9

The Capital Asset Pricing Model

Knowing how risk (market beta) and reward (expected rate of return) are measured, you are now ready to proceed to the punchline: a formula that tells you how much reward your investment projects have to offer to compensate your investors for their risks. If you can judge the risk of new corporate investment projects, you can then determine the appropriate costs of capital that you should use in your project's NPV calculations. Alas, like NPV, the formula may be simple, but the application is hard. The devil is in the details.

We will first briefly review what you already know. Then you will learn all about this new model—the CAPM. Finally, you will get to apply it.

One apology in advance: In this chapter, I do not fully explain where all the formulas come from. This is because it really takes a full investments course to derive them. (The appendix goes into more detail, but if you really want to learn about investments, you need to take a full course on the subject.)

9.1 What You Already Know and What You Want to Know

You are still after an estimate for your opportunity cost of capital.

Let's take stock. First, you already know the right train of thought for capital budgeting purposes: As a corporate manager, your task is to determine whether you should accept or reject a project. You make this decision with the NPV formula. To determine the discount factor in the NPV formula, you need to estimate an appropriate cost of capital—or, more precisely, the *opportunity* cost of capital for your investors. This means that you need to judge what a fair expected rate of return, $\mathcal{E}(r)$, for your project is, given your project's risk characteristics. If your project offers a lower expected return than what your investors can earn elsewhere in similarly risky projects, then you should not put your investors' money into your project but instead return their money to them. If your project offers more expected return, then you should go ahead and invest their money into your project. Put differently, your goal is to learn what your investors, if asked, would have wanted you to invest in on their behalves.

Assume perfect markets, that investors dislike risk and like reward, and more.

Second, the perfect market assumptions are not enough to proceed. We must assume that investors like overall portfolio reward (expected return) and dislike overall portfolio risk (variance or standard deviation of return). We also assume that investors are smart. Presumably, this means that they diversify, hopefully holding many assets and be reasonably close to the market portfolio. Somewhat less appealing, we also must assume that investors all have access to exactly the same set of assets. (This means we are ignoring investments in people's own houses or education, for example.) And finally, mostly for convenience, we assume that they want to maximize their wealth in the market for only one period.

This allows you to figure out how they—and you—should measure project risk and reward.

Third, for investors with these preferences and who are therefore already holding the overall market portfolio, you can follow their trains of thought. You can infer how they should view the risk and reward of your individual projects. Their reward is their expected rate of return. Their risk is their overall portfolio risk, *not* your project's own standard-deviation risk. Your project's contribution to your investors' overall portfolio risk is the market beta of your project—think of it as a measure of your project's "toxicity." A project that decreases in value when the market decreases in value, and increases when the market increases, has a positive market beta. It's toxic—investors don't like it. A project that increases in value when the market decreases in value, and vice versa, has a negative market beta. It's less toxic—investors like it more. That is, a project with a low market beta helps an investor who holds a portfolio similar to the market portfolio to reduce the overall investment risk.

This gives you a trade-off between risk and reward "in equilibrium."

You can also draw some additional conclusions without any math. In our assumed perfect world, you can guess that investors will have already snatched up the best projects—those that have low risk and high expected rates of return. In fact, anyone selling projects with lower risk contributions can sell them for higher prices, which in turn immediately drives down their expected rates of return. Consequently, what is available for purchase in the real world must be subject to some trade-off: Projects that have more market-risk contribution must offer a higher expected rate of return if their sellers want to convince investors to purchase them. But what *exactly* does this relationship between risk and reward look like? This is the subject of this chapter—it is the domain of the capital asset pricing model, the CAPM.

Q 9.1. What are the assumptions underlying the CAPM? Are the perfect market assumptions among them? Are there more?

9.2 Using The Capital Asset Pricing Model (CAPM)

The CAPM gives you the cost of capital if you give it the risk-free rate, the expected rate of return on the market, and your project's market beta.

The **capital asset pricing model** (**CAPM**) is a model that gives you an appropriate expected rate of return (cost of capital) for each project if you give it the project's relevant risk characteristics. The model states that an investment's cost of capital is lower when it offers better diversification benefits for an investor who holds the overall market portfolio—less required reward for less risk contribution. Market beta is its measure of risk contribution. Projects contributing more risk (market beta) require a higher expected rate of return for you to want them; projects contributing less risk require a lower expected rate of return for you to want them. This is the precise relationship that the CAPM gives you.

IMPORTANT

To estimate the required expected rate of return for a project or firm—that is, the cost of capital—according to the CAPM, you need three inputs:

1. The risk-free rate of return, r_F
2. The expected rate of return on the overall market, $\mathcal{E}(r_M)$
3. A firm's or project's beta with respect to the market, β_i

The CAPM formula is

$$\mathcal{E}(r_i) = r_F + [\mathcal{E}(r_M) - r_F] \cdot \beta_i \tag{9.1}$$

where i is the name of your project and $\mathcal{E}(r_i)$ is your project's expected rate of return.

The difference between the expected rate of return on the risky (stock) market and the risk-free investment, $[\mathcal{E}(r_M) - r_F]$, is called the **equity premium** or **market risk premium**, discussed in more detail later.

You need to memorize the CAPM formula. It is *the* standard model in the finance.

A first quick use of the CAPM formula.

Let's use the formula. If you believe that the risk-free rate is 3% and the expected rate of return on the market is 7%, then the CAPM states that

$$\mathcal{E}(r_i) = 3\% + (7\% - 3\%) \cdot \beta_i = 3\% + 4\% \cdot \beta_i$$
$$\mathcal{E}(r_i) = r_F + [\mathcal{E}(r_M) - r_F] \cdot \beta_i$$

Therefore, a project with a beta of 0.5 should have a cost of capital of $3\%+4\%\cdot 0.5 = 5\%$, and a project with a beta of 2.0 should have a cost of capital of $3\% + 4\% \cdot 2.0 = 11\%$. The CAPM gives an opportunity cost for your investors' capital: If the project with the beta of 2.0 cannot earn an expected rate of return of 11%, you should not take this project and instead return the money to your investors. Your project would add too much risk for its reward. Your investors have better opportunities elsewhere.

It is easier to work in required returns than in prices.

The CAPM is called an **asset-pricing model**, even though it is most often expressed in terms of a required expected rate of return rather than in terms of an appropriate project price. Fortunately, though messy, the two are equivalent—you can always work with the CAPM return first, and discount the expected cash flow into an appropriate price second. A given expected rate of return implies a given price. (If you do not know the fair price, you will however have to take two aspirins and work with a more difficult version of the CAPM formula. It is called **certainty equivalence** and explained in the chapter appendix.)

➤ *Certainty equivalence CAPM form*, *Sect.* App.9.A (Companion), *Pg.*≈*51*.

The CAPM formula tells you what investors care about: comovement with the market.

The CAPM specifically ignores the standard deviation of individual projects' rates of return. That is, the model posits that investors do not care about it, because they are smart enough to diversify away any idiosyncratic risk. The CAPM posits that investors instead care about the project market betas, because these measure the risk components that investors holding the market portfolio cannot diversify away. (This makes a lot of sense for highly-diversified investors, though not for liquidity-constrained entrepreneurs.)

The CAPM has three inputs. We will cover them in detail.

For the three CAPM inputs, as always, you are really interested in the future: the future expected rate of return on the market and the future beta of your firm/project with respect to the market. You really don't care about the past average rates of return or the past market betas. But, as usual, you often have no choice other than to rely on estimates that are based at least partly on historical data. In Section 9.4, you will learn how to estimate each CAPM input. But let's explore the model itself first, assuming that you know all the inputs.

➤ *Will history repeat itself?*, Sect. 7.1, Pg.175.

