performance and Predictability." *Journal of Financial Markets*, Vol. 4, No. 1, P. 1.

218 Pierce, P., ed. 1982. *The Dow Jones Averages 1885-1980* (Homewood, Ill.: Dow Jones-Irwin).

real total return of slightly over 7%. However, as Exhibit 11.8 shows, it was a very bumpy ride with a number of major drawdowns, some of which can be linked with specific economic and political events.

Exhibit 11.8 shows the growth of $1.00 invested in the U.S. stock market at the end of 1870 through December 2020 in *real* terms, along with a line that shows the highest level that the index had achieved as of that date (shown in gray).[219] Whenever this line is above the cumulative value line (shown in red), the index was below its most recent peak. The bigger the gap, the more severe the decline; the wider the gap, the longer the time until the index returned to its peak. Wherever this line coincides with the index line, the index was climbing to a new peak. The market crash in the first quarter of 2020 that was precipitated by the spread of the COVID 19 virus (when measured on a monthly basis) was significantly shorter and less acute than several of the drawdowns illustrated in Exhibit 11.8.

**Exhibit 11.8:** Large-cap Stocks: Real Return Index 1870–2020




**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll.

[219] Beginning with the *2017 SBBI®* Yearbook, we changed Exhibit 11.8 compared to previous editions. The information in Exhibit 11.8 first appeared in the *2010 SBBI® Yearbook*. In the 2010 through 2016 *SBBI® Yearbooks*, the graph shown in Exhibit 11.8 included both *annual* returns (for years 1871–1885) and *monthly* data (for years 1886–present), which distorted the graph slightly. Beginning with the 2017 version, Exhibit 11.8 includes only *monthly* data points over the entire time horizon (1871–2020). For each of the first 15 years of the graph (1871–1885), the annual returns were converted to "monthly" returns by calculating the single monthly return that could be applied to each standard 12-month period (January through December) that would result in an annual geometric annual return matching the original study. For example, for year 1871 the original Morningstar study reported an annual return of 7.56%. The single value calculated for the imputed "monthly" returns for January 1871 through December 1871 was therefore 0.609% (1+7.56%)^(1/12).

Exhibit 11.9 lists all of the drawdowns that exceeded 20%. There were 17 such declines, including the most recent one that ended in May 2013. Not surprisingly, the largest of all market declines started just before the Crash of 1929 and did not recover until toward the end of 1936. The U.S. stock market lost 79% of its real value in less than three years and took more than five years to recover. The most recent drawdown, the global financial crisis, was the second greatest decline, and it lasted nearly a decade. The combined effect of the crash of the Internet bubble in 2000 and the global financial crisis of 2008 caused the U.S. stock market to lose 54% of its real value from August of 2000 to February 2009.

The history of stock market drawdowns presented here shows that investing in stocks can be very risky, and the most recent crisis was hardly a "once-in-a-century" event. We should use this long-run data to better gauge the potential risks and long-term rewards of investing in risky assets such as stocks.

**Exhibit 11.9:** Largest Declines in U.S. Stock Market History, in Real Total Return Terms 1870–2020

| Peak | Trough | Decline (%) | Recovery | Event(s) |
|---|---|---|---|---|
| Aug. 1929 | May 1932 | 79.00 | Nov. 1936 | Crash of 1929, 1st part of Great Depression |
| Aug. 2000 | Feb. 2009 | 54.00 | May 2013 | Dot-com bubble burst (00-02), Crash 07-09 |
| Dec.1972 | Sep. 1974 | 51.86 | Dec, 1984 | Inflationary Bear Market, Vietnam, Watergate |
| Jun.1911 | Dec. 1920 | 50.96 | Dec. 1924 | WWI, Post-war Auto Bubble Burst |
| Feb.1937 | Mar. 1938 | 49.93 | Feb. 1945 | 2nd part of Great Depression, WWII |
| May 1946 | Feb. 1948 | 37.18 | Oct. 1950 | Post-war Bear Market |
| Nov.1968 | Jun. 1970 | 35.46 | Nov. 1972 | Start of Inflationary Bear Market |
| Jan.1906 | Oct. 1907 | 34.22 | Aug. 1908 | Panic of 1907 |
| Apr.1899 | Jun. 1900 | 30.41 | Mar. 1901 | Cornering of Northern Pacific Stock |
| Aug.1987 | Nov. 1987 | 30.16 | Jul. 1989 | Black Monday |
| Oct.1892 | Jul. 1893 | 27.32 | Mar. 1894 | Silver Agitation |
| Dec.1961 | Jun. 1962 | 22.80 | Apr. 1963 | Height of the Cold War, Cuban Missile Crisis |
| Nov.1886 | Mar. 1888 | 22.04 | May 1889 | Depression, Railroad Strikes |
| Apr.1903 | Sep. 1903 | 21.67 | Nov. 1904 | Rich Man's Panic |
| Aug.1897 | Mar. 1898 | 21.13 | Aug. 1898 | Outbreak of Boer War |
| Sep.1909 | Jul. 1910 | 20.55 | Feb. 1911 | Enforcement of the Sherman Anti-Trust Act |
| May 1890 | Jul. 1891 | 20.11 | Feb. 1892 | Baring Brothers Crisis |

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. In Exhibit 11.1, the Ibbotson Associates "Large Company Stocks" series represents U.S. equities for all dates from January 1926 forward. The Ibbotson Associates "Large Company Stocks" series is essentially the S&P 500 index.

In the fall of 2018, U.S. equity indices experienced significant declines. The S&P price index peaked at 2,930.75 on September 20, 2018; By December 24, 2018, the S&P 500 price index had declined to 2,351.10, or –19.8%, just short of the –20% threshold necessary to qualify it to appear in Exhibit 11.8.

Most recently, the market crash in the first quarter of 2020 that was precipitated by the spread of the COVID-19 virus (when measured on a monthly basis, as the analysis presented in Exhibit 11.8 is) also did not meet the 20% threshold. As of December 31, 2019 the S&P 500 total return index ($1.00 = December 31, 1925) was $9,243.90, a record high. By the end of March 2020 the index had fallen to $7,432.28, a drop of 19.6%. On a daily basis, however, the S&P price index was 3,386.15 on February 19, 2020. By March 23, 2020 this index had fallen to 2,237.40, representing a 33.9% decline.[220]

Traditional measures of risk, such as standard deviation, can underestimate the risk of drawdowns that are many standard deviations away from the mean (i.e., on the left tail of a distribution). We suggest that these traditional measures of risk be supplemented with measures that better capture the "fat tailed" nature of the historical returns and drawdowns as presented here. A complete discussion of incorporating fat-tailed distributions into risk measures is found in Chapter 10.

## Reaching Back Beyond 1926

Collection efforts have yielded a comprehensive database of NYSE security prices for nearly the entire history of the exchange. The goal of these studies is to assemble a NYSE database for the period prior to 1926. The 1926 starting date was approximately when high-quality financial data came into existence. However, with a pre-1926 database assembled, researchers can expand their analyses back to the early 1800s. It is our hope that the long time series outlined in this chapter will lead to a better understanding of how the U.S. stock market evolved from an emerging market at the turn of the 18th century to the largest capital market in the world today.

## The Origin of Market Bubbles

As we've seen so far in this chapter, we have witnessed many asset-price bubbles. In each case, the story seems to be the same: Positive feedback and herding among speculative investors produce runaway prices until the deviation from equilibrium is so large that the market becomes unstable, creating a high probability (or an inevitability) of a crash. This raises the question: Do asset-price bubbles typically share the same characteristics and do all bubbles originate in the same manner? If yes, can we identify these factors beforehand and predict when a bubble will burst? James Xiong, head of quantitative research at Morningstar Investment Management, addressed these questions in an article in Morningstar magazine, "The Chinese Art Market and the Origin of Bubbles."[221] The rest of this section has been written by Xiong and adapted from his article.

---

220 Source of daily S&P 500 price index data: Yahoo Finance at https://finance.yahoo.com/.

221 Xiong, J. 2012. "The Chinese Art Market and the Origin of Bubbles." *Morningstar magazine*, August/September, P. 64.

## Herd Behavior and Market Bubbles

A number of studies have considered herd behavior as a possible explanation for the excessive volatility observed in financial markets.[222] The thinking behind this approach is simple: Interaction of market participants through herding can lead to large fluctuations in aggregate demand, leading to heavy tails in the distribution of returns. In the popular literature, "crowd effects" often have been associated with large fluctuations in market prices of financial assets.

Robert Shiller provides evidence to support his argument that "irrational exuberance" played a role in producing the ups and downs of the stock and real estate markets.[223] He listed 12 precipitating factors that gave rise to the booms in the stock markets and housing markets. These factors are amplified via feedback loops and naturally occurring Ponzi schemes, aided by the media, and can ultimately lead to market crashes.

Shiller also demonstrates that psychological factors, such as herd behavior and epidemics, are exerting important effects. For example, the influence of authority over people can be enormous; people are ready to believe authorities even when they plainly contradict matter-of-fact judgment.

He cites many other factors, including that people tend to follow other people and choose not to exercise their own judgment about the market; also, most people purchase stocks based on direct interpersonal communication instead of independent research.

Rama Cont and Jean-Philippe Bouchaud[224] provide a mathematical model to link two well-known market phenomena: the heavy tails observed in the distribution of stock market returns on one hand and herding behavior in financial markets on the other hand.

## Predicting Crashes

In the 1990s, two groups of researchers[225] independently discovered an apparent tendency of stock prices to exhibit log-periodic power laws (LPPL) before a crash. The fundamental hypothesis of the model is that financial crashes are macroscopic examples of critical phenomena. A critical phenomenon indicates a highly correlated unstable market. In other words, as some traders say, "In a market crisis, all correlations jump to one."

