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ON THE ASSESSMENT OF RISK

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INTRODUCTION

THE CONCEPT OF RISK has so permeated the financial community that no one needs to be convinced of the necessity of including risk in investment analysis. Still of controversy is what constitutes risk and how it should be measured. This paper examines the statistical properties of one measure of risk which has had wide acceptance in the academic community: namely the coefficient of non-diversifiable risk or more simply the beta coefficient in the market model.

The next section defines this beta coefficient and presents a brief non-rigorous justification of its use as a measure of risk. After discussing the sample and its basic properties in Section III, Section IV examines the stationarity of this beta coefficient over time and proposes a method of obtaining improved assessments of this measure of risk.

II. THE RATIONALE OF BETA AS A MEASURE OF RISK

The interpretation of the beta coefficient as a measure of risk rests upon the empirical validity of the market model. This model asserts that the return from time (t-1) to t on asset i, $\tilde{R}_{it}$,[1] is a linear function of a market factor common to all assets $\tilde{M}_t$, and independent factors unique to asset i, $\tilde{\varepsilon}_{it}$. Symbolically, this relationship takes the form

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{M}_t + \tilde{\varepsilon}_{it}, \tag{1}$$

where the tilde indicates a random variable, α_i is a parameter whose value is such that the expected value of $\tilde{\varepsilon}_{it}$ is zero, and β_i is a parameter appropriate to asset i.[2] That the random variables $\tilde{\varepsilon}_{it}$ are assumed to be independent and

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1. In this paper, return will be measured as the ratio of the value of the investment at time t with dividends reinvested to the value of the investment at time (t-1). Dividends are assumed reinvested at time t.

2. The parameter β_i is defined as Cov $(\tilde{R}_i, \tilde{M})$/Var $(\tilde{M})$.

unique to asset i implies that Cov $(\tilde{\varepsilon}_{it}, \tilde{M}_t)$ is zero and that Cov $(\tilde{\varepsilon}_{it}, \tilde{\varepsilon}_{jt})$, $i \neq j$, are zero. This last conclusion is tantamount to assuming the absence of industry effects.

The empirical validity of the market model as it applies to common stocks listed on the NYSE has been examined extensively in the literature.[3] The principal conclusions are: (1) The linearity assumption of the model is adequate.[4] (2) The variables $\tilde{\varepsilon}_{it}$ cannot be assumed independent between securities because of the existence of industry effects. However, these industry effects, as documented by King,[5] probably account for only about ten percent of the variation in returns, so that as a first approximation they can be ignored. (3) The unique factors $\tilde{\varepsilon}_{it}$ correspond more closely to non-normal stable variates than to normal ones. This conclusion means that variances and covariances of the unique factors do not exist. Nonetheless, this paper will make the more common assumption of the existence of these statistics in justifying the beta coefficient as a measure of risk since Fama[6] and Jensen[7] have shown that this coefficient can still be interpreted as a measure of risk under the assumption that the $\tilde{\varepsilon}_{it}$'s are non-normal stable variates.

That the beta coefficient, β_i, in the market model can be interpreted as a measure of risk will be justified in two different ways: the portfolio approach and the equilibrium approach.

A. *The Portfolio Approach*

The important assumption underlying the portfolio approach is that individuals evaluate the risk of a portfolio as a whole rather than the risk of each asset individually. An example will illustrate the meaning of this statement. Consider two assets, each of which by itself is extremely risky. If, however, it is always the case that when one of the assets has a high return, the other has a low return, the return on a combination of these two assets in a portfolio may be constant. Thus, the return on the portfolio may be risk free whereas each of the assets has a highly uncertain return. The discussion of such an

3. See Marshall E. Blume, "Portfolio Theory: A Step Towards Its Practical Application," forthcoming *Journal of Business;* Eugene F. Fama, "The Behavior of Stock Market Prices," *Journal of Business* (1965), 34-105; Eugene F. Fama, Lawrence Fisher, Michael Jensen, and Richard Roll, "The Adjustment of Stock Prices to New Information," *International Economic Review* (1969), 1-21; Michael Jensen, "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios," *Journal of Business* (1969), 167-247; Benjamin F. King, "Market and Industry Factors in Stock Price Behavior," *Journal of Business* (1966), 139-90; and William F. Sharpe, "Mutual Fund Performance," *Journal of Business* (1966), 119-38.