The Security Market Line (SML)

Examples of CAPM rates of return that individual securities should offer.

Let's apply the CAPM in a specific example. Assume that the risk-free rate is 3% per year and that the market offers an expected rate of return of 8% per year. The CAPM formula then states that a stock with a beta of 1 should offer an expected rate of return of $3\% + (8\% - 3\%) \cdot 1 = 8\%$ per year; that a stock with a beta of 0 should offer an expected rate of return of $3\% + (8\% - 3\%) \cdot 0 = 3\%$ per year; that a stock with a beta of $1/2$ should offer an expected rate of return of $3\% + (8\% - 3\%) \cdot 0.5 = 5.5\%$ per year; that a stock with a beta of 2 should offer an expected rate of return of $3\% + (8\% - 3\%) \cdot 2 = 13\%$ per year; and so on.

The SML is just a graphical representation of the CAPM formula.

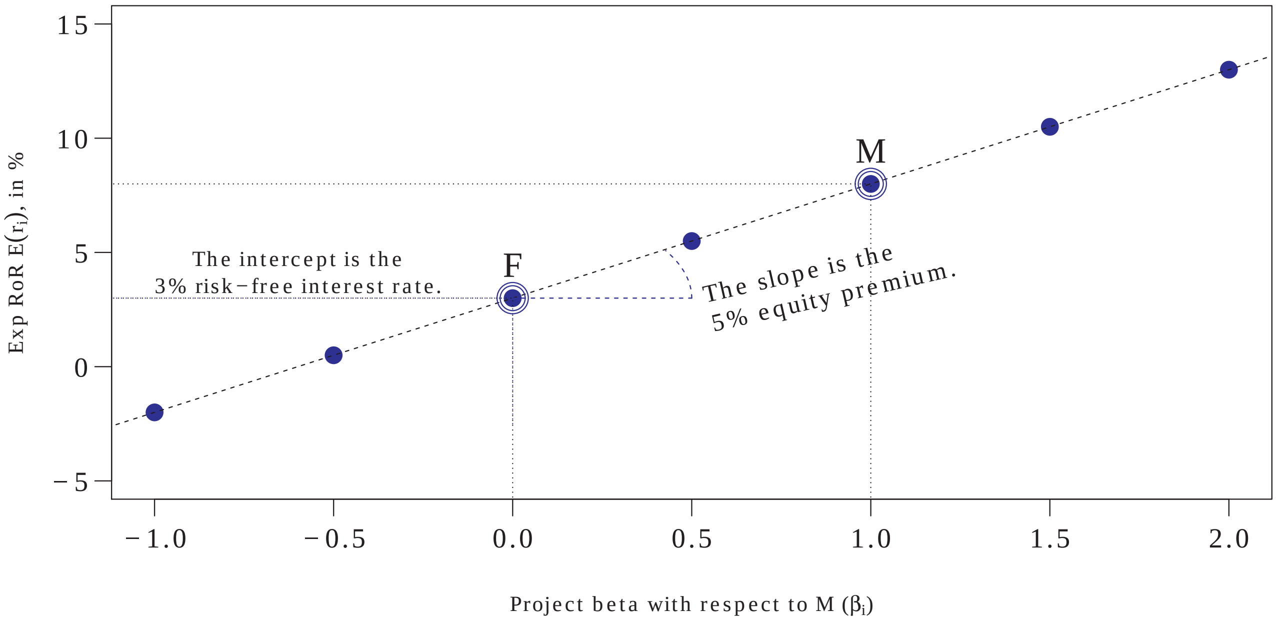
The CAPM formula is often graphed as the **security market line** (**SML**), which shows the relationship between the expected rate of return of a project and its beta. Exhibit 9.1 draws a first security market line for seven assets. Each investment asset (such as a stock or a project) is a point in this coordinate system. Because all assets properly follow the CAPM formula in our example, they must lie on a straight line. In other words, the SML is just a graphical representation of the CAPM formula. The slope of this line is the equity premium, $\mathscr{E}(r_M) - r_F$, and the intercept is the risk-free rate, r_F.

If you know the inputs, the SML is a sharp line; if you estimate them, it is a scatterplot.

Alas, in the real world, even if the CAPM holds, you would not have the data to draw Exhibit 9.1. The reason is that you do not know true expected returns and true market betas. Exhibit 9.2 plots two graphs in a perfect CAPM world. The top graph repeats Exhibit 9.1 and falsely presumes that you know CAPM inputs—the true market betas and true expected rates of return. This line is perfectly straight. In the bottom graph, you have to rely only on observables—estimates of expected returns and betas, presumably based mostly on historical data averages. Now you can only fit an "estimated security market line," not the "true security market line." Of course, you hope that your historical data provides good, unbiased estimates of true market beta and true expected rates of return (and this is a big assumption), so that your fitted line will look at least approximately straight. A workable version of the CAPM thus can only state that there should roughly be a linear relationship between the data-estimated market betas and the data-estimated expected rates of return, just as drawn here.

Q 9.2. The risk-free rate is 4%. The expected rate of return on the market is 7%. What is the appropriate cost of capital for a project that has a beta of 3?

Q 9.3. The risk-free rate is 4%. The expected rate of return on the market is 12%. What is the cost of capital for a project that has a beta of 3?



		Investment Asset						
		A	B	F	C	M	D	E
Market Beta	β_i	–1.0	-0.5	0.0	0.5	1.0	1.5	2.0
Expected Rate of Return	$\mathcal{E}(r_i)$	–2.0%	0.5%	3.0%	5.5%	8.0%	10.5%	13.0%

Exhibit 9.1: *The Security Market Line.* This graph plots the CAPM relation $\mathcal{E}(r_i) = r_F + [\mathcal{E}(r_M) - r_F] \cdot \beta_i = 3\% + (8\% - 3\%) \cdot \beta_i$, where β_i is the beta of an individual asset with respect to the market. In this graph, we assume that the risk-free rate is 3% and the equity premium is 5%. Each point is one asset (such as a stock, a project, or a mutual fund). The point M in this graph could also be any other security with a $\beta_i = 1$. F could be the risk-free asset or any other security with a $\beta_i = 0$.

Q 9.4. The risk-free rate is 4%. The expected rate of return on the market is 12%. What is the cost of capital for a project that has a beta of –3? Does this make economic sense?

Q 9.5. Is the real-world SML with historical data a perfect straight line?

Q 9.6. The risk-free rate is 4%. The expected rate of return on the market is 7%. A corporation intends to issue publicly-traded bonds that *promise* a rate of return of 6% and offer an *expected* rate of return of 5%. What is the implicit beta of the bonds?

Q 9.7. Draw the SML if the risk-free rate is 5% and the equity premium is 9%.

Q 9.8. What is the equity premium, both mathematically and intuitively?

