BEFORE THE WASHINGTON

UTILITIES & TRANSPORTATION COMMISSION

WASHINGTON UTILITIES AND TRANSPORTATION COMMISSION

Complainant,

v.

PUGET SOUND ENERGY,

Respondent.

DOCKET UG-230968

**CROSS-ANSWERING TESTIMONY OF DR. ROBERT L. EARLE
ON BEHALF OF THE
WASHINGTON STATE OFFICE OF THE ATTORNEY GENERAL
PUBLIC COUNSEL UNIT**


EXHIBIT RLE-4C

Technical Note on Use of the Normal Distribution

September 12, 2024

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PSE proposes (and Staff and JEA adopt this approach) to use z-scores to determine the 75th and 97.5[th] percentile levels. These z-scores, or 'normal z-scores,' are based on "the value from the standard normal distribution for the desired percentile."[1] In its calculations for the example in its testimony adopted by Staff,[2] PSE appropriately uses the actual percentiles from market data, however PSE's presentation suggests that it proposes to use normal z-scores that are based on allowance prices being normally distributed, that is shaped like the familiar bell curve. Figure 1 shows a normal distribution with the mean and variance equal to the mean and variance of allowance prices provided by PSE.

Figure 1
**Staff Proposes to Model Allowance Prices with Normal Distribution**
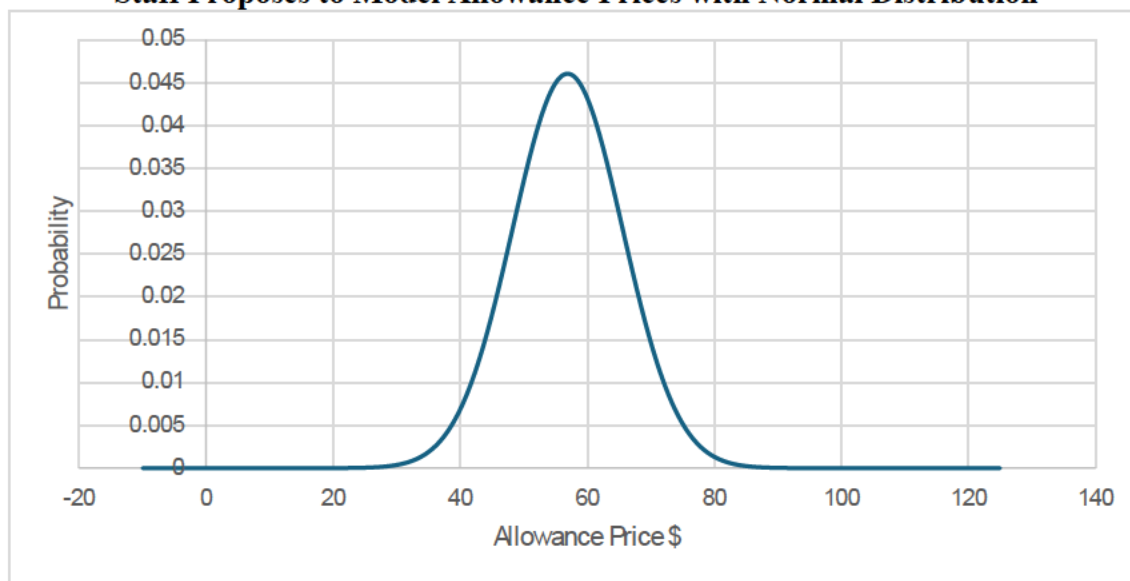
It would be very surprising if allowance prices were normally distributed. Thus far, from the prices available in the market, allowance prices are far from

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[1] Direct Test. of Mickelson, Exh. CTM-1CT at 3:6–11
[2] Mickelson, Exh. CTM-3C.