4. The linearity assumption of the model should not be confused with the equilibrium requirement of William F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," *Journal of Finance* (1964), 425-42, which states that $\alpha_i = (1 - \beta_i)\ R_F$, where R_F is the risk free rate. It is quite possible that this equality does not hold and at the same time that the market model is linear.

5. King, *op. cit.*

6. Eugene F. Fama, "Risk, Return, and Equilibrium" (Report No. 6831, University of Chicago, Center for Mathematical Studies in Business and Economics, June, 1968).

7. Jensen, *op. cit.*

obvious point may seem unwarranted, but there is very little empirical work which indicates that people do in fact behave according to it.

Now if an individual is willing to judge the risk inherent in a portfolio solely in terms of the variance of the future aggregate returns, the risk of a portfolio of n securities with an equal amount invested in each, according to the market model, will be given by

$$\mathrm{Var}\,(\tilde{W}_t) = \left(\sum_{i=1}^{n} \frac{1}{n}\beta_i\right)^2 \mathrm{Var}\,(\tilde{M}_t) + \sum_{i=1}^{n}\left(\frac{1}{n}\right)^2 \mathrm{Var}\,(\tilde{\varepsilon}_{it}) \qquad (2)$$

where $\tilde{W}_t$ is the return on the portfolio. Equation (2) can be rewritten as

$$\mathrm{Var}\,(\tilde{W}_t) = \bar{\beta}^2\,\mathrm{Var}\,(\tilde{M}_t) + \frac{\overline{\mathrm{Var}\,(\tilde{\varepsilon})}}{n} \qquad (3)$$

where the bar indicates an average. As one diversifies by increasing the number of securities n, the last term in equation (3) will decrease. Evans and Archer[8] have shown empirically that this process of diversification proceeds quite rapidly, and with ten or more securities most of the effect of diversification has taken place. For a well diversified portfolio, Var $(\tilde{W}_t)$ will approximate $\bar{\beta}^2$ Var $(\tilde{M}_t)$. Since Var $(\tilde{M}_t)$ is the same for all securities, $\bar{\beta}$ becomes a measure of risk for a portfolio and thus β_i, as it contributes to the value of $\bar{\beta}$, is a measure of risk for a security. The larger the value of β_i, the more risk the security will contribute to a portfolio.[9]

B. *The Equilibrium Approach*

Using the market model, Sharpe[10] and Lintner,[11] as clarified by Fama,[12] have developed a theory of equilibrium in the capital markets. This theory relates the risk premium for an individual security, $E(\tilde{R}_{it}) - R_F$, where R_F is the risk free rate, to the risk premium of the market, $E(\tilde{M}_t) - R_F$, by the formula

$$E(\tilde{R}_{it}) - R_F = \beta_i[E(\tilde{M}_t) - R_F]. \qquad (4)$$

The risk premium for an individual security is proportional to the risk premium for the market. The constant of proportionality β_i can therefore be interpreted as a measure of risk for individual securities.

8. John L. Evans and Stephan H. Archer, "Diversification and the Reduction of Dispersion: An Empirical Analysis," *Journal of Finance* (1968), 761-68.

9. This argument has been extended to a non-Gaussian, symmetric stable world by E. F. Fama, "Portfolio Analysis in a Stable Paretian Market," *Management Science* (1965), 404-19; and P. A. Samuelson, "Efficient Portfolio Selection for Pareto-Levy Investments," *Journal of Financial and Quantitative Analysis* (1967), 107-22.