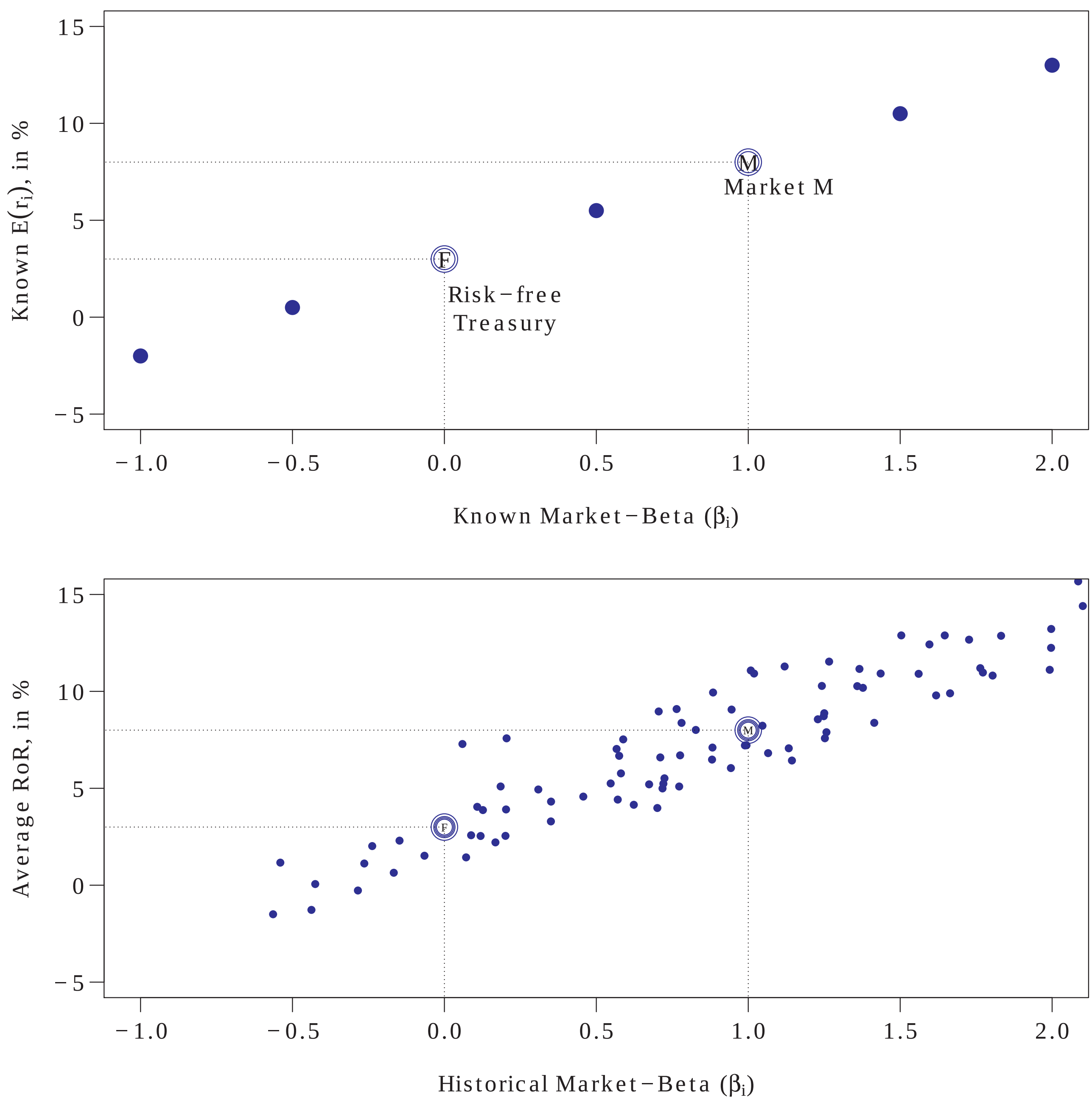


Exhibit 9.2: *The Security Market Line in an Ideal CAPM World.* The lower panel shows what we are usually confronted with: Historical average returns and historical betas are just estimates from the data. We hope that they are representative of the true underlying mean returns and true betas, which in turn would mean that they will also be representative of the future means and betas.

9.3 The CAPM Cost of Capital in the Present Value Formula

We usually use the CAPM output, the expected rate of return, as our discount rate.

For a corporate manager, the CAPM is needed to get the denominator in the NPV formula, the opportunity cost of capital, $\mathcal{E}(r)$:

$$\text{NPV} = C_0 + \frac{\mathcal{E}(C_1)}{1+\mathcal{E}(r_1)} + \frac{\mathcal{E}(C_2)}{1+\mathcal{E}(r_2)} + \cdots$$

Together, the CAPM and the NPV formulas tell you again that cash flows that correlate more with the overall market are of less value to your investors and therefore require higher expected rates of return ($\mathcal{E}(r)$) in order to pass muster (well, the hurdle rate, which is determined by the alternative opportunities that your model presumes your investors have).

Deconstructing Quoted Rates of Return— Risk Premiums

Reminder: Stated bond yields contain time and default premiums.

Let me return to the subject of Section 6.2. You learned that in a perfect and risk-neutral world, stated rates of return consist of a time premium and a default premium. On average, the default premium is zero, so the expected rate of return is just the time premium.

➤ *Time and default premiums*, Sect. 6.2, Pg. 129.

The CAPM gives you the time and risk premiums.

The CAPM extends the expected rate of return to a world in which investors are risk averse. It gives you an expected rate of return that adds a **risk premium** (as a reward for your willingness to absorb risk) to the time premium.

$$\begin{aligned} \text{Promised Rate of Return} &= \text{Time Premium} + \text{Default Premium} + \text{Risk Premium} \\ \text{Actual Earned Rate} &= \text{Time Premium} + \text{Default Realization} + \text{Risk Premium} \\ \underbrace{\text{Expected Rate of Return}}_{\text{provided by the CAPM}} &= \text{Time Premium} + \text{Expected Risk Premium} \end{aligned}$$

In the risk-neutral perfect world, there were no differences in *expected* rates of return across assets. There were only differences in *stated* rates of return. The CAPM changes all this—different assets can now also have different *expected* rates of return.

Important: *The CAPM ignores default risk and, thus, does not provide a default premium. You must take care of it yourself!*

However, the CAPM does *not* take default risk into account, much less give you an appropriate stated rate of return. You should therefore wonder: How do you find the appropriate quoted rate of return in the real world? After all, it is this stated rate of return that is usually publicly posted, not the expected rate of return. Put differently, how do you put the default risk and CAPM risk into one valuation?

A specific bond example: First compute the price necessary to make you "even" relative to the Treasury if you are risk-neutral. This price is based on the time premium and the default premium.

Here is an example. Say you want to determine the PV of a corporate zero-bond that has a beta of 0.25 and promises to deliver \$200 next year. This bond pays off 95% of the time, and 5% of the time it totally defaults. Assume that the risk-free rate of return is 6% per annum and that the expected rate of return on the market is 10%. Therefore, the CAPM states that the expected rate of return on your bond must be

$$\begin{aligned} \mathcal{E}(r_{\text{Bond}}) &= 6\% + 4\% \cdot 0.25 = 7\% \\ &= r_F + [\mathcal{E}(r_M) - r_F] \cdot \beta_{\text{Bond}} \end{aligned}$$

This takes care of the time and risk premiums. To take the bond's default risk into account, you must still find the numerator. You cannot use the promised payment. You must adjust it for the probability of default. You expect to receive not $200, but

$$\mathscr{E}(C_{\text{Bond}}) = 95\% \cdot \$200 + 5\% \cdot 0 = \$190$$
$$= \mathscr{P}rob(\text{No Default}) \cdot \text{Promise} + \mathscr{P}rob(\text{Default}) \cdot \text{Nothing}$$

Therefore, the present value formula states that the value of the bond is

$$PV_{\text{Bond}} = \frac{\mathscr{E}(C_{\text{Bond}})}{1 + \mathscr{E}(r_{\text{Bond}})} = \frac{\$190}{1 + 7\%} \approx \$177.57$$

Given this price, you can now compute the promised (or quoted) rate of return on this bond:

$$\frac{\$200 - \$177.57}{\$177.57} \approx 12.6\%$$
$$\frac{\text{Promised Cash Flow} - \text{PV}}{\text{PV}} = \text{Promised Rate of Return}$$

The risk premium is above and beyond the time and default premiums. On average, risky investments earn more than risk-free investments now.