normally distributed as can be seen in the histogram in Figure 2 with the normal

distribution overlayed. Visual inspection of the figure shows that allowance prices

are not at all close to being normally distributed.[3]
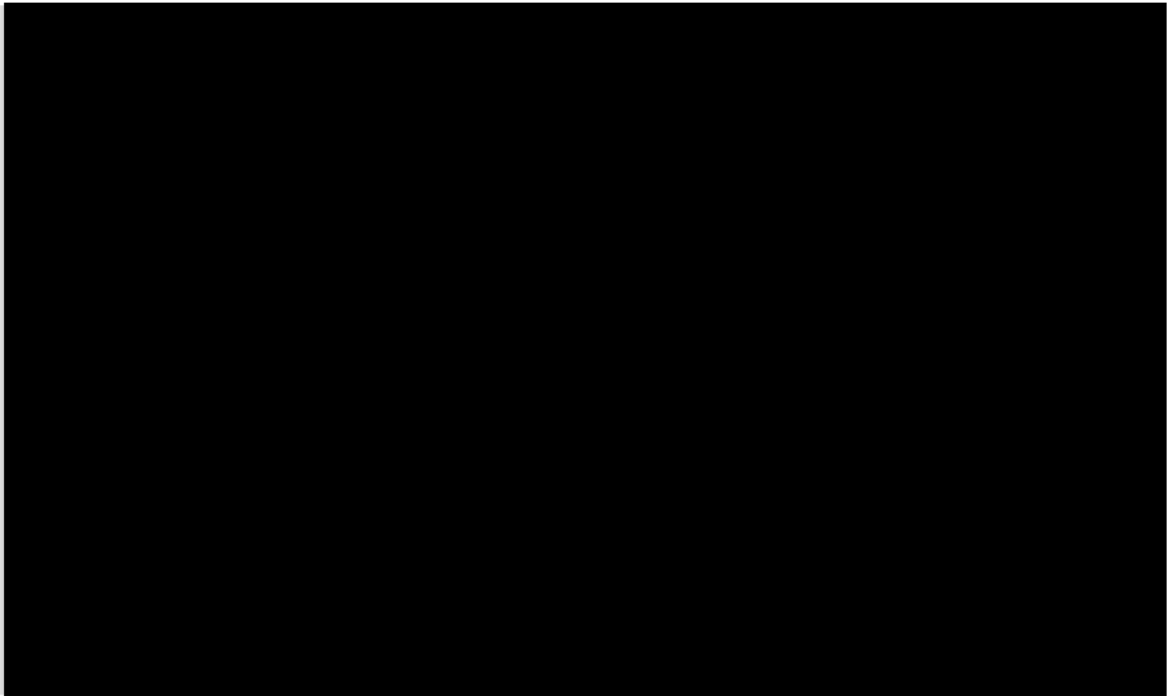
It is not surprising that allowance prices are not normally distributed. If a

parametric distribution is used,[4] commodity and equity prices are more typically

modelled with some variant of the lognormal not the normal distribution. One

reason for this is the left skewness of commodity and equity prices because

negative prices are not generally possible.

In defense of its use of normal z-scores, Staff adapting PSE's proposal

claims that "with a sufficiently large dataset, such as a 4-year compliance period

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[3] The allowance prices also fail the Shapiro-Wilk and d'Agostino-Pearson tests for normality. A visual inspection of the QQ plot also shows that allowance prices are not normally distributed.

[4] A parametric distribution is one in which a fixed number of parameters and a set formula completely define the distribution. For example, the normal distribution is completely defined by the mean and the variance and its formula. In contrast to parametric distributions, one need not specify parameters, but base calculations on the underlying data. Using the data itself as the distribution, sometimes called the empirical distribution, has the advantage of not needing to assume any particular underlying shape as is required with parametric distributions.

with daily prices, then the distribution of allowance prices can begin to approximate a normal distribution."[5] PSE goes on to state "with a sufficiently large sample size, the sample distribution will approximate a normal distribution, and the sample mean will approach the population mean."[6] While it is true that the sample mean will approach the population mean with a large enough sample size, it is not true at all that the sample distribution will approximate the normal distribution unless the underlying distribution for the entire population is approximately normal. Consider a variable that is uniformly distributed as shown in Figure 3. Below one it has zero probability of occurring, and above two it has zero probability of occurring. Between one and two, this uniformly distributed variable has an equal probability of occurring. No matter how much one samples the uniformly distributed variable, the sample distribution will never approximate the normal distribution.