Collective behaviors in people emerge through the forces of imitation, which leads to herding. Herding behavior of investors can result in a significant deviation of financial prices from their

---

222 See three references: Bannerjee, A.V. 1992. "A Simple Model of Herd Behavior," *Quarterly Journal of Economics*, Vol. 107, P. 797. Topol, R. 1991. "Bubbles and Volatility of Stock Prices: Effect of Mimetic Contagion," *The Economic Journal*, Vol. 101, P. 786. Shiller, R.J. 1989. *Market Volatility* (Cambridge, Mass.: MIT Press).

223 Shiller, R.J. 2005. *Irrational Exuberance*, 2nd ed. (Princeton, N.J: Princeton University Press).

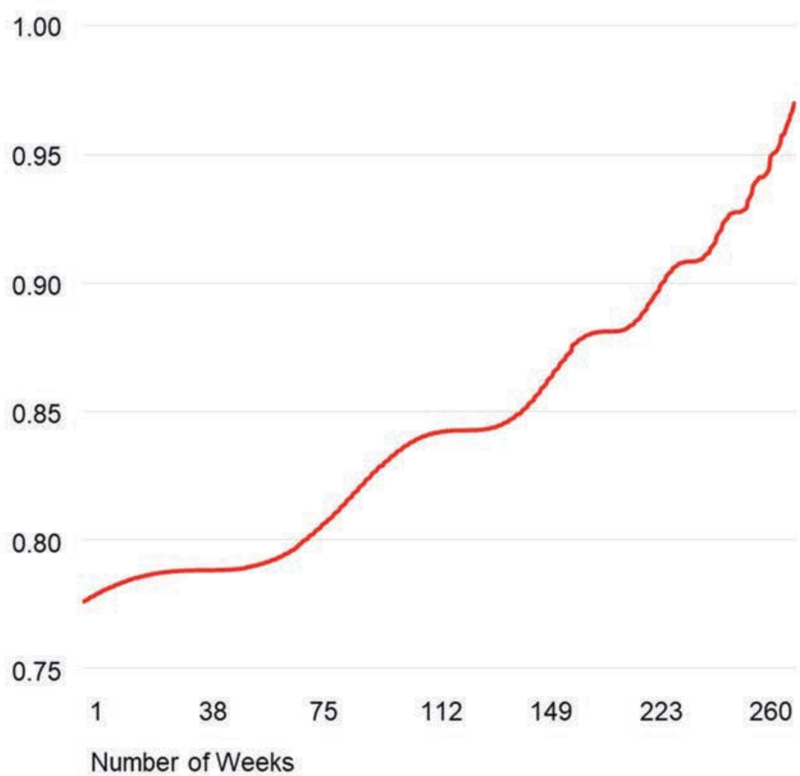
224 Cont, R. & Bouchaud, J.-P. 2000. "Herd Behavior and Aggregate Fluctuations in Financial Markets," *Journal of Macroeconomic Dynamics*, Vol. 4, P. 170.

225 See two references: Sornette, D., Johansen, A. & Bouchaud, J.-P. 1996. "Stock Market Crashes, Precursors and Replicas," *J. Phys. I.* (France), Vol. 6, P. 167. Feigenbaum, J. & Freund, P.G.O. 1996. "Discrete Scale Invariance in Stock Markets Before Crashes," *International Journal of Modern Physics B*, Vol. 10, P. 3737.

fundamental values. A speculative bubble, which is caused by a positive feedback investing style, also leads to a faster-than exponential power law growth of prices.[226] The competition between such nonlinear positive feedbacks and negative feedbacks contributes to nonlinear oscillations. For example, technical investors who have a positive view of the market bid up prices at the expense of fundamental investors, who view the market as ridiculously overpriced. The result is that a log-periodic modulation of the price accelerates up to the crash point. Exhibit 11.10 shows an example of what smooth log-periodic oscillations look like. Notice how the oscillations and the index value increase at an increasing rate as the date gets closer to the crash date.

Like any other models, the LPPL model has been debated and challenged, and we will not attempt to discuss that here. Major stock market crashes around the world, however, can be quantitatively explained by this model. These crashes include the 1929 crash, the 1987 crash, the crash of the Russian market in 1998, the 1990 Japanese Nikkei Index crash, several Hong Kong crashes in the 1990s, the Internet bubble crash in 2000, the financial crisis of 2008–2009, and more than 20 emerging-markets crashes. All of these market bubbles appeared to show the similar LPPL before they crashed.

**Exhibit 11.10:** Example of Log-Periodic Oscillations




226 Sornette, D. 2003. *Why Stock Markets Crash: Critical Events in Complex Financial Systems* (Princeton, N.J.: Princeton University Press).

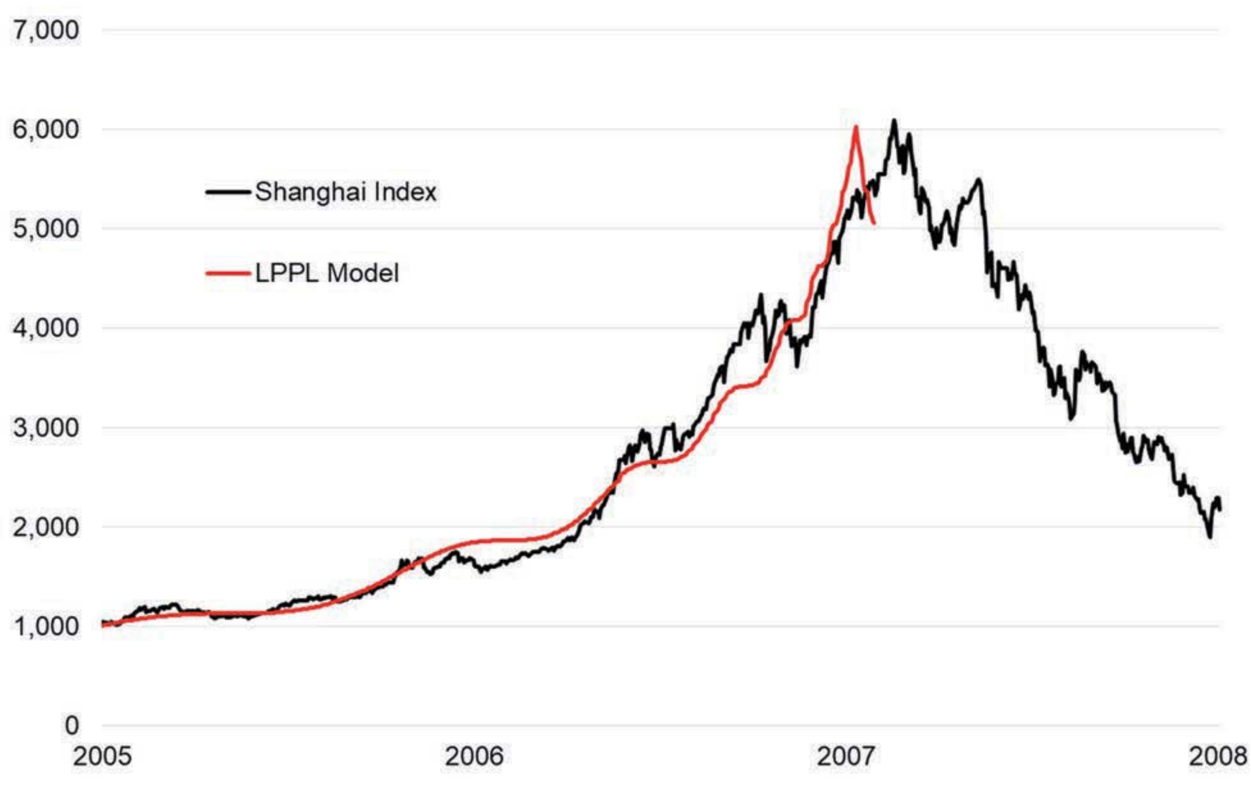
**Chinese Stock Market Crash of 2007**

Greed and fear are rooted in human nature, so it is unlikely that people will change anytime soon. Greediness and fear also drive herding and positive feedbacks, so investors should expect these factors to remain in markets. The latest herding example occurred not too long ago, in 2007. In particular, we'll look at the Chinese stock market crash.

We use the Shanghai Stock Exchange Composite Index to represent the Chinese stock market. The Chinese stock market is dominated by individual investors, unlike equity markets in developed countries where a form of polarization exists between individual and institutional investors. Millions of new Chinese small investors flooded into the booming Chinese stock market from 2005 to 2007, indicating a strong herd behavior. The bubble burst in October 2007. A year after the crash, the Shanghai Composite had lost about 64% of its value, a classic example of herd behavior leading to a market crash in an emerging market.

Using the LPPL model, Exhibit 11.11 shows that the Chinese stock market crash in 2007 was predictable. The gray line charts the price of the index. The red line is the calculated curve based on the LPPL model. The out-of-sample test was made Sept. 25, 2007. The model predicted a crash date of Sept. 5, 2007. The actual crash started Oct. 17, 2007, 42 days later than predicted. The time series price index is reasonably fitted by the log periodic power law model; we can see the precursors of log periodic oscillations before the crash occurred.