You can now quantify the three components in this example. For this bond, the time premium of money is 6% per annum—it is the rate of return that an equivalent-term Treasury offers. The time premium plus the risk premium is provided by the CAPM, and it is 7% per annum. Therefore, 1% per annum is your "average" compensation for your willingness to hold this risky bond instead of the risk-free Treasury. The remaining $12.6\% - 7\% = 5.6\%$ per annum is the default premium: You do not expect to earn money from this default premium "on average." You only earn it if the bond does not default.

$$12.6\% = 6\% + 5.6\% + 1\%$$
$$\text{Promised Interest Rate} = \text{Time Premium} + \text{Default Premium} + \text{Risk Premium}$$

In the real world, most bonds have fairly small market betas (often much smaller than 0.25) and thus fairly low risk premiums. Instead, most of the premium that ordinary corporate bonds quote above equivalent risk-free Treasury rates is not due to the risk premium, but due to the default premium. They simply won't pay as much as they promise, on average. However, for corporate projects and equity shares, the risk premium can be quite large. (Watch out—there are also some important imperfect market premiums that you will only learn in the next chapter.)

IMPORTANT

Never forget:

- The CAPM provides an expected rate of return.
- This return is not a stated (promised, quoted) rate of return, because it does not include a default premium.
- The probability of default must be handled in the NPV numerator (through the expected cash flow), and not in the NPV denominator (through the expected rate of return).

Q 9.9. A corporate bond with a beta of 0.2 will pay off next year with 99% probability. The risk-free rate is 3% per annum, and the equity premium is 5% per annum.

1. What is the price of this bond?
2. What is its promised rate of return?
3. Decompose the bond's quoted rate of return into its components.

Q 9.10. Going to your school has total additional and opportunity costs of $30,000 *this year and up-front*. With 90% probability, you are likely to graduate from your school. If you do not graduate, you have lost the entire sum. Graduating from the school will increase your 40-year lifetime annual salary by roughly $5,000 per year, but more so when the market rate of return is high than when it is low. For argument's sake, assume that your extra-income beta is 1.5. Assume the risk-free rate is 3%, and the equity premium is 5%. What is the value of your education?

9.4 Estimating the CAPM Inputs

How can you obtain reasonable estimates of the three inputs into the CAPM formula $\mathscr{E}(r_i) = r_F + [\mathscr{E}(r_M) - r_F] \cdot \beta_i$?

The Risk-Free Rate and Multi-Year Term-Structure Considerations

Which risk-free rate?

The first input into the CAPM formula is the risk-free rate of return (r_F). First, don't forget to use nominal rates to discount nominal expected cash flows. Now, this nominal risk-free rate is relatively easy to obtain from U.S. Treasuries. There is one small issue, though—which Treasury? What if the yield curve is upward sloping (as it usually does) and Treasuries yield 1% per year over one year, 3% per year over ten years, and 5% per year over thirty years? Which risk-free interest rate should go into the CAPM?

➤ *US Treasuries*, *Sect.* 5.3, *Pg.97*.

Advice: Pick the interest rate for a Treasury that is "most similar" to your project.

➤ *Yield Curve*, *Chapter* 5, *Pg.85*.

Unfortunately, the CAPM offers no guidance, because it has no concept of more than one single time period and thus no concept of a yield curve. However, from a practical perspective, it makes sense to match projects to similar risk-free bond benchmarks. That is, pick the risk-free zero-bond yield that is closest to each of your project's specific expected cash flows at the same time. For example, to value a machine that operates for three years, use the 1-year T-bond yield to discount the expected cash flow in the first year's NPV term, the 2-year T-bond yield for the second year's NPV term, and the 3-year T-bond yield for the third year's NPV term. If you had to use just one risk-free rate for multiple cash flows (because your pointy-haired boss says so), choose an average of the three rates or simply the 2-year bond. (There are better ways to do this, but the extra precision is rarely worth it.)

But don't we need formal guidance? Isn't this violating the letter of the law?

You may think this is a pretty loose method to handle an important question, and you would be right. However, it is also a reasonable method. Think about the opportunity cost of capital for a small investment with a market-beta of 0. If your corporation's investors are willing to commit their money for ten years, they could earn the yield on a ten-year risk-free Treasury bond instead. It is this ten-year rate that would then be the opportunity cost of capital on your own project cash flow that will materialize in ten years. If your project's cash flow will occur in three months, your investors could only earn the rate of return on a three-month T-bill instead. Indeed, there is almost universal agreement that companies should use a risk-free rate lined up with the project cash flow timing in the first part of the CAPM formula (where r_F appears by itself).

Q 9.11. What is today's risk-free rate for a 1-year project? For a 10-year project?

Q 9.12. If you can use only one Treasury, which risk-free rate should you use for a project that will yield $5 million each year for 10 years?

The Equity Premium

You want to know the equity premium, regardless of the CAPM

Your second CAPM input, the equity premium ($\mathcal{E}(r_M) - r_F$), is much more difficult to estimate. It is the extra expected rate of return that risky equity projects have to offer above and beyond what risk-free bond projects are offering. (It is a difference, so you can use either two nominal or two real rates.) By the way, regardless of whether the CAPM holds or not, this is a number of first-order importance to *you*—it helps you decide whether you should invest your own money in risky equities or in safer bonds.

You must provide the CAPM with the equity premium. *Good luck!*

The theoretical CAPM model assumes that you already know the *expected* rate of return on the market perfectly, not that you have to estimate it. But in real life, the equity premium is not posted anywhere, and *no one really knows the correct number*. Worse: Not only is it difficult to estimate, but your estimate often has a large influence over the CAPM's estimated cost of capital. C'est la vis.

Do not use a short-term-Treasury based equity premium for benchmarking your far-into-the-future cash flows.

Many other finance text books quote just one equity-premium estimate, and it is often the expected rate of return on stocks relative to the short-term Treasury yield. This choice can be reasonable if your own cash flows (that you want to discount) are also very short-horizon. Stock market investors, who can buy one day and sell the next, can defend this practice. It also means that an investment in a project with a beta of 1 has an expected rate of return equal to that in the stock market, because the risk-free rates in the intercept and slope cancel. Unfortunately, corporate-finance executives can rarely move in and out of projects on a moment's notice. They usually need to use the CAPM to decide on investments that have cash flows expected to materialize only many years into the future. In this case, everyone agrees that your CAPM equity premium should not be expected stock returns above short-term Treasuries. Instead, you should use the same equivalent-term-to-your-project-cash-flows Treasury rate in your estimate of the equity premium that you used as your risk-free Treasury in the constant term in the CAPM formula. (In fact, there is even a second argument to use long-term risk-free rates in the equity premium: equities are long-term investments, so you should always net out the long-term Treasury rate from expected stock returns, regardless of your own cash flows' horizons.)

There are a number of methods to guesstimate the equity premium. Unfortunately, for many decades, these methods have not tended to agree with one another. It should thus not come as a surprise that practitioners, instructors, finance textbook authors have also been confused and confusing. Exhibit 9.3 shows that each text book seems to have had its own estimate. (Fortunately, both the disagreement and the average recommended estimate seem to be slowly declining.)

Should I just give it to you?