10. Sharpe, "Capital Asset Prices," *op. cit.*

11. John Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics* (1965), 13-37.

12. Eugene F. Fama, "Risk, Return, and Equilibrium: Some Clarifying Comments," *Journal of Finance* (1968), 29-40.

This theory of equilibrium, although theoretically sound, is based upon numerous assumptions which obviously do not hold in the real world. A theoretical model, however, should not be judged by the accuracy of its assumptions but rather by the accuracy of its predictions. The empirical work of Friend and Blume[13] suggests that the predictions of this model are seriously biased and that this bias is primarily attributable to the inaccuracy of one key assumption, namely that the borrowing and lending rates are equal and the same for all investors. Therefore, although Sharpe's and Lintner's theory of equilibrium can be used as a justification for β_i as measure of risk, it is a weaker and considerably less robust justification than that provided by the portfolio approach.

III. The Sample and Its Properties

The sample was taken from the updated Price Relative File of the Center for Research in Security Prices at the Graduate School of Business, University of Chicago. This file contains the monthly investment relatives, adjusted for dividends and capital changes of all common stocks listed on the New York Stock Exchange during any part of the period from January 1926 through June 1968, for the months in which they were listed. Six equal time periods beginning in July 1926 and ending in June 1968 were examined. Table 1 lists these six periods and the number of companies in each for which there was a complete history of monthly return data. This number ranged from 415 to 890.

The investment relatives for a particular security and a particular period were regressed[14] upon the corresponding combination market link relatives, which were originally prepared by Fisher[15] as a measure of the market factor. This process was repeated for each security and each period, yielding, for instance, in the July 1926 through June 1933 period, 415 separate regressions. The average coefficient of determination of these 415 regressions was 0.51. The corresponding average coefficients of determination for the next five periods were, respectively, 0.49, 0.36, 0.32, 0.25, and 0.28. These figures are consistent with King's findings[16] in that the proportion of the variance of returns explained by the market declined steadily until 1960 when his sample terminated. Since 1960, the importance of the market factor has increased slightly according to these figures.

Table 1, besides giving the number of companies analyzed, summarizes the distributions of the estimated beta coefficients in terms of the means, standard deviations, and various fractiles of these distributions. In addition, the number of estimated betas which were less than zero is given. In three of the periods,

13. Irwin Friend and Marshall Blume, "Measurement of Portfolio Performance Under Uncertainty," *American Economic Review* (1970), 561-75.

14. John Wise, "Linear Estimators for Linear Regression Systems Having Infinite Variances," (Berkeley-Stanford Mathematics-Economics Seminar, October, 1963) has given some justification for the use of least squares in estimating coefficients of regressions in which the disturbances are non-normal symmetric stable variates.

15. Lawrence Fisher, "Some New Stock-Market Indexes," *Journal of Business* (1966), 191-225.

16. King, *op. cit.*

TABLE 1

DESCRIPTIVE SUMMARY OF ESTIMATED BETA COEFFICIENTS

Period	Number of Companies	Mean	Standard Deviation	Number of BETAS less than Zero	Fractiles				
					.10	.25	.50	.75	.90
7/26-6/33	415	1.051	0.462	1	0.498	0.711	1.023	1.352	1.616
7/33-6/40	604	1.036	0.474	0	0.436	0.701	1.015	1.349	1.581
7/40-6/47	731	0.990	0.504	0	0.500	0.643	0.872	1.186	1.606
7/47-6/54	870	1.010	0.409	2	0.473	0.727	0.996	1.263	1.565
7/54-6/61	890	0.998	0.423	0	0.458	0.678	0.984	1.250	1.558
7/61-6/68	847	0.962	0.390	4	0.475	0.681	0.934	1.199	1.491

none of the estimated betas was negative. Of the 4357 betas estimated in all six periods, only seven or 0.16 per cent were negative. This means that although the inclusion of a stock which moves counter to the market can reduce the risk of a portfolio substantially, there are virtually no opportunities to do this. Nearly every stock appears to move with the market.[17]