**Exhibit 11.11:** Chinese Stock Market Crash Predicted by LPPL Model



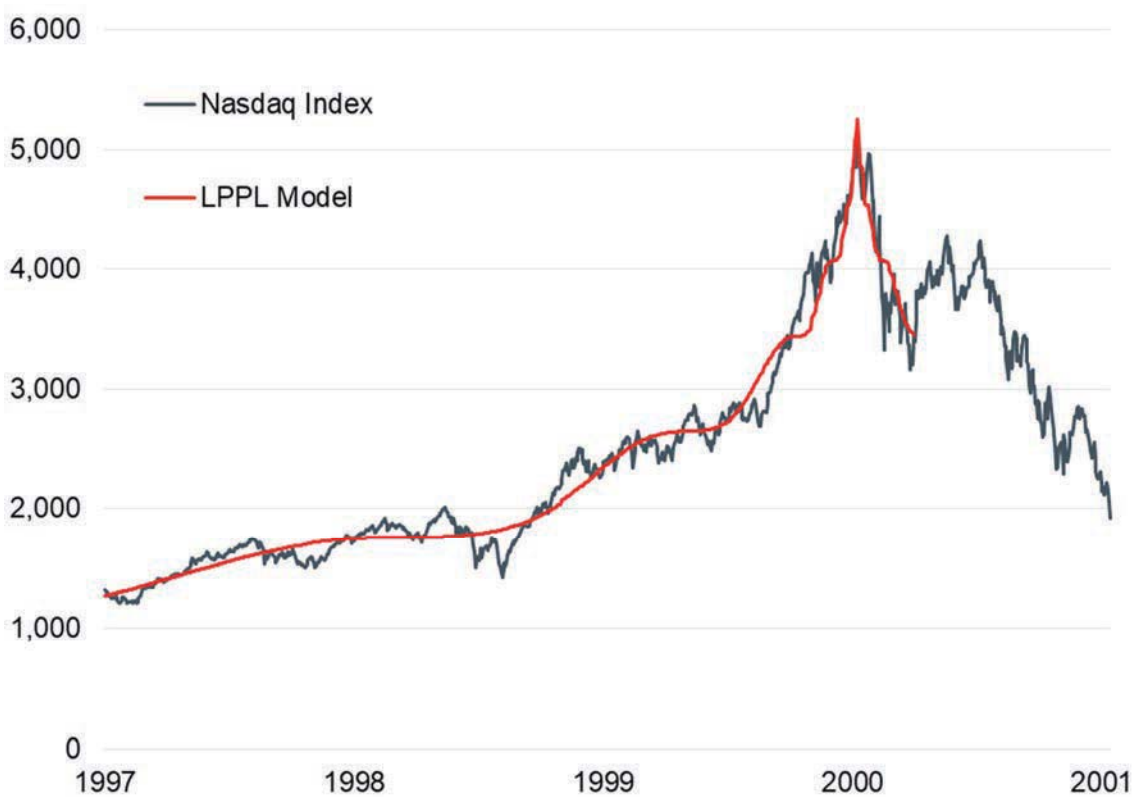
**NASDAQ Crash of 2000**

History provides many examples of bubbles driven by unrealistic expectations of future earnings. These types of bubbles do not just occur in developing markets. An example is the NASDAQ crash of 2000.

The NASDAQ Composite Index consists mainly of technology stocks, such as Internet, e-commerce, software, computer hardware, and telecommunications names. When the NASDAQ closed at a high of 5,049 on March 10, 2000, many stocks were trading at four-digit price/earnings (P/E) ratios.[227] Brocade Communications Systems, for example, had a P/E of 6,185; Trend Micro ADR had a P/E of 4,350; and SeaChange International traded at a P/E of 3,765. Investors in these companies seemed to be focusing on high future earnings and seemingly did not focus on other economic fundamentals.

Exhibit 11.12 shows the bubble phase of the NASDAQ. The red line stands for the price of the index. The red line is based on the LPPL model. Again, the model clearly picked up the signals of an impending crash and almost perfectly predicted it.

**Exhibit 11.12:** NASDAQ Market Crash Predicted by LPPL Model




[227] The March 10, 2000 closing level (5,049) was an all-time high close for the NASDAQ at the time. The NASDAQ did not close above this price until over 14 years later, April 23, 2014, when the index closed at 5,056.06.

## The LPPL Model

The log-periodic power law can be quantified as:[228]

$$\ln[p(t)] = A - B\tau^m + C\tau^m \cos[\omega \ln(\tau) - \varphi]$$

Where:

- $p(t)$ = price
- $A$ = The peak value of $\ln(p(t))$
- $B$ = Base for the slope of the logarithmic curve
- $\tau$ = $t_c - t$; which is the distance to the end of the bubble
- $m$ = Growth accelerator; must be $0 < m < 1$
- $C$ = Base for the oscillations; must be > 0
- $\omega$ = Angular log-frequency
- $\varphi$ = Arbitrary phase determining the unit of time

A geometric description for LPPL Model is that a log-periodic modulation of the ln(price) accelerates up to the crash point. The combination of *B* with a value greater than 0 and *m* with a value between 0 and 1 accelerates the slope so that it is faster than a typical exponential acceleration. The combination of *C* and the cosine segment determines the amplitude and frequency of the log-periodic oscillations.

We used the Levenberg Marquardt algorithm to predict the crash for the two bubbles (Exhibits 11.11 and 11.12). The fitted parameters are exhibited in Exhibit 11.13.

**Exhibit 11.13:** Best Fitted Parameters for the Shanghai Composite Index and the NASDAQ Index

| Stock | $t_c$ | $m$ | $w$ | $\phi$ | $A$ | $B$ | $C$ |
|---|---|---|---|---|---|---|---|
| Shanghai Index | September 2007 | 0.64 | 10.90 | 4.91 | 2.17 | 0.15 | -0.01 |
| NASDAQ Index | March 2000 | 0.45 | 6.45 | 5.26 | 8.61 | 0.88 | 0.06 |

228 Sornette, D. 2003. *Why Stock Markets Crash: Critical Events in Complex Financial Systems* (Princeton, N.J.: Princeton University Press).

**Power of the Model**

We showed that two recent market bubbles displayed the same LPPL signature before they crashed. Our analyses indicate that all the bubbles have the same origins and similarly move toward a crash.

Positive feedback and herding produce runaway prices until the deviation from equilibrium is so large that the market is unstable and has a high probability to crash. When the stock price accelerates at a much faster rate than the exponential growth rate, the skyrocketing return will always come with an increased crash hazard rate.

Financial markets are complex systems. In such systems, a speculative bubble can easily be created through positive feedback. What is more challenging is that, as complex systems grow, two things happen.[229] These systems require exponentially greater amounts of energy to keep operating, and they become vastly more risky and prone to catastrophic failure.

[229] Rickards, J. 2011. *Currency Wars: The Making of the Next Global Crisis* (New York: Portfolio/Penguin).

# Chapter 12
# International Equity Investing[230]

International investment opportunities are growing rapidly, encouraged by open markets and the accelerating economies of many nations. The evidence in favor of taking a global approach to investing is plentiful, as are the possible rewards an investor can reap.

However, significant risks are present as well – risks that apply strictly to the international marketplace. In this chapter, we consider both the rewards and the risks associated with international investments.

## Construction of the International Indexes

Our analysis of international investing uses the indexes created by Morgan Stanley Capital International, Inc. The MSCI® indexes are designed to measure the performance of the developed and emerging stock markets, reflecting the performance of the entire range of stocks available to investors in each local market.[231,232]

From January 1970 to October 2001, inclusion in the MSCI indexes was based upon market capitalization. Stocks chosen for the indexes were required to have a target market representation of 60% of total market capitalization.

MSCI has enhanced its index construction methodology by free-float-adjusting constituents' index weights and increasing the target market representation. Target market representation increased

230 This chapter is an overview of international equity investing that is limited to analyzing the relative historical performance of international (versus U.S.) equities, and does not include the much-expanded analyses of country-level risks and industry level risks (on a global scale) that are available in the D&P/Kroll online Cost of Capital Navigator platform's (i) International Cost of Capital Module, and (ii) International Industry Benchmarking Module. To learn more about the Capital Navigator, visit dpcostofcapital.com. These two resources are summarized as follows: International Cost of Capital Module: Provides measures of relative country risk for over 175 countries from the perspective of investors based in over 50 countries. Other data includes equity risk premia for 16 countries, risk-free rates for developed markets, industry betas for a global index as well as for developed markets, and long-term inflation expectations and corporate income tax rates for over 175 countries. Full country risk premia (CRPs) and relative volatility (RV) factor Tables by country (depending on subscription level). International Industry Benchmarking Module: Provides industry-level cost of capital estimates (cost of equity capital, cost of debt capital, and weighted average cost of capital, or WACC) plus detailed industry-level statistics for sales, market capitalization, capital structure, levered and unlevered betas, valuation multiples, financial and profitability ratios, equity returns, aggregate forward-looking earnings-per share (EPS) growth rates, and more. Over 300 critical industry-level data points are calculated for each industry (depending on data availability). Industries are organized by global industry classification standard (GICS) code. The International Industry Benchmarking Module can be used to benchmark, augment, and support the analyst's own custom analysis of the industry in which a subject business, business ownership interest, security, or intangible asset resides. The Cost of Capital Navigator also has two U.S.-centric modules: the U.S. Cost of Capital Module and the U.S. Industry Benchmarking Module. For more information about the Cost of Capital Navigator visit dpcostofcapital.com.

231 The international stock series presented throughout this chapter is represented by the MSCI EAFE® equities index. The MSCI EAFE Index is an equity index which captures large- and mid-cap representation across Developed Markets countries around the world, excluding the US and Canada. With 918 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in each country. To learn more about MSCI, visit msci.com.

232 All returns and statistics in this chapter are expressed in $USD, unless otherwise noted.

from 60% of total market capitalization to 85% of free-float-adjusted market cap within each industry group, within each country. MSCI defines the free float of a security as the proportion of shares outstanding that is deemed to be available for purchase in the public equity markets by international investors.