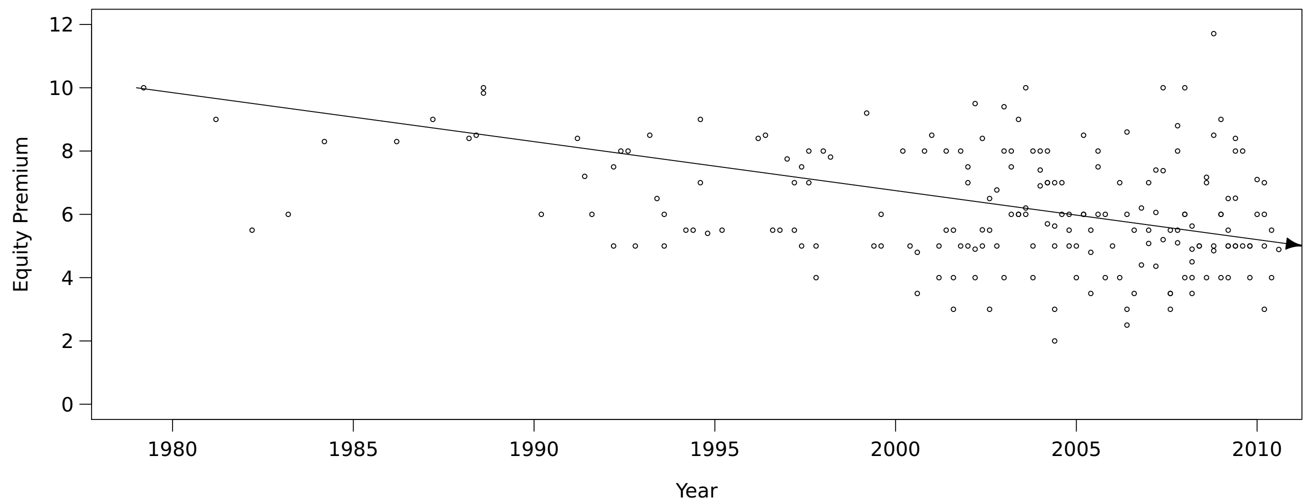


Exhibit 9.3: *Equity Premia from Different Textbooks. Source: Pablo Fernandez, SSRN, 2013..*

Ultimately, we finance-textbook authors have two choices: The first is to throw you one estimate, pretend it is the correct one, and hope you forget to ask hard questions. If you like a formulaic painting-by-numbers approach, this would leave you (wrongly) satisfied. The second is to tell you about the different methods that lead to different estimates. This is the route I will take—explaining different reasoning behind different estimates—if only because the first would eventually leave you startled to discover that your boss is using some other equity-premium and therefore has come up with a different cost-of-capital estimate. I will both explain the intuition behind the most-common methods and describe the magnitude that each suggests nowadays. You can make up your own mind what you deem to be the best estimate. (I will tell you my own personal estimate only at the end.)

Let's show you how people are reasoning.

Historical Averages I

Here are the historical numbers.

The first and most common guesstimation method is to assume that whatever the equity premium was in the past will also be the case in the future. Let's look at the historical performance of stocks vs. bonds in two different time samples, 1926-2012 and 1970-2012:

	1926-2012			1970-2012		
	Ari	Geo	$\mathscr{S}dv$	Ari	Geo	$\mathscr{S}dv$
Value-Weighed Stock Market	11.6%	9.7%	19.8%	11.3%	9.8%	17.3%
net of 1-Year Treasuries	**7.9%**	6.1%	20%	**5.8%**	4.4%	17%
net of 30-Year Treasuries	5.5%	**4.0%**	22%	1.7%	**0.8%**	20%
net of Long-Term Corporates	5.2%	3.6%	20%	1.7%	0.7%	18%

Stocks returned about 11.5% in arithmetic terms with a standard deviation of about 17-20% per year. (The value-weighted stock market is actually the correct portfolio from a CAPM perspective, but it wouldn't be much different if you used the S&P 500 instead.) The geometric return of about 9.5% was in line with the rule-of-thumb formula on Page 162. Although the stock market rate of return was pretty much the same in both samples, the equity premium was not: bond returns were higher after 1970, especially the long-term Treasuries. Thus, the historical equity premium you would want to use depends on the (matched) duration of your own project cash flow, not only for the aforementioned r_F, but also for the $\mathscr{E}(r_M) - r_F$ term.

PS: 30-Year Treasuries had market betas of < 0.1.

➤ *Morningstar Ibbotson Averages*, *Exhibit* 7.5, *Pg.168*.

We can roughly reconcile the difference between the highest equity-premium figure of 7.9% and the lowest figure of 0.7% in the table as follows:

Arithmetic Equity Premium 1926 to 2012 vs. Short-Term Bonds	≈ 8%
Minus Later Sample Period 1970 to 2012	–2%
Minus Long-Term T-Bonds Instead of Short-Term T-Bills	–2%
Minus Use of Geometric Return	–2%
Minus Cross-Product of Above Three	–1%
Geometric Equity Premium 1970-2012 vs. Long-Term Bonds	≈ 1%

Earlier textbooks touted the equivalent of the 7.9% figure, which thus etched itself into the minds of generations of students, practitioners, and finance professors. (*In fact, many other finance textbooks still etch it, without a second thought!*) But 7.9% is not necessarily the right one to use. Let's go through the three differences one by one:

1. **Sample Period?:** You have to judge what historical sample is appropriate. You probably want to end the sample recently (say 2012). But it is not clear whether you should start, say, in 1926 (when most of our data series become available) or in 1970 (about half-way). Although your estimate can seem statistically more reliable if you use more years, using the long sample means that you are then leaning more heavily on the (heroic) assumption that the world has not changed. Is the world really still the same in 2013 as it was in 1926? (And is the United States really the right country to consider alone? Maybe it just had an unusually lucky streak during (first half of) the "American Century," which is unlikely to

repeat. In this case, the average country's experience may be a better forecast for today's U.S., too.) No one knows the correct choice. I prefer the latter sample, and more so not because (noisier) stocks have performed differently, but because (less noisy) Treasuries have performed better—and continue to perform better.

➤ *CAPM Term Structure, Sect. 9.4, Pg.227.*

2. **Long-Term or Short-Term Bonds?:** You have to judge whether short-term or long-term bonds are the appropriate benchmark. As already mentioned, the CAPM theory itself does not understand the concept of a term structure (Chapter 5). Thus, it does not understand yield differentials for cash flows over different horizons. And thus, it offers you no easy guidance which one you should use. As with our choice for the risk-free rate in the first term of the CAPM, we have no theory guidance. We need a reasonable approach here, too.

   Again, from the perspective of an investor who can make monthly decisions and shift effortlessly between risk-free bonds and stocks, using short bonds as your benchmark makes sense. From the perspective of a manager who needs to decide on a short-term project, using short T-bills as your benchmark can also make sense. However, from the perspective of a manager who needs to commit funds to a long-term project with cash flows over decades, it does not. If all investors can earn a higher yield in Treasuries if they commit their money for 20 years, and if your own project requires them to commit their money for 20 years, too, then your project should also be benchmarked to this long-term expected rate of return.

   Conveniently, we already know a reasonable approximation of the term premium that your firm has to offer for your own longer-term projects vs. your shorter-term projects: the prevailing yield differential that similar-horizon long-term Treasuries are offering over short-term Treasuries. And, better yet, you can use the yield curve to (simultaneously) reduce your equity premium estimate and raise your risk-free rate. And, more better yet, for projects with betas around 1, this means that risk-free rates cancel and you would expect a rate of return similar to that of the overall stock market. Just don't commit the mistake of using a (high) long-term risk-free rate in the first CAPM term, and a (high) equity premium over the short-term T-bill rate in the second CAPM term.

3. **Geometric or Arithmetic?:** Should you use geometric or arithmetic rates of return in your benchmark cost of capital in the NPV formula? The answer is not clear, as you can may recall from Section 7.1. There was a convention of assuming that past returns represent equally likely future outcomes, many CAPM users compound the annual arithmetic average stock return or equity premium. However, doing so means that you expect the future multi-year stock performance relative to bonds to be better than it was in the past.