Figure 3
Uniform Distribution on the Interval between One and Two

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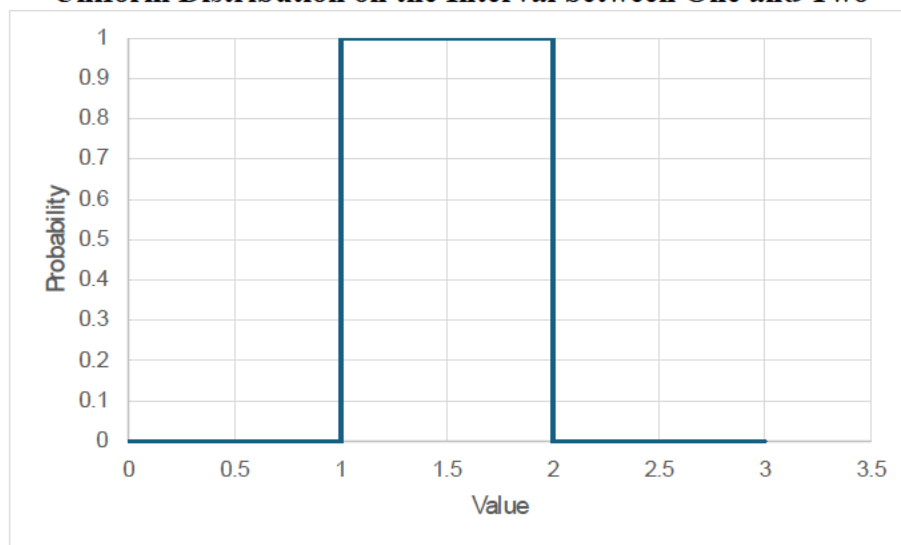
[5] Robert L. Earle, Exh. RLE-6 (PSE Response to Public Counsel Data Request No. 3).
[6] *Id.*

This holds true for any variable that is not approximately normally distributed. Its sample distribution will converge to the underlying distribution for the entire population as the sample size grows larger, not to the normal distribution.

PSE further claims "Although PSE does not assume a normal distribution for allowance prices, z-scores are used as a standard statistical tool to determine deviations from the mean. This method provides a consistent framework for identifying significant variations in allowance prices and establishing cost-sharing thresholds."[7] PSE is in error. While the use of normal z-scores is a standard statistical tool, the use of such tools is only valid when the distribution is approximately normal. If the distribution is not normal, the use of normal z-scores will result in errors. Non-normal distributions call for non-normal z-scores, or a direct calculation of percentiles as PSE actually did.

For example, consider the uniform distribution pictured in Figure 3. The mean is 1.5 and the standard deviation is .2887. The 75th and 97.5th percentiles occur at 1.75 and 1.9725, respectively. Using normal z-scores to calculate the 75th and 97.5th percentiles yields 1.69 and 2.07. Note that the probability of 2.07 is zero under this uniform distribution. Applying methods that are valid when the distribution is approximately normal yields errors when the distribution is not approximately normal. The current evidence is that allowance prices are not approximately normal.

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[7] *Id.*

The percentiles that PSE actually calculates in its example based on percentiles from the actual data is the correct approach and free of assumptions about the underlying distribution of allowance prices. Using the normal z-score method results in \$62.76 and \$73.89 for the 75th and 97.5th percentiles compared to \$65.25 and \$69.00 values that PSE calculates. Moreover, the normal z-score 97.5th percentile, \$73.89, is larger than the maximum price in the secondary market.