IV. The Stationarity of Beta over Time

No economic variable including the beta coefficient is constant over time. Yet for some purposes, an individual might be willing to act *as if* the values of beta for individual securities were constant or stationary over time. For example, a person who wishes to assess the future risk of a well diversified portfolio is really interested in the behavior of averages of the β_i's over time and not directly in the values for individual securities. For the purposes of evaluating a portfolio, it may be sufficient that the historical values of β_i be unbiased estimates of the future values for an individual to act *as if* the values of the β_i's for individual securities are stationary over time. This is because the errors in the assessment of an average will tend to be less than those of the components of the average providing that the errors in the assessments of the components are independent of each other.[18] Yet, a statistician or a person who wishes to assess the risk of an individual security may have completely different standards in determining whether he would act as if the β_i's are constant over time. The remainder of the paper examines the stationarity of the β_i's from the point of view of a person who wishes to analyze a portfolio.

A. *Correlations*

To examine the empirical behavior of the risk measures for portfolios over time, arbitrary portfolios of n securities were selected as follows: The estimates of β_i were derived using data from the first period, July 1926 through June 1933, and were then ranked in ascending order.[19] The first portfolio of n securities consisted of those securities with the n smallest estimates of β_i. The second portfolio consisted of those securities with the next n smallest estimates of β_i, and so on until the number of securities remaining was less than n. The number of securities n was allowed to vary over 1, 2, 4, 7, 10, 20, 35, 50, 75, and 100. This process was repeated for each of the next four periods.

Table 2 presents the product moment and rank order correlation coefficients between the risk measures for portfolios of n securities assuming an equal investment in each security estimated in one period and the corresponding risk

17. The use of considerably less than seven years of monthly data such as two or three years to estimate the beta coefficient results in a larger proportion of negative estimates. This larger proportion is probably due to sampling errors which, as documented in Richard Roll, "The Efficient Market Model Applied to U. S. Treasury Bill Rates," (Unpublished Ph.D. thesis, Graduate School of Business, University of Chicago, 1968) may be quite large for models with non-normal symmetric stable disturbances.

18. This property of averages does not hold for all distributions (*cf.* Eugene F. Fama, "Portfolio Analysis in a Stable Paretian Market"), but for the distributions associated with stock market returns it almost certainly holds.

19. Only securities which also had complete data in the next seven year period were included in this ranking.

measure for the same portfolio estimated in the next period.[20] The risk measure calculated using the earlier data might be regarded as an individual's assessment of the future risk, and the measure calculated using the later data can be regarded as the realized risk. Thus, these correlation coefficients can be interpreted as a measure of the accuracy of one's assessments, which in this case are simple extrapolations of historical data.

TABLE 2

PRODUCT MOMENT AND RANK ORDER CORRELATION COEFFICIENTS OF BETAS FOR PORTFOLIOS OF N SECURITIES

Number of Securities per Portfolio	7/26-6/33 and 7/33-6/40		7/33-6/40 and 7/40-6/47		7/40-6/47 and 7/47-6/54		7/47-6/54 and 7/54-6/61		7/54-6/61 and 7/61-6/68	
	P.M.	Rank	P.M.	Rank	P.M.	Rank	P.M.	Rank	P.M.	Rank
1	0.63	0.69	0.62	0.73	0.59	0.65	0.65	0.67	0.60	0.62
2	0.71	0.75	0.76	0.83	0.72	0.79	0.76	0.76	0.73	0.74
4	0.80	0.84	0.85	0.90	0.81	0.89	0.84	0.84	0.84	0.85
7	0.86	0.90	0.91	0.93	0.88	0.93	0.87	0.88	0.88	0.89
10	0.89	0.93	0.94	0.95	0.90	0.95	0.92	0.93	0.92	0.93
20	0.93	0.99	0.97	0.98	0.95	0.98	0.95	0.96	0.97	0.98
35	0.96	1.00	0.98	0.99	0.95	0.99	0.97	0.98	0.97	0.97
50	0.98	1.00	0.99	0.98	0.98	0.99	0.98	0.98	0.98	0.97