➤ *Geometric vs. Arithmetic Returns and Extrapolation, Sect. 7.1, Pg.161.*

   You should probably compound an equity premium estimate somewhere in between the arithmetic and geometric averages. (The correct value depends on your own cash flow's duration. Besides, your own expected future cash flows are normally geometric, too. If you think in terms of arithmetic expected cash flows compounded over many periods—i.e., if you consider the expected cash flow on a project that first earns +200% and then –100% [for a complete overall loss] to be a positive, then you should use the arithmetic average. Hardly anyone thinks this way.)

My recommendation.

My own preference is to use the later 40 years, to use bonds with similar maturity as the cash flow that are discounted, and to use an average between the arithmetic and geometric historical average stock returns. Thus, to discount expected cash flows that will occur in about 10 years and beyond, my own equity-premium estimate is around 1.5%—which is much lower than the 3-5% that would be touted in other books. Conveniently, my way of estimating means that I can also use the same risk-free rate in both the first and the second term of the CAPM. It also means that **my equity premium estimate is lower for longer-term cash flows, but my cost of capital estimate is usually not**. I still assign higher costs of capital to Longer-term cash flows, but this just manifests itself more through the first term (the risk-free rate) than the second term (the equity premium).

Yet another problem: your margin of error.

We are not done with all the problems. Small (and often seemingly innocuous) variations in how you estimate the CAPM inputs can lead to very different cost-of-capital estimates—think 3% vs 5%. Even if the CAPM were correct under one definition, neither you nor I nor anyone else know exactly which one it is. And besides the problem of assessing the expected equity premium point estimate, there is also the problem of the fairly large margin of error. The standard deviation of annual returns of 20% translates into a standard error of error of about $0.2/\sqrt{86} \approx 2\%$ over 86 years and $0.2/\sqrt{43} \approx 3\%$ over 43 years. If you are willing to assume that nothing has changed over the sample, then you can use some additional statistical artillery: You are then about 95% sure (a confidence range popular in statistics) that the mean geometric stock return over long bonds was between 0% and 8% from 1926 to 2012. From 1970 to 2012, you are about 95% sure that the same number was between –2% and +7%. Frankly, this large a range doesn't tell you much. We already knew, or at least believed, that the equity premium should not have been negative.

Peso Problems

To make matters even more complex, some economists believe that the historical data are not telling the full story. There are tiny probability of desasters that just happened not to happen. (This is sometimes called a **Peso problem**, based on a similar unobserved crash situation first described in an otherwise obscure academic paper about the Mexican Peso.) If you might have lost all your money, it's no wonder that you would have earned more in the scenario in which this big disaster did not occur. We just happened to have lived in this world, and so we now see superior returns when we look back. There is some empirical evidence that investors behave exactly as if they fear such a crash—but we do not know whether such a fear is (or was) rational and we are not sure how much of the historical or future equity premium such fear can explain. A reasonable order of magnitude is that extra compensation for crash risk could account for no more than a 1% equity premium per annum and perhaps for nothing (given that stock investors lost more than a third of their investments from 2000-2002 and in 2008 alone).

A sarcastic view: It ain't great!

If your estimate of the forward-looking equity premium is based on the "historical averages I" method, then you can defend a choice of 1% (for long-term cash flows). If you are aggressive, you can defend even a choice of 8% (for short-term cash flows), and ranges from 0% to beyond 10% if need be (or, more cynically, if you are an expert witness paid to opine so). Are you in awe (or disgust) of the wide possible range here?

Historical averages II

The second method is to look at historical equity premiums in the opposite light. If stocks have become more desirable, perhaps this is because investors have become less risk averse, because more investors thus competed to own stocks, drove up the prices, and thereby lowered their future expected rates of return. High historical rates of return would then be indicative of low future expected rates of return.

Method 2: Inverse historical averages.

An even more extreme version of this argument suggests that high past equity returns could have been not just due to high ex-ante equity premiums, but due to historical "bubbles" in the stock market. The proponents of the bubble view usually cannot quantify the appropriate equity premium, but they do argue that it is lower after recent market run-ups—exactly the opposite of what proponents of the historical averages I method argue.

However, you should be aware that not everyone believes that there were bubbles in the stock-market.

Current predictive ratios

The third method is to try to predict the stock market rate of return actively with historical dividend yields (i.e., the dividend payments received by stockholders). Higher dividend yields should make stocks more attractive and therefore predict higher future equity premiums. The equity premium estimation is usually done in two steps: First, you must estimate a statistical regression that predicts next year's equity premium with this year's dividend yield; then, you substitute the currently prevailing dividend yield into your estimated regression to get a prediction. Sometimes, as in 2008, current dividend yields were so low that the predicted equity premium was negative—which would make no sense. Variations of this method have used interest rates or earnings yields, typically with similar results. In any case, the empirical evidence suggests that this method does not yield great predictions—for example, it predicted low equity premiums in the 1990s, which was a period of superb stock market performance.

Method 3: Dividend or earnings yields.

Philosophical prediction

The fourth method is to wonder how much rate of return is required to entice reasonable investors to switch from bonds into stocks. Even with an equity premium as low as 3%, over 25 years, an equity investor would end up with more than twice the money of a bond investor. Naturally, in a perfect market, nothing should come for free, and the reward for risk-taking should be just about fair. Therefore, equity premiums of 6-8% just seem too high for the amount of risk observed in the stock market. This philosophical method generally suggests equity premiums of about **1%** to **3%**.

Method 4: Introspection and philosophy.

Sidenote: A **bubble** is a runaway market, in which rationality has temporarily disappeared. There is a lot of debate as to whether bubbles in the stock market ever occurred. A strong case can be made that technology stocks experienced a bubble from around 1998 to 2000. It is often called the **dot-com bubble**, the **internet bubble**, or simply the **tech bubble**. There is no convincing explanation based on fundamentals that can explain *both* why the NASDAQ Index climbed from 2,280 in March 1999 to 5,000 by March 2000, *and* why it then dropped back to 1,640 by April 2001.

Consensus survey

Method 5: Just ask!

What to choose? Welcome to the club! No one knows the true equity premium. So, the fifth method is to ask the experts—or anyone else who may *or may not* know. It's the blind leading the blind. The ranges of estimates have varied widely (and they are often also conveniently tilted in the interest of those giving them):

- The Social Security Administration uses an estimate of around 4%.
- The consulting firm McKinsey uses a standard of around 5%.

Analysts' estimates are all over the map, too. Estimates between 2% and 6% per annum seem reasonable.

- Around the turn of the millenium, the most common equity premium estimates recommended by professors of finance were **5%** for a 1-year horizon and **6%** for a 30-year horizon, both with a range from 3% to 8%. The estimates were generally similar in the U.S., Spain, Germany, and the UK.
- On Monday, February 28, 2005, the *Wall Street Journal* reported the following average *after-inflation* forecasts from then to 2050 (per annum):

		Government		Corp.	Equity Premium	
Name	**Organization**	**Stocks**	**Bonds**	Bonds	**Rel Gov**	**Rel Corp**
William Dudley	Goldman Sachs	5.0%	2.0%	2.5%	3.0%	2.5%
Jeremy Siegel	Wharton	6.0%	1.8%	2.3%	4.2%	3.7%
David Rosenberg	Merrill Lynch	4.0%	3.0%	4.0%	1.0%	0.0%
Ethan Harris	Lehman Brothers	4.0%	3.5%	2.5%	0.5%	1.5%
Robert Shiller	Yale	4.6%	2.2%	2.7%	2.4%	1.9%
Robert LaVorgna	Deutsche Bank	6.5%	4.0%	5.0%	2.5%	1.5%
Parul Jain	Nomura	4.5%	3.5%	4.0%	1.0%	0.5%
John Lonski	Moody's	4.0%	2.0%	3.0%	2.0%	1.0%
David Malpass	Bear Stearns	5.5%	3.5%	4.3%	2.0%	1.2%
Jim Glassman	JP Morgan	4.0%	2.5%	3.5%	1.5%	0.5%
				Arithmetic Average (Difference):	2.0%	1.4%
				Volatility-Adjusted Geometric Average ≈ –1% :	1.0%	0.4%

As you already know, it matters (a) whether you quote geometric or arithmetic averages; and (b) whether you quote the equity premium with respect to a short-term or a long-term interest rate. If you want to use the short rate, then you need to add another 1-2% to the equity-premium estimates in this table. (Unrelated, for the equity premium, it does not matter whether equity premium numbers are inflation adjusted. Inflation cancels out, because the equity premium is itself a difference in nominal rates.)