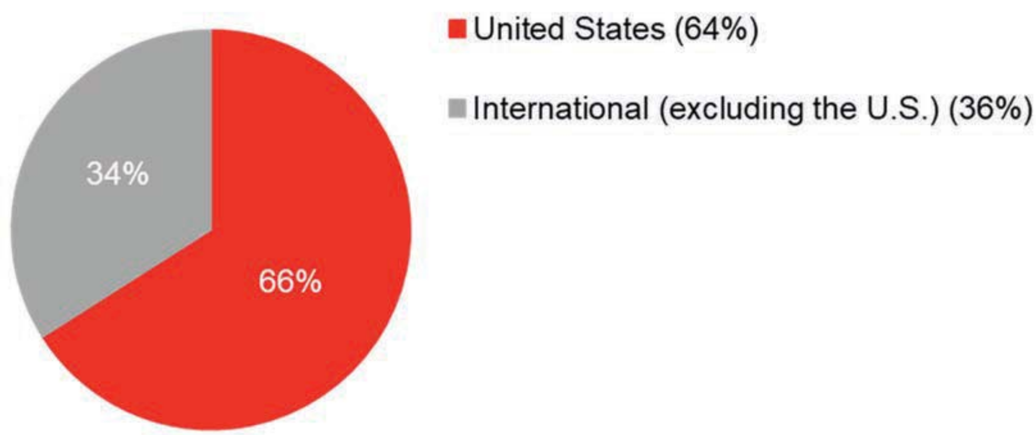
## Benefits of Investing Internationally

The arguments for investing internationally can be powerful. Examples may include (i) participation in the more than half of the world's investable assets that exist outside the U.S., (ii) growth potential, (iii) diversification, and (iv) potential improvement of the risk/reward trade-off.

### Investment Opportunities

An investor who chooses to ignore investment opportunities outside of the U.S. is missing out on a significant percentage of the investable developed stock market opportunities in the world. Exhibit 12.1 presents the relative size of international and domestic developed markets as of February 2021. As of February 2021, the total developed world stock market capitalization was $52.1 trillion, with $17.7 trillion representing international stock market capitalization.[233]

**Exhibit 12.1:** MSCI World Stock Market Capitalization: $52.1 Trillion
February 2021




233 **Source:** MSCI World Index Equity Fact Sheet. For more information, visit: msci.com.

### Growth Potential

Exhibit 12.2 Illustrates the growth of $1.00 invested in international stocks (as represented by the MSCI EAFE index), and U.S. large-cap stocks (i.e., the S&P 500 total return index), long-term government bonds, U.S. Treasury Bills, and a hypothetical asset returning the inflation rate over the period from the end of 1969 to the end of 2020.[234] Of the asset classes shown in Exhibit 12.2, the $1.00 invested at year-end 1969 in U.S. large-cap stocks grew the most by year-end 2020 (over $180), followed by International Stocks (over $90).

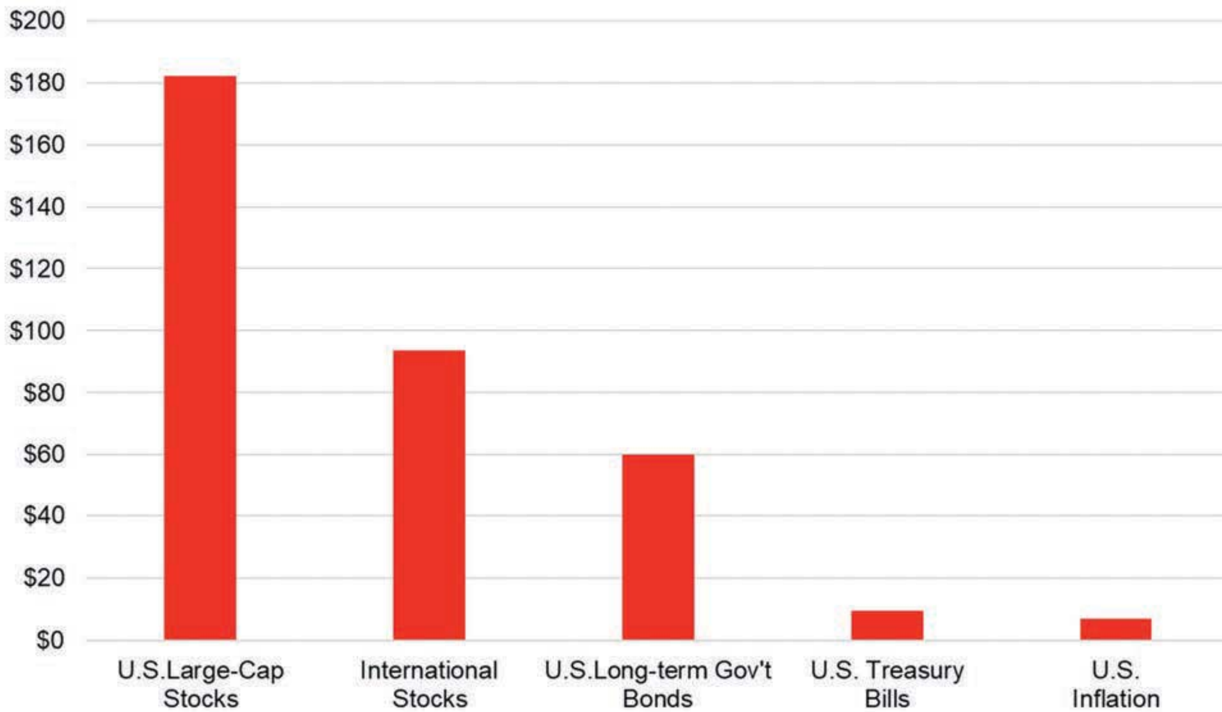
In the time horizon over which this analysis is performed (1970–2020), international stocks generally outperformed U.S. large-cap stocks from 1970 through the late 1990s, but in more recent years U.S. large-cap stocks have generally outperformed international stocks.

To illustrate this seeming reversal of relative performance in more recent years, consider that a $1.00 investment at year-end 1969 in U.S. large-cap stocks would have grown to nearly $19 by end of 1995, but the same dollar invested in international stocks would have grown to nearly $25. However, a $1.00 investment at year-end 1995 in U.S. large-cap stocks would have grown to nearly $10 by the end of 2020 (25 years), but the same dollar invested in international stocks would have grown to slightly a little more than $3.70.

Both U.S. and international stocks were affected by the 2008 financial crisis. In 2008, U.S. large-cap stocks fell nearly 37% and international stocks fell over 43%. In the twelve-year period after 2008, both U.S. large-cap stocks and international stocks have recovered, with U.S. large-cap stocks producing an approximate 15% annual return, significantly outperforming international stocks which produced an annual return of just over 8%.

234 In this chapter, the "U.S." series used are the same "SBBI" series used throughout the rest of this book. "U.S." is added to these series' names in this chapter only to differentiate them from the MSCI EAFE equities index, which is used to represent "international" equities in this chapter.

**Exhibit 12.2:** Global Investing
Index (Year-end 1969 = $1.00) 1970–2020

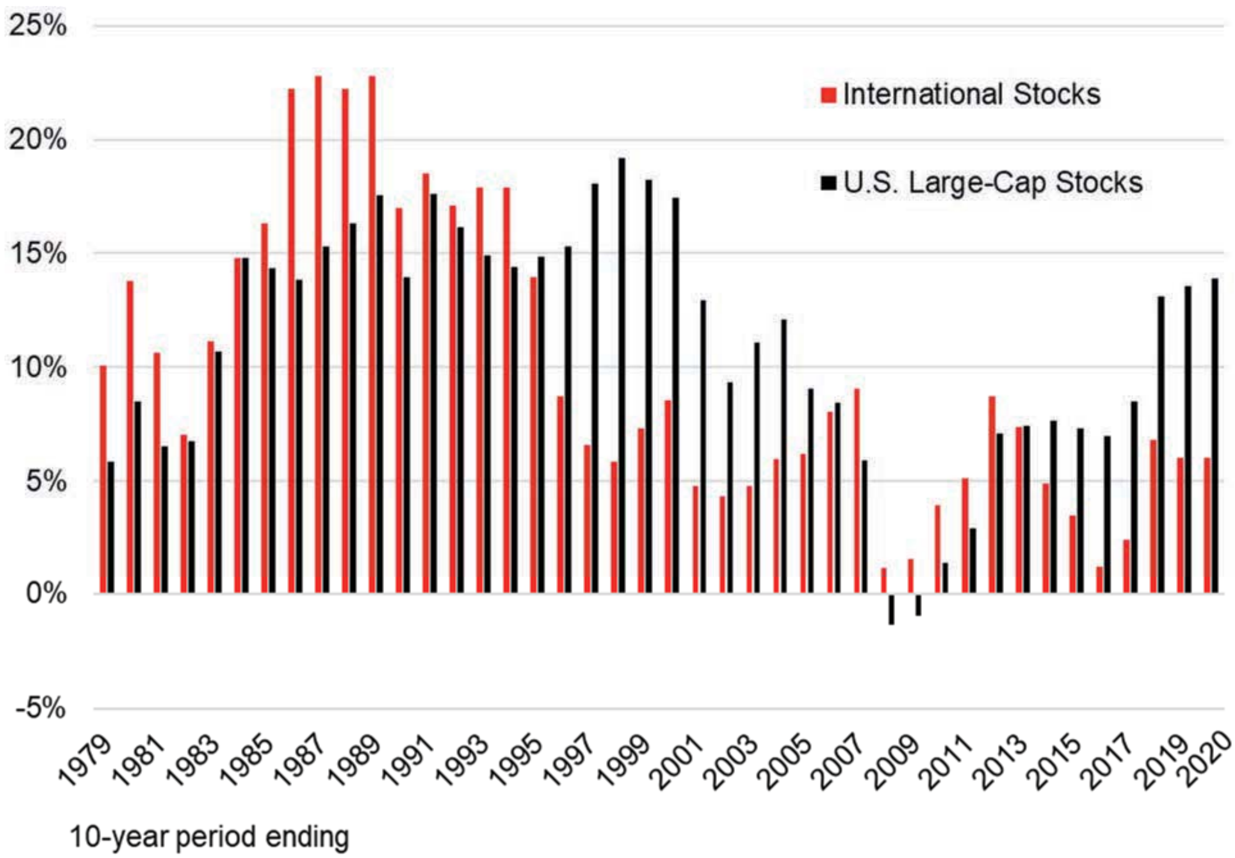



**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (iii) (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** The international stock series is represented by the MSCI EAFE® equities index. To learn more about MSCI, visit msci.com.