The values of these correlation coefficients are striking. For the assessments based upon the data from July 1926 through June 1933 and evaluated using data from July 1933 through June 1940, the product moment correlations varied from 0.63 for single securities to 0.98 for portfolios of 50 securities. The high value of the latter coefficient indicates that substantially all of the variation in the risk among portfolios of 50 securities can be explained by assessments based upon previous data. The former correlation suggests that assessments for individual securities derived from historical data can explain roughly 36 per cent of the variation in the future estimated values, leaving about 64 per cent unexplained.[21]

These results, which are typical of the other periods, suggest that at least as measured by the correlation coefficients, naively extrapolated assessments of future risk for larger portfolios are remarkably accurate, whereas extrapolated assessments of future risk for individual securities and smaller portfolios are of some, but limited value in forecasting the future.

B. *A Closer Examination*

Table 3 presents the actual estimates of the risk parameters for portfolios of 100 securities for successive periods. For all five different sets of portfolios, the rank order correlations between the successive estimates are one, but there is obviously some tendency for the estimated values of the risk parameter to

20. Because of the small number of portfolios of 100 securities, correlations are not presented in Table 2 for these portfolios.

21. This large magnitude of unexplained variation may make the beta coefficient an inadequate measure of risk for analyzing the cost of equity for an individual firm although it may be adequate for cross-section analyses of cost of equity.

TABLE 3

ESTIMATED BETA COEFFICIENTS FOR PORTFOLIOS OF 100 SECURITIES IN TWO SUCCESSIVE PERIODS

Portfolio	7/26-6/33	7/33-6/40	7/33-6/40	7/40-6/47	7/40-6/47	7/47-6/54	7/47-6/54	7/54-6/61	7/54-6/61	7/61-6/68
1	0.528	0.610	0.394	0.573	0.442	0.593	0.385	0.553	0.393	0.620
2	0.898	1.004	0.708	0.784	0.615	0.776	0.654	0.748	0.612	0.707
3	1.225	1.296	0.925	0.902	0.746	0.887	0.832	0.971	0.810	0.861
4			1.177	1.145	0.876	1.008	0.967	1.010	0.987	0.914
5			1.403	1.354	1.037	1.124	1.093	1.095	1.138	0.995
6					1.282	1.251	1.245	1.243	1.337	1.169

change gradually over time. This tendency is most pronounced in the lowest risk portfolios, for which the estimated risk in the second period is invariably higher than that estimated in the first period. There is some tendency for the high risk portfolios to have lower estimated risk coefficients in the second period than in those estimated in the first. Therefore, the estimated values of the risk coefficients in one period are biased assessments of the future values, and furthermore the values of the risk coefficients as measured by the estimates of β_i tend to regress towards the means with this tendency stronger for the lower risk portfolios than the higher risk portfolios.

C. *A Method of Correction*

In so far as the rate of regression towards the mean is stationary over time, one can in principle correct for this tendency in forming one's assessments. An obvious method is to regress the estimated values of β_i in one period on the values estimated in a previous period and to use this estimated relationship to modify one's assessments of the future.

Table 4 presents these regressions for five successive periods of time for individual securities.[22] The slope coefficients are all less than one in agreement with the regression tendency, observed above. The coefficients themselves do change over time, so that the use of the historical rate of regression to correct

TABLE 4

MEASUREMENT OF REGRESSION TENDENCY OF ESTIMATED BETA COEFFICIENTS FOR INDIVIDUAL SECURITIES

Regression Tendency Implied Between Periods	$\beta_2 = a + b\beta_1$
7/33-6/40 and 7/26-6/33	$\beta_2 = 0.320 + 0.714\beta_1$
7/40-6/47 and 7/33-6/40	$\beta_2 = 0.265 + 0.750\beta_1$
7/47-6/54 and 7/40-6/47	$\beta_2 = 0.526 + 0.489\beta_1$
7/54-6/61 and 7/47-6/54	$\beta_2 = 0.343 + 0.677\beta_1$
7/61-6/68 and 7/54-6/61	$\beta_2 = 0.399 + 0.546\beta_1$