- In 2005, a poll by Graham and Harvey (from Duke) and *CFO Magazine* reported an average equity premium estimate of CFOs of around 3%.
- In mid-2008, Merrill Lynch's survey of 300 institutional investors reported 3%.
- In 2012, Fernandez reported that analysts and companies in the U.S., Spain, Germany and the U.K. all used average estimates between 5% and 6%—just like finance professors, and with the same typical range from about 3% to 8%.

Of course, these estimates are themselves based on the first four methods, they do not take your own cash flow duration into account, and they occur in echo chambers—they are what analysts, companies, consultants, students, and professors have been reading in corporate finance textbooks (like this one) for many years now.

One aspect that does not make sense is that these estimates seem to correlate too strongly with very recent stock market returns. For example, in late 2000, right after a huge run-up in the stock market, surveys by *Fortune* or *Gallup/Paine Webber* had investors expecting equity premiums as high as 15% per year. (They were acutely disappointed: The stock market dropped by as much as 30% over the following two years. Maybe they just got the sign wrong?!)

Internal Cost of Capital (ICC)

Method 6: Ask and Use!

A hybrid method combining survey methods and analysis is the "Internal Cost of Capital." Basically, this uses analysts' consensus projections about S&P 500 earnings over the next few years, and then uses a perpetuity model to back out the cost of capital that makes the price equal to the analysts discounted future earnings. These estimates vary over the business cycle, which is why one usually uses an average ICC over many years. The estimates that come out of these models are about 2.5%-3% per annum relative to 10-year bonds in arithmetic terms, and about 1.5% in geometric terms. (And, as with historical estimates, different variants can give estimates with a much larger range, say from 0% all the way to 7%.)

Conclusion

Remain consistent: Don't use different equity premium estimates for different projects.

You now know that no one can tell you the authoritative number for the equity premium. Such authority does not exist. Everyone is guessing, but there is no way around it—you have to take a stance on the equity premium. I could not shield you from this problem. I could only give you the arguments that you should contemplate when you are picking *your* number. My own take is this: First, I have my doubts that equity premiums will be 8% in the future. (The twentieth century was the "American Century" for a good reason: There were a lot of positive surprises for American investors.) I personally prefer equity premium estimates around **2%**, and this is actually in line with the majority of methods mentioned above. But realize that reasonable expert witnesses can cherry-pick equity premium estimates as low as 1% or as high as 8%. Of course, I personally find their estimates less believable the farther they are from my own personal estimate. And I find anything outside this 1% to 8% range just too tough to swallow. Second, whatever equity premium you do choose, *be consistent*. Do not use 3% for investing in one project and 8% for investing in another similarly-timed project. And do not use a risk-free rate based on long-term bonds as your risk-free rate in the CAPM and an equity premium estimate based on short-term bills. Being consistent can sometimes reduce your relative mistakes in choosing one project over another.

The equity premium is an extremely important number, even without the CAPM.

Yes, the equity premium is difficult to estimate, but there is really no way around your taking a stance. Even if you had never heard of the CAPM, you would still consider the equity premium to be one of the two most important numbers in finance (together with the risk-free rate, the other CAPM input). **If you believe that the equity premium is**

ANECDOTE **Was the 20th Century Really the "American Century?"**

The compound rate of return in the United States was about 8% per year from 1920 to 1995. Adjusted for inflation, it was about 6%. In contrast, an investor who had invested in Romania in 1937 experienced not only the German invasion and Soviet domination, but also a real annual capital appreciation of about –27% per annum over its 4 years of stock market existence (1937–1941). Similar fates befell many other Eastern European countries, but even countries not experiencing political disasters often proved to be less than stellar investments. For example, Argentina had a stock market from 1947 to 1965, even though its only function seems to have been to wipe out its investors. Peru tried three times: From 1941 to 1953 and from 1957 to 1977, its stock market investors lost *all* their money. But the third time was the charm: From 1988 to 1995, its investors earned a whopping 63% real rate of return. India's stock market started in 1940 and offered its investors a real rate of return of just about –1% per annum. Pakistan started in 1960 and offered about –0.1% per annum.

Even European countries with long stock market histories and no political trouble did not perform as well as the United States. For example, Switzerland and Denmark earned nominal rates of return of about 5% per annum from 1920 to 1995, while the United States earned about 8% per annum. A book by Dimson, Marsh, and Staunton looks at 101 years of global investment returns and argue that measurement and hindsight biases can account for much of this superior return.

Nevertheless, the United States stock market was an unusual above-average performer in most of the twentieth century. Will the twenty-first century be the Chinese century? And do Chinese asset prices already reflect this? Or already reflect *too much* of this?

Goetzmann and Jorion (1999)

high, you would want to allocate a lot of your personal assets to stocks. Otherwise, you would allocate more to bonds. You really do need to know the equity premium even for basic investing purposes, too—no escape possible.

The CAPM is about relative pricing, not absolute pricing.

In a corporate context, like every other corporate manager, you cannot let your limited knowledge of the equity premium stop you from making investment decisions. In order to use the CAPM, you do need to judge the appropriate reward for risky projects relative to risk-free projects. Indeed, you can think of the CAPM as telling you the *relative* expected rate of return for projects, not the *absolute* expected rate of return. Given *your* estimate of how much risky average stock market projects should earn relative to safe projects, the CAPM can tell you the costs of capital for projects of a specific beta. But the basic judgment of the appropriate spread between high-beta and low-beta projects is left up to you.

Q 9.13. What are appropriate equity premium estimates? What are not? What kind of reasoning are you relying on?

Investment Projects' Market Betas

Unlike the risk-free rate and the equity premium, beta is specific to each project.

Your third CAPM input is your project's **market beta** (β_i). It measures how the rate of return of your project fluctuates with that of the overall market. Unlike the previous two inputs, which are the same for every project in the economy, the beta input depends on your specific project characteristics: Different investments have different betas.

The Implications of Beta for a Project's Risk and Reward

Projects with higher betas have more market risk, so their own idiosyncratic variances tend to be higher, too.

You already understand the role of market beta in determining the expected rate of return for an asset. This is the security market line—that is, the CAPM formula itself is an upward-sloping line when the expected rate of return is plotted against beta. But market beta also has implications for the standard deviation of assets. First, note that assets with a low beta are not very exposed to market risk. Thus, assets that have either a very high or a very low market beta tend to have higher standard deviation. Second, note that you can only learn much about an asset's market-beta in months in which the market does not turn in the same performance as the risk-free security. If the market and the risk-free asset turn in the same performance in a given month, then *any* asset's expected rate of return is just the risk-free rate, regardless of its market-beta.

Beta Estimation

Ways to estimate beta.

How do you find good forward-looking market-beta estimates for your own project? As usual, when we do not know the input, we rely on statistical analysis of past data. The mechanics of finding the beta for a stock are easy. You run a **market-model** regression on historical stock returns. The independent variable is the rate of return on the stock-market (the S&P500 percent change, even without dividends, is usually good enough). The dependent variable is the rate of return on your project. Usually, you should run such regressions with daily rather than with monthly returns and you should use about 3-5 years of data. Any statistical package (and common computer spreadsheet programs) readily give you the regression coefficients. The slope is the historical market-beta.