An additional perspective of the relative returns of U.S. large-cap stocks and international stocks is provided in Exhibit 12.3, which shows the annual compound performance of international and U.S. large-cap stocks over rolling 10-year holding periods ending 1979 through 2020.

International stocks outperformed in each of the 10-year periods ending 1979 through 1994, but U.S. large-cap stocks outperformed International stocks in 20 out of the 26 10-year periods ending 1995 through 2020, sometimes quite significantly. For example, in the twelve-year period since 2008, U.S. large-cap stocks have outperformed international stocks by a factor of two (approximately 15% annual compound return versus just over 8% annual compound return, respectively).

**Exhibit 12.3:** U.S. Large-Cap Stocks and International Stocks, 10-Year Holding Period Compound Annual Total Returns (%) 1970–2020

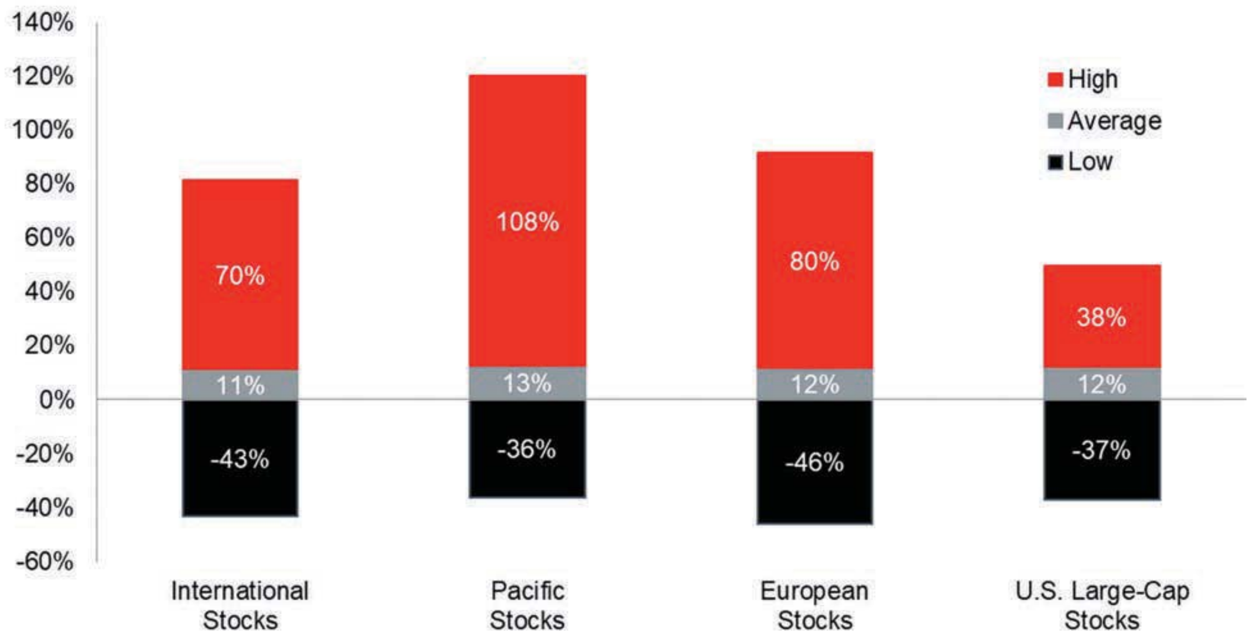



**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** The international stock series is represented by the MSCI EAFE® equities index. To learn more about MSCI, visit msci.com.

Just as U.S. stock prices fluctuate from one period to the next, prices of international stocks are subject to significant gains and declines. However, past returns from international stocks have fluctuated even more so than the returns of U.S. stocks. Annual ranges of returns provide an indication of the historical volatility (risk) experienced by investments in various markets.

Exhibit 12.4 illustrates the range of annual returns for U.S. large-cap stocks and international stocks, as well as European and Pacific regional equity composites, over the period 1970 through 2020. Although all of the composites have similar compound returns over the period, the three international composites exhibit greater volatility than the U.S. composite. All investments have the potential of dramatic ups and downs; however, a long-term approach to investing may help reduce the pain of volatility.

**Exhibit 12.4:** Global Stock Market Returns: Annual Ranges of Returns (%) 1970–2020



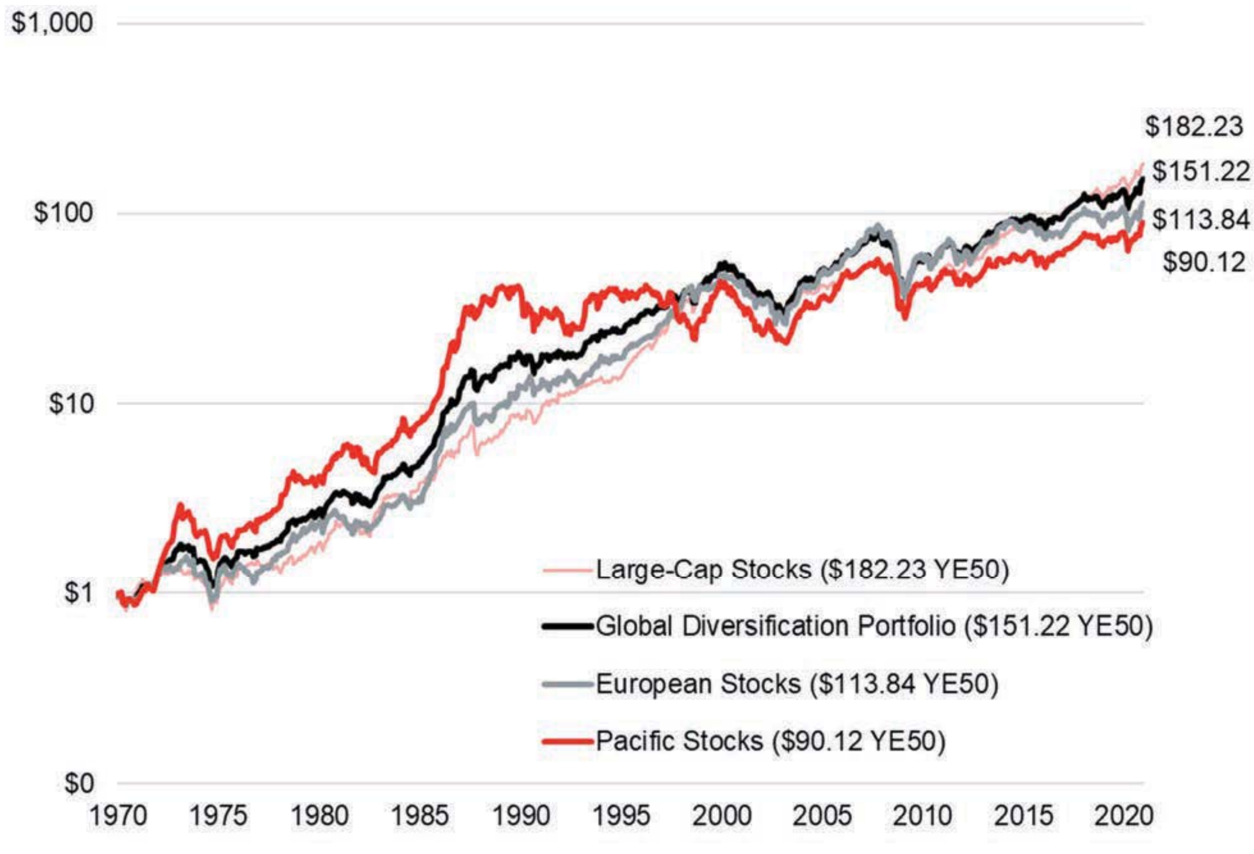

**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** International stocks are represented by the MSCI EAFE® equities index. Pacific stocks are represented by the MSCI Pacific GR USD index. European stocks are represented by the MSCI Europe GR USD index. To learn more about MSCI, visit msci.com.

## Diversification

Diversification can be another important benefit of international investing. By spreading risks among foreign and U.S. stocks, investors can potentially lower overall investment risk and/or improve investment returns. Fluctuations may occur at different times for different markets, and if growth is slow in one country, global investing provides a means of possibly participating in stronger market returns elsewhere. Investing abroad may help an investor balance such fluctuations. Because it is almost impossible to forecast which markets will be top performers in any given year, it can be very valuable to be invested in a portfolio diversified across several countries.

Exhibit 12.5 depicts the growth of $1.00 invested at year-end 1969 in U.S. large-cap stocks, European, and Pacific stocks as well as a "global diversification portfolio" that is comprised of an equally weighted mix of the U.S. large-cap stocks, European, and Pacific stocks. Notice that the U.S. large-cap stocks index was the top performer, followed (in order of performance) by the global diversification portfolio, Europe, and Pacific indexes at the end of the 51-year period.