22. The reader should not think of these regressions as a test of the stationarity of the risk of securities over time but rather merely as a test of the accuracy of the assessments of future risk which happen to be derived as historical estimates. In this test of accuracy, the independent variable in these regressions is measured without error, so that the estimated coefficients are unbiased. In the test of the stationarity of the risk measures over time, the independent variable would be measured with error, so that the coefficients in Table 4 would be biased.

for the future rate will not perfectly adjust the assessments and may even overcorrect by introducing larger errors into the assessments than were present in the unadjusted data.

To examine the efficacy of using historical rates of regression to correct one's assessments, the estimated risk coefficients for the individual securities for the period from July 1933 through June 1940 were modified using the first equation in Table 4 to obtain adjusted risk coefficients under the assumption that the future rate of regression will be the same as the past. This process was repeated for each of the next three periods using respectively the next three equations in Table 4 to estimate the rate of regression.

Table 5 compares these adjusted assessments with the unadjusted assessments which were used in Tables 2 and 3. For the portfolios selected previously using the data from July 1933 through June 1940, both the unadjusted

TABLE 5

MEAN SQUARE ERRORS BETWEEN ASSESSMENTS AND FUTURE ESTIMATED VALUES

Number of Sec./ Port.	Assessments Based Upon							
	7/33-6/40		7/40-6/47		7/47-6/54		7/54-6/61	
	unadjusted	adjusted	unadjusted	adjusted	unadjusted	adjusted	unadjusted	adjusted
1	0.1929	0.1808	0.1747	0.1261	0.1203	0.1087	0.1305	0.1013
2	0.0915	0.0813	0.1218	0.0736	0.0729	0.0614	0.0827	0.0535
4	0.0538	0.0453	0.0958	0.0483	0.0495	0.0381	0.0587	0.0296
7	0.0323	0.0247	0.0631	0.0276	0.0387	0.0281	0.0523	0.0231
10	0.0243	0.0174	0.0535	0.0220	0.0305	0.0189	0.0430	0.0169
20	0.0160	0.0090	0.0328	0.0106	0.0258	0.0139	0.0291	0.0089
35	0.0120	0.0055	0.0266	0.0080	0.0197	0.0101	0.0302	0.0089
50	0.0096	0.0046	0.0192	0.0046	0.0122	0.0097	0.0237	0.0064
75	0.0081	0.0035	0.0269	0.0067	0.0112	0.0078	0.0193	0.0056
100	0.0084	0.0020	0.0157	0.0035	0.0114	0.0084	0.0195	0.0056

and adjusted assessments of future risk were obtained. The accuracy of these two alternative methods of assessment were compared through the mean squared errors of the assessments versus the estimated risk coefficients in the next period, July 1940 through June 1947.[23] This process was repeated for each of the next three periods.

For individual securities as well as portfolios of two or more securities, the assessments adjusted for the historical rate of regression are more accurate than the unadjusted or naive assessments. Thus, an improvement in the accuracy of one's assessments of risk can be obtained by adjusting for the historical rate of regression even though the rate of regression over time is not strictly stationary.

23. The mean square error was calculated by $\frac{\Sigma(\beta_1 - \beta_2)^2}{n}$ where β_1 is the assessed value of the future risk, β_2 is the estimated value of the risk, and n is the number of portfolios. In using an estimate of beta rather than the actual value, the mean square error will be biased upwards, but the effect of this bias will be the same for both the adjusted and unadjusted assessments.

V. Conclusion

This paper examined the empirical behavior of one measure of risk over time. There was some tendency for the estimated values of these risk measures to regress towards the mean over time. Correcting for this regression tendency resulted in considerably more accurate assessments of the future values of risk.