**Exhibit 12.5:** Benefits of Global Diversification Index (Year-end 1969 = $1.00) 1970–2020

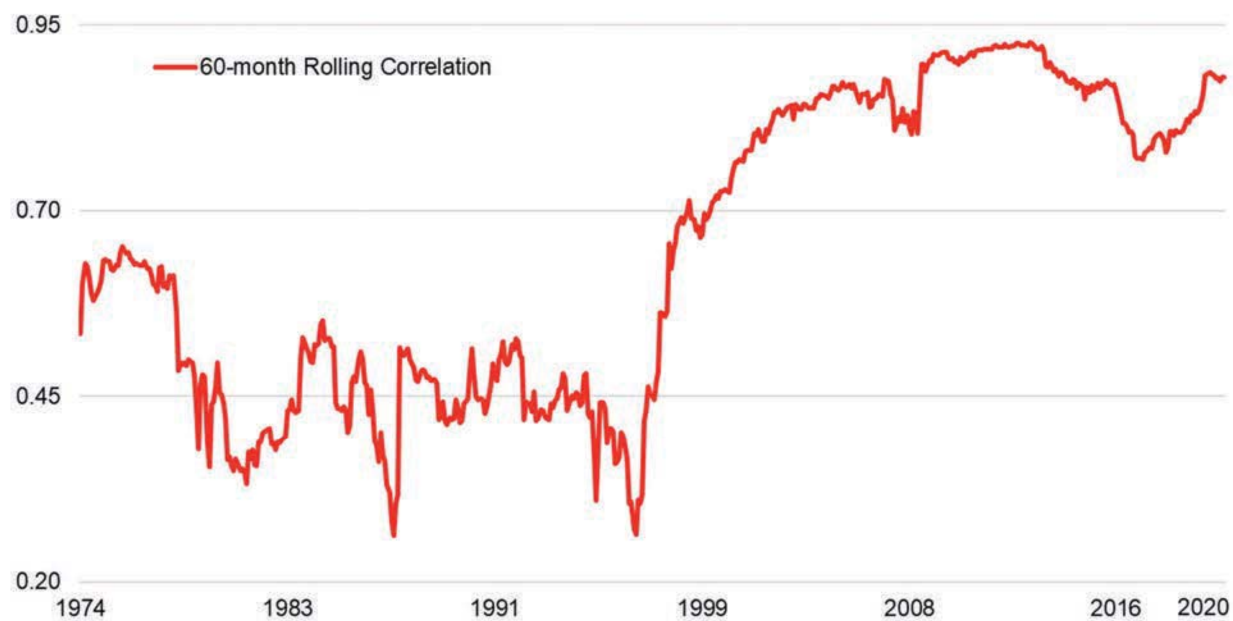



**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** International stocks are represented by the MSCI EAFE® equities index. Pacific stocks are represented by the MSCI Pacific GR USD index. European stocks are represented by the MSCI Europe GR USD index. To learn more about MSCI, visit msci.com.

The cross-correlation coefficient between two series, covered in Chapter 6, measures the extent to which they are linearly related. The correlation coefficient measures the sensitivity of returns on one asset class or portfolio to the returns of another.

Exhibit 12.6 examines the 60-month rolling period correlation between international and U.S. large-cap stocks. Exhibit 12.6 illustrates the recent rise in cross-correlation between the two, suggesting that the benefit of diversification has suffered in recent years. The maximum benefit to an investor would have come in the 60-month period ending July 1987 where the cross-correlation was 0.26. The least amount of diversification benefit would have come in the 60-month period ending February 2013 where the cross correlation was 0.93. The monthly average over the entire period was 0.65.

**Exhibit 12.6:** Rolling 60-Month Correlations: U.S. Large-Cap Stocks and International Stocks 1970–2020




**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** International stocks are represented by the MSCI EAFE® equities index. To learn more about MSCI, visit msci.com.

As discussed previously in regard to REITs (see Chapter 2), diversification is "spreading a portfolio over many investments to avoid excessive exposure to any one source of risk."[235] Put simply, diversification is "not putting all your eggs in one basket." Diversification offers the potential of higher returns for the same level of risk, or lower risk for the same level of return.

A low correlation between assets in a portfolio allows for the possibility of an increase in returns without a corresponding increase in risk, or alternatively, a reduction in risk without a corresponding decrease in return.

235 Cara Griffith, "Practical Tax Considerations for Working with REITs," State Tax Notes (October 31, 2011): 315–320, quoting Jennifer Weiss: 316. In 2009, the IRS issued guidance that indicates that the distributions may be in the form of cash or stock in certain instances.

## Risks Typically Associated with International Investment[236]

The risks associated with international investing can largely be characterized as *financial, economic,* or *political.* Many of these are the types of risks associated with investing in general – the possibility of loan default, the possibility of delayed payments of suppliers' credits, the possibility of inefficiencies brought about by the work of complying with unfamiliar (or burdensome) regulation, unexpected increases in taxes and transaction fees, differences in information availability, and liquidity issues, to name just a few. Some risks, however, are typically associated more with global investing – currency risk, lack of good accounting information, poorly developed legal systems, and even expropriation, government instability, or war.

### Financial Risks

Financial risks typically entail an issue that is specifically money-centric (e.g., loan default, inability to easily repatriate profits to the home country, etc.). Among these types of risks, currency risk is probably the most familiar. Currency risk is the *financial* risk that exchange rates (the value of one currency versus another) will change unexpectedly.

For example, when a French investor invests in Brazil, he or she must first convert Euros into the local currency, in this case the Brazilian Real (BRL). The returns that the French investor experiences in local currency terms are identical to the returns that a Brazilian investor would experience, but the French investor faces an additional risk in the form of currency risk when returns are "brought home" and must be converted back to Euros.[237]

Expected changes in exchange rates can often be hedged. However, even when currency hedging is used, exchange rate risk often remains. To the extent the Euro unexpectedly *increases* in value versus the Real (i.e., the Euro appreciates against the Real), the French investor is able to purchase fewer Euros for each Real he realized in the Brazilian investment when returns from the investment are repatriated, and his return is thus *diminished*.[238,239]

Conversely, to the extent the Euro unexpectedly *decreases* in value versus the Real (i.e., the Euro depreciates against the Real), the French investor is able to purchase more Euros for each

---

236 The following section is largely excerpted from the D&P/Kroll online Cost of Capital Navigator's International Cost of Capital Module's "Resources" section. For more information and to purchase the Cost of Capital Navigator's International Cost of Capital Module, visit dpcostofcapital.com.

237 For this example, we assume that the French and local investor are both subject to the same regulations, taxes, and local risks when investing in the same local asset.

238 We say "unexpectedly" for a reason. If the investor had been able to predict (at the time of investing) the precise exchange rate at which he/she would be repatriating his/her returns, these "expected" changes to the exchange rate would have been reflected in the expected cash flows of the investment at inception.

239 For example, say the French investor had achieved a 10% return in local (Brazilian) terms on his investment in a given year, but the Euro had unexpectedly appreciated by 3% in value relative to the Real over the same period. When the returns are repatriated, the French investor's overall return is diminished to approximately 6.7% [(1+10%)*(1-3%)-1] in Euro terms. Conversely, had the Euro depreciated in value versus the Real by 3%, the repatriated returns would be enhanced to approximately 13.3% [(1+10%)*(1+3%)-1]

Real he realized in the Brazilian investment when returns from the investment are repatriated, and his return is thus *enhanced*.

For example, in 2007 Brazilian equities returned an astonishing 50% return in local terms (see Exhibit 12.7). Because the Euro *depreciated* against the Real in 2007, French-based investors in Brazilian stocks experienced an even *higher* return (62%) when they repatriated their returns and converted them to Euros. Similarly, in 2009 the Euro *depreciated* relative to the South African Rand (ZAR), and French-based investors realized higher returns in Euros once again versus the local South African investors. In a more recent example, U.S.-based investors investing in U.S. equities realized an approximate return of just 1.0% in 2015, but French investors making a similar investment in the U.S. realized an approximate 13% return when they repatriated their returns and converted them to Euros (the Euro *depreciated* against the U.S. Dollar in 2015, so the French investors could purchase *more* Euros with their Dollars when they repatriated their returns).

It is important to note that currency conversion effects can also work to *diminish* realized returns. For example, in 2015 Brazilian equities returned -12% in local terms. Because the Euro *appreciated* against the Real in 2015, French-based investors in Brazilian stocks experienced an even *lower* return (-34%) when they repatriated their returns and converted them to Euros.

**Exhibit 12.7:** Currency Conversion Effects

| Year | Currency | Return in Local Terms | Return to French Investors (EUR) | Currency Conversion Effect |
|---|---|---|---|---|
| 2007 | Brazil (BRL) | 50% | 62% | 12% |
| 2009 | South Africa (ZAR) | 26% | 53% | 27% |
| 2015 | Japan (JPY) | 10% | 22% | 12% |
| 2015 | Switzerland (CHF) | 2% | 13% | 11% |
| 2015 | Brazil (BRL) | -12% | -34% | -22% |
| 2015 | Argentina (ARS) | 52% | 11% | -41% |
| 2015 | United States (USD) | 1% | 13% | 12% |
| 2016 | United Kingdom (GBP) | 19% | 3% | -16% |

**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** Morgan Stanley Capital International (MSCI) Brazil, South Africa, Japan, Switzerland, Brazil, and Argentina, gross return (GR) equity indices. For more information about MSCI, visit msci.com.

A common misstep we often encounter is companies constructing forward looking budgets or projection analyses in local currencies, and then converting these projections to the currency of the parent company using the spot rate.

This mistakenly assumes that the exchange rate will not change in the future. Projections, which are inherently forward-looking, need to embody expected currency conversion rates. We are interested in currency risks over the period of the projected net cash flows, not just in the spot market. Even then, these are merely estimates of future currency exchange rates and the actual exchange rate can vary from these estimates.

Does currency risk affect the cost of capital? One team of researchers found that emerging market exchange risks have a significant impact on risk premiums and are time varying (for countries in the sample). They found that exchange risks affect risk premiums as a separate risk factor and represent more than 50% of total risk premiums for investments in emerging market equities. The exchange risk from investments in emerging markets was found to even affect the risk premiums for investments in developed market equities.[240]

While exchange rate volatility appears to be partly systematic, researchers have found that despite not being a constant, the currency risk premium is small and seems to fluctuate around zero.[241] A recently published academic paper set out to study whether corporate managers should include foreign exchange risk premia in cost of equity estimations. The authors empirically estimated the differences between the cost of equity estimates of several risk-return models, including some models that have an explicit currency risk premia and others that do not. They found that adjusting for currency risk makes little difference, on average, in the cost of equity estimates, even for small firms and for firms with extreme currency exposure estimates. The authors concluded that, at a minimum, these results applied to U.S. companies, but future research would still have to be conducted for other countries.[242]

Rather than attempting to quantify and add a currency risk premium to the discount rate, using expected or forward exchange rates to translate projected cash flows into the home currency will inherently capture the currency risk, if any, priced by market participants.[243]

### Economic Risks

Global investors may also be exposed to *economic* risks associated with international investing. These risks may include the volatility of a country's economy as reflected in the current (and expected) inflation rate, the current account balance as a percentage of goods and services, burdensome regulation, and labor rules, among others. In the current environment, an economic risk that has come to the forefront is the sovereign debt crisis. The recent economic and financial crisis in Greece, for example, has prompted many governments around the world to re-think their

---

[240] Francesca Carrieri, Vihang Errunza, and Basma Majerbi, ''Does Emerging Market Exchange Risk Affect Global Equity Prices?'' *Journal of Financial Quantitative Analysis* (September 2006): 511–540.

[241] Sercu, Piet (2009), *International Finance: Theory into Practice*, Princeton, NJ: Princeton University Press, Chapter 19.

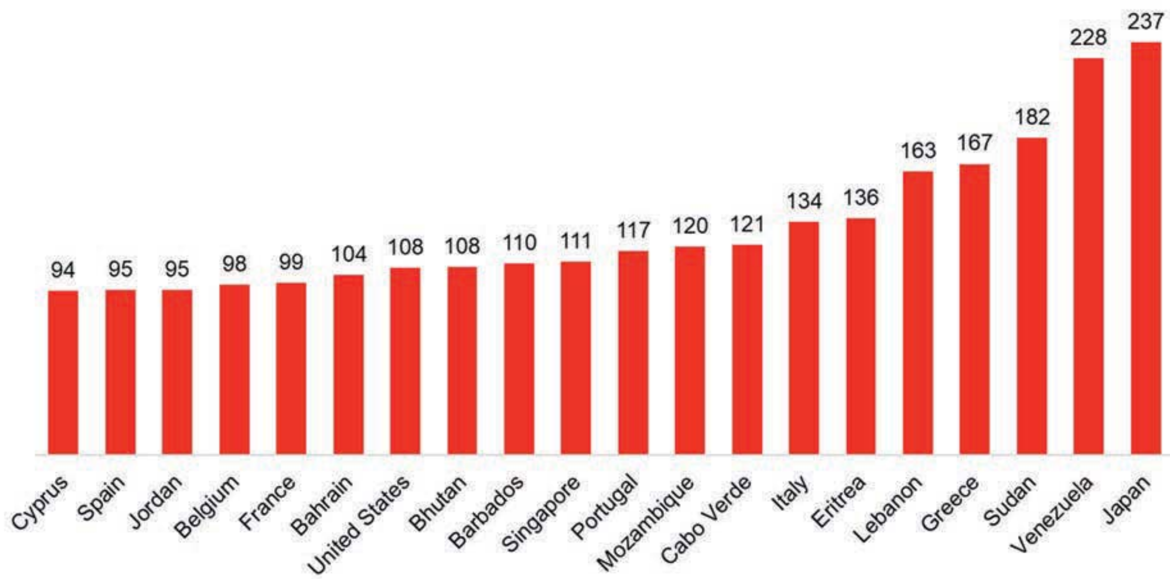
[242] Krapl, A. and O'Brien, T. J. (2016), "Estimating Cost of Equity: Do You Need to Adjust for Foreign Exchange Risk?," *Journal of International Financial Management & Accounting*, 27: 5–25.

[243] This assumes that the valuation is being conducted in the home currency, by discounting projected cash flows denominated in the home currency, with a discount rate also denominated in home currency. Alternatively, the analyst can conduct the entire valuation in foreign currency terms (projected cash flows and discount rate are both in foreign currency terms), in which case the estimated value would be translated into the home currency using a spot exchange rate.

own fiscal policies as it becomes evident that current debt loads are likely unsustainable in many of these countries.

In Exhibit 12.8a, the 20 countries with the *overall* highest estimated government debt-to-GDP ratios are shown (regardless of the size of their economies), as of 2020. For example, the United States has a debt-to-GDP ratio of 108% (i.e., the United States' government debt is 8% *larger* than the United States' annual GDP), and France has a debt-to-GDP ratio of 99% (i.e., France's government debt is 1% *less* than France's annual GDP).

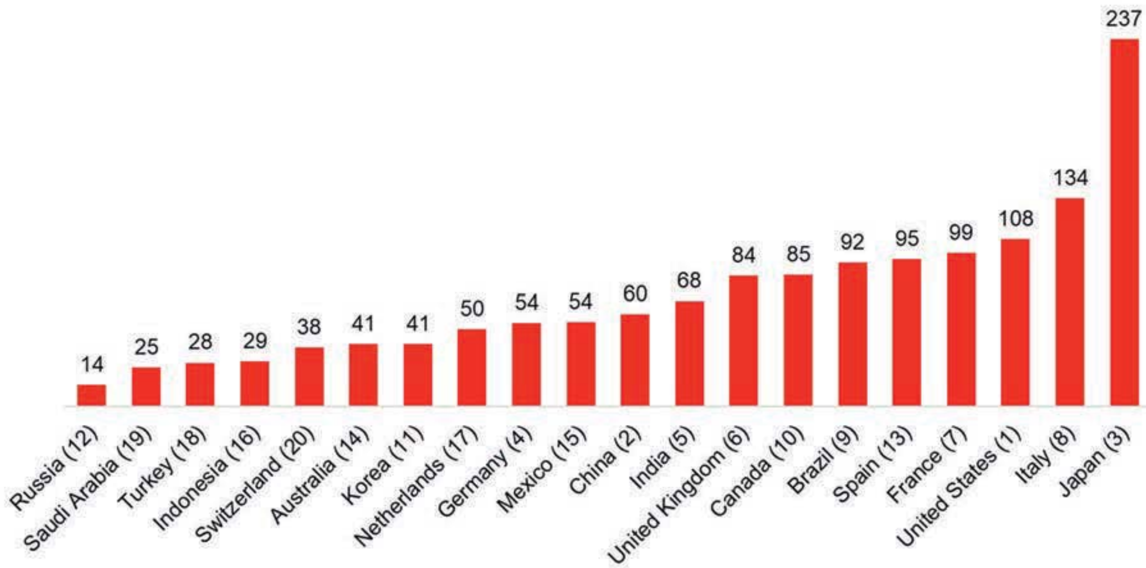
**Exhibit 12.8a:** 2020 Government Debt-to-GDP (in percent)




**Source of underlying data:** World Economic Outlook Database from the International Monetary Fund (IMF). For additional information, please visit: http://www.imf.org/external/pubs/ft/weo/2019/02/weodata/download.aspx.

In Exhibit 12.8b, the estimated government debt-to-GDP ratios for the 20 countries with the *largest* economies (as measured by GDP) are shown, also as of calendar year 2020. The rank of GDP size is shown in parentheses after each country's name. Switzerland (with a ranking of "20") is the smallest GDP, and the United States (with a ranking of "1") is the largest GDP.

**Exhibit 12.8b:** 2020 Government Debt-to-GDP (in percent), 20 countries with largest GDP




**Source of underlying data:** World Economic Outlook Database from the International Monetary Fund (IMF). For additional information, please visit: http://www.imf.org/external/pubs/ft/weo/2019/02/weodata/download.aspx.

There are costs that tend to go hand-in-hand with what might be considered unsustainable debt levels by governments. Lenders may demand a higher expected return to compensate them for additional default risk when investing not only in the country's sovereign debt, but also in businesses operating in those countries.

Governments may decide to increase the money supply in an effort to inflate their way out of debt. Ultimately, some governments may decide on outright currency devaluation or even a repudiation of debt (i.e., defaulting on their debt obligations). These risks are not entirely limited to less developed countries, but less developed countries may be more willing to resort to these extreme measures than developed countries.

## Political Risks

*Political* risks can include government instability, expropriation, bureaucratic inefficiency, corruption, and even war. A relatively recent example of the effects of political risk is Venezuela's expropriation of various foreign owned oil, gas, and mining interests. These actions tend to reduce Venezuela's attractiveness to foreign investors who will likely demand a significantly higher expected return in exchange for future investment in the country – in effect raising their cost of capital estimates for projects located in Venezuela.

Exhibit 12.9 summarizes some of the risks that investors may view as unique or country-specific.

**Exhibit 12.9:** Reasons Typically Cited for Adding a Country Risk Premium Adjustment

**Political Risks**

- Repudiation of contracts by governments
- Expropriation of private investments in total or part through change in taxation
- Economic planning failures
- Political leadership and frequency of change
- External conflict
- Corruption in government
- Military in politics
- Organized religion in politics
- Lack of law-and-order tradition
- Racial and national tensions
- Civil war
- Poor quality of the bureaucracy
- Poorly developed legal system
- Political terrorism

**Financial Risks**

- Currency volatility plus the inability to convert, hedge, or repatriate profits
- Loan default or unfavorable loan restructuring
- Delayed payment of suppliers' credits
- Losses from exchange controls
- Foreign trade collection experience

**Economic Risks**

- Volatility of the economy
- Unexpected changes in inflation
- Debt service as a percentage of exports of goods and services
- Current account balance of the country in which the subject company operates as a percentage of goods and services
- Parallel foreign exchange rate market indicators
- Labor issues

## International and Domestic Series Summary Data

Exhibit 12.10 shows summary statistics of annual total returns for various international regions and composites. The summary statistics presented are geometric mean, arithmetic mean, and standard deviation. From 1970 to 2020, the Pacific regional composite was the riskiest, with a standard deviation of 28.5 percentage points. The annual geometric mean of the Pacific regional composite over the 1920–2020 time period was 9.2%, less than the other composite analyzed, which were considerably less risky.[244]

244 At the 2-digit level, the Pacific regional composite's annual geometric mean over the 1970–2020 time period was 9.17%, and Canada's annual geometric mean was 9.18%.

**Exhibit 12.10:** Summary Statistics of Annual Returns
1970–2020 (%)

| Series | Geometric Mean | Arithemtic Mean | Standard Deviation |
|---|---|---|---|
| EAFE | 9.3 | 11.4 | 21.5 |
| Pacific | 9.2 | 12.6 | 28.5 |
| Europe | 9.7 | 11.8 | 21.2 |
| World | 9.8 | 11.3 | 17.4 |
| Canada | 9.1 | 11.3 | 21.3 |
| U.S. | 10.7 | 12.1 | 16.9 |

**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** Morgan Stanley Capital International (MSCI) Europe, Australasia and Far East (EAFE) index, and MSCI Pacific, Europe, World, and Canada GR indices. To learn more about MSCI, visit msci.com.

Exhibit 12.11 ranks the performance (as measured by compound annual rates of return) of U.S., EAFE, Pacific, Europe, World, and Canada equities for each decade from best performer (at top) to worst performer (at bottom). For example, in the 2010s the best performer was U.S. Large-Cap Stocks, and the worst performer was Canada.

**Exhibit 12.11:** The Relative Performance of U.S., EAFE, Pacific, Europe, World, and Canada Equities by Decade (Best Performer at Top, Worst Performer at bottom)

| 1970s | 1980s | 1990s | 2000s | 2010s |
|---|---|---|---|---|
| Pacific | Pacific | U.S. | Canada | U.S. |
| Canada | EAFE | Europe | Europe | World |
| EAFE | World | World | EAFE | Pacific |
| Europe | Europe | Canada | World | EAFE |
| World | U.S. | EAFE | Pacific | Europe |
| U.S. | Canada | Pacific | U.S. | Canada |

**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** Morgan Stanley Capital International (MSCI) Europe, Australasia and Far East (EAFE) index, and MSCI Pacific, Europe, World, and Canada GR indices. To learn more about MSCI, visit msci.com.

Exhibit 12.12 shows the annualized monthly standard deviations by decade for the various international regions and composites.

The World composite was the least risky in the 1970s, 1980s, and the 1990s. The Canadian index was the riskiest in the 2000s, while Europe was the riskiest in the most recent decade. The Pacific regional composite was the least risky in the most recent decade.[245]

**Exhibit 12.12:** Annualized Monthly Standard Deviation by Decade (%)

| Series | 1970s | 1980s | 1990s | 2000s | 2010s |
|---|---|---|---|---|---|
| EAFE | 17.4 | 21.6 | 18.7 | 18.5 | 15.6 |
| Pacific | 22.1 | 26.6 | 24.8 | 18.2 | 14.1 |
| Europe | 18.6 | 21.5 | 16.8 | 20.4 | 17.5 |
| World | 15.1 | 17.6 | 15.7 | 16.9 | 14.4 |
| Canada | 20.6 | 24.8 | 18.6 | 25.9 | 16.3 |
| U.S. | 17.1 | 19.4 | 15.9 | 16.3 | 14.1 |

**Source 1 of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** Morgan Stanley Capital International (MSCI) Europe, Australasia and Far East (EAFE) index, and MSCI Pacific, Europe, World, and Canada GR indices. To learn more about MSCI, visit msci.com.

Exhibit 12.13 presents annual cross-correlations and serial correlations from 1970 to 2020 for the six basic SBBI® series and inflation as well as international stocks, as defined by the MSCI EAFE Index. International stocks, when compared to U.S. large-cap stocks, provided a higher cross-correlation than when compared to U.S. small-cap stocks. The serial correlation of international stocks suggests no pattern, and the return from period to period can best be interpreted as random or unpredictable.

[245] At the 2-digit level, the Pacific regional composite's annualized monthly standard deviation over the 1970–2019 time period was 14.11%, and the U.S. large stock composite's annualized monthly standard deviation was 14.14%.

**Exhibit 12.13:** Basic Series and International Stocks: Serial and Cross-Correlations of Historical Annual Returns 1970–2020

| | Int'l Stocks | Large-Cap Stocks | Small-Cap Stocks | Long-term Corp Bonds | Long-term Gov't Bonds | Inter-term Gov't Bonds | U.S. Treasury Bills | Inflation |
|---|---|---|---|---|---|---|---|---|
| International Stocks | 1.00 | | | | | | | |
| Large-Cap Stocks | 0.67 | 1.00 | | | | | | |
| Small-Cap Stocks | 0.52 | 0.72 | 1.00 | | | | | |
| Long-term Corp Bonds | 0.06 | 0.27 | 0.09 | 1.00 | | | | |
| Long-term Gov't Bonds | -0.11 | 0.04 | -0.13 | 0.89 | 1.00 | | | |
| Inter-term Gov't Bonds | -0.11 | 0.03 | -0.08 | 0.82 | 0.85 | 1.00 | | |
| U.S. Treasury Bills | 0.03 | 0.03 | 0.04 | 0.04 | 0.08 | 0.43 | 1.00 | |
| Inflation | -0.05 | -0.12 | 0.06 | -0.31 | -0.26 | -0.01 | 0.70 | 1.00 |
| Serial Correlation | 0.04 | -0.02 | -0.01 | -0.11 | -0.28 | 0.10 | 0.89 | 0.75 |

**Source of underlying data:** Morningstar, Inc. Used with permission. All rights reserved. Calculations by D&P/Kroll. Asset classes and inflation represented by the Ibbotson Associates (IA) Stocks, Bonds, Bills, and Inflation® (SBBI®) series, as follows: (i) Large-Cap Stocks: IA SBBI® US Large Stock TR USD Ext, (ii) Small-Cap Stocks: IA SBBI® US Small Stock TR USD, (iii) Long-term (i.e., 20-year) Corporate Bonds: IA SBBI® US LT Corp TR USD, (iv) Long-Term (i.e. 20-year) Government Bonds: IA SBBI® US LT Govt TR USD, (v) Intermediate-term (i.e., 5-year) Government Bonds: IA SBBI® US IT Govt TR USD, (vi) U.S. (30-day) Treasury Bills: IA SBBI® US 30 Day TBill TR USD, and (vii) Inflation: IA SBBI® US Inflation. For a detailed description of the SBBI® series, see Chapter 3, "Description of the Basic Series". "Stocks, Bonds, Bills, and Inflation" and "SBBI" are registered trademarks of Morningstar, Inc. All rights reserved. Used with permission. **Source 2 of underlying data:** The international stock series is represented by the MSCI EAFE® equities index. The MSCI EAFE Index is an equity index which captures large- and mid-cap representation across Developed Markets countries around the world, excluding the US and Canada. With 918 constituents, the index covers approximately 85% of the free float-adjusted market capitalization in each country. To learn more about MSCI, visit msci.com.

## Conclusion

Country risk is generally described as financial, economic, or political in nature. These rules may create incremental complexities when developing cost of capital estimates for a business, business ownership interest, security, or an intangible asset based outside of a mature market such as the United States.

International investments are no different from any other investment when it comes to information gathering. Investors interested in or already taking part in the international marketplace should learn as much as possible about the corresponding significant rewards and risks. International investments are not for everyone, and the most appropriate mix for an individual investor depends on his or her risk tolerance, investment goals, time horizon, and financial resources.

## Named Endowments

CFA Institute Research Foundation acknowledges with sincere gratitude the generous contributions of the Named Endowment participants listed below.

Gifts of at least US$100,000 qualify donors for membership in the Named Endowment category, which recognizes in perpetuity the commitment toward unbiased, practitioner-oriented, relevant research that these firms and individuals have expressed through their generous support of the CFA Institute Research Foundation.

- Ameritech
- Anonymous
- Robert D. Arnott
- Theodore R. Aronson, CFA
- Asahi Mutual Life Insurance Company
- Batterymarch Financial Management
- Boston Company
- Boston Partners Asset Management, L.P.
- Gary P. Brinson, CFA
- Brinson Partners, Inc.
- Capital Group International, Inc.
- Concord Capital Management
- Dai-Ichi Life Insurance Company
- Daiwa Securities
- Mr. and Mrs. Jeffrey Diermeier
- Gifford Fong Associates
- Investment Counsel Association of America, Inc.
- Jacobs Levy Equity Management
- John A. Gunn, CFA
- John B. Neff, CFA
- Jon L. Hagler Foundation
- Long-Term Credit Bank of Japan, Ltd.
- Lynch, Jones & Ryan, LLC
- Meiji Mutual Life Insurance Company
- Miller Anderson & Sherrerd, LLP
- Nikko Securities Co., Ltd.
- Nippon Life Insurance Company of Japan
- Nomura Securities Co., Ltd.
- Payden & Rygel
- Provident National Bank
- Frank K. Reilly, CFA
- Salomon Brothers
- Sassoon Holdings Pte. Ltd.
- Scudder Stevens & Clark
- Security Analysts Association of Japan
- Shaw Data Securities, Inc.
- Sit Investment Associates, Inc.
- Standish, Ayer & Wood, Inc.
- State Farm Insurance Company
- Sumitomo Life America, Inc.
- T. Rowe Price Associates, Inc.
- Templeton Investment Counsel Inc.
- Frank Trainer, CFA
- Travelers Insurance Co.
- USF&G Companies
- Yamaichi Securities Co., Ltd

## Senior Research Fellows

Financial Services Analyst Association

For more on upcoming Research Foundation publications and webcasts, please visit:

https://www.cfainstitute.org/en/research/foundation/publications

CFA Institute
Research
Foundation

Available online at www.cfainstitute.org