Unfortunately, although estimates of future betas are better than estimates of the future equity premium, they are still not great. The reason is that stock returns are very, very noisy. (And projects are rarely the same as stock, and project and stocks both often change their characteristics over time, too, but let's ignore this for the moment.) Thus, statisticians recommend that you should "shrink" your beta estimates further. **Shrinking** comes in two forms:

- Instead of using your own historical rates of returns, use the historical rates of return on a broader portfolio. For example, if you want to estimate the future market-beta of AMD, do not use the historical rates of return of AMD in your market-model, but those of the "computer hardware sector" instead. In other words, assume that all computer hardware makers have about the same stock market beta, and that AMD's own future beta will look more like that of its sector in the past than like that of its own past.
- Instead of using the coefficient estimate from the regression, use an average between the regression estimate and the number "1" (which is the average of

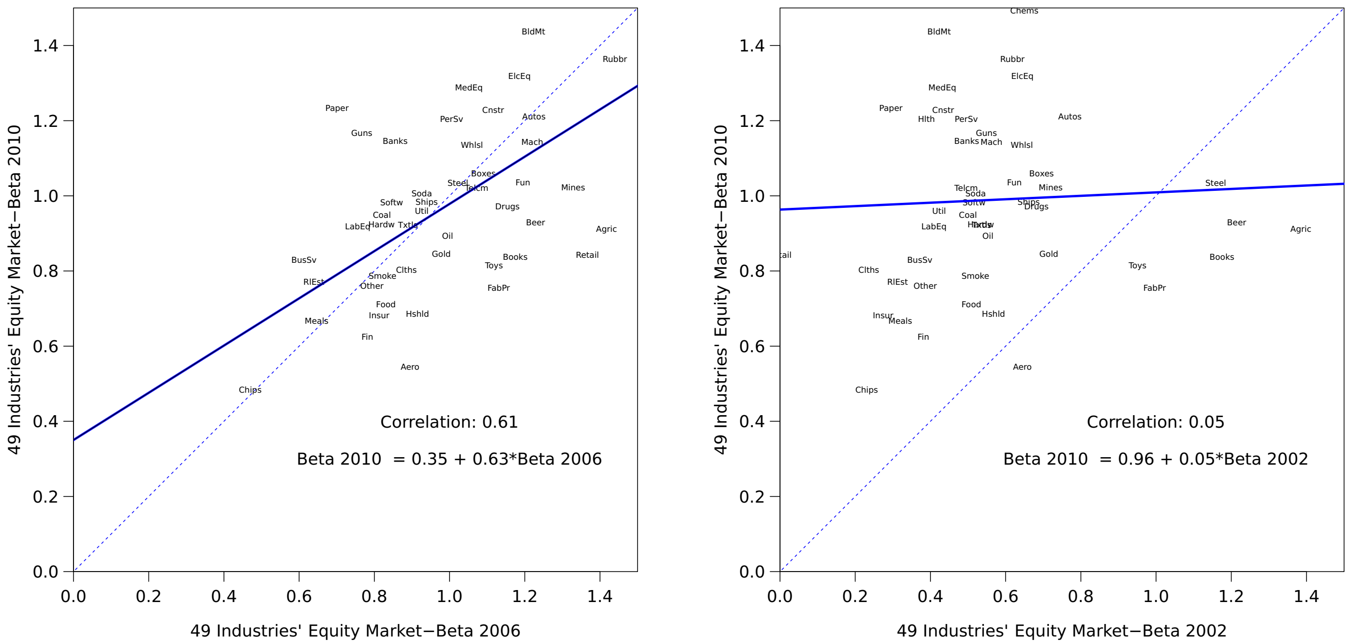


Exhibit 9.4: *Betas For 49 Industries Far Into The Future.* These figures plot industry market betas at the end of 2010 against their own value a few years earlier. Industries that had high market-betas in 2006 still tended to have high market-betas in 2010—although you should have not have used your exact estimates but shrunk them towards 1 to reflect their tendency to mean-revert. In contrast, industries that had high market-betas in 2002 unfortunately did not have high market-betas in 2010. If you had to guess market-betas in 2002 for 2010, you may as well have guessed the same value for every industry, ignoring the prevailing 2002 market-betas. The 0.05 coefficient is unusually low. In other eight-year samples, it was more like 0.3. Data Sources: 49 industries from Fama-French. Betas from 3 years of daily data.

all stock's market-betas). For example, if your market-model coefficient estimate based on past data is 2.6, use $1/2 \cdot 2.6 + 1/2 \cdot 1.0 = 1.8$ for your estimate of the future. Many studies have confirmed that such shrunk market-betas perform better in predicting subsequent market-betas than the unshrunk coefficient estimates themselves. The market-betas that are posted on many websites, such as YAHOO! FINANCE, are also shrunk.

Unfortunately, while these two shrinkages combined work reasonably well for predicting stock market-betas over the next quarter, they do not work so well for predicting stock market betas for cash flows that will occur in many years. Figure 9.4 shows how the stock market-betas for 49 different industries and then shrunk again. These industry betas typically range from about 0.3 to about 1.5, but change over time. The left panel shows that 2006 market-betas were still similar to those in 2010. The right panel shows that 2002 market-betas were not. (The left panel was better than usual, the right panel was worse than usual.) Based on a more detailed statistical study, my advice is to shrink the market-betas for cash flows in more than 2-5 years a second time. In our example of an industry market-beta of 2.6, shrunk once to 1.8 for cash flows that occur within the next year, if you had to assess the market betas of cash flows in about 5 to 15 years, you would shrink your beta a second time, say to $1/2 \cdot 1.8 + 1/2 \cdot 1.0 = 1.4$.

Unfortunately, as a corporate manager, you are rarely interested in the market-beta of an industry or even a stock. Usually, you are interested in the market-beta of a new project that you are considering. Sometimes, your firm is not even publicly traded, so you would not even have historical data if you wanted to. (And, if not publicly traded, then it is quite possible that your investors would not have been fully diversified, which is an essential assumption in the CAPM. If your main investor is undiversified, you may care about idiosyncratic standard deviation more than about the market-beta.) In this case, corporate CAPM users must thus rely more on economic intuition than pure statistics. You can rearrange the CAPM formula to obtain a beta estimate. Now, do you think your project cash flows and its future project value (which is influenced by changes in the economy) is likely to move more or less with the overall stock market (and, possibly, the overall economy)?

$$\mathscr{E}(r_i) = r_F + [\mathscr{E}(r_M) - r_F] \cdot \beta_i \iff \beta_i = \frac{\mathscr{E}(r_i) - r_F}{\mathscr{E}(r_M) - r_F}$$

The right side of this formula helps translate your intuition into a beta estimate. What rate of return (above the risk-free rate) will your project have if the market were to have +10% or –10% rate of return (above the risk-free rate)? Clearly, such guesswork is difficult and error-prone—but it can provide a beta estimate when no other is available. Or, perhaps you can "start" with an industry market-beta and shrink it appropriately, perhaps adjusting for the fact that some (smaller) firms typically have higher betas?

Equity and Asset Betas Revisited

Don't use the equity beta to estimate your project's hurdle rate. Use the asset beta instead.

No matter how good your estimates of your stock betas are, it is important that you always distinguish between asset betas and equity betas. Let me remind you with an example. Assume that the risk-free rate is 4% and the equity premium is 5%. You own a $100 million project with an asset beta of 2.0 that you can finance with $20 million of risk-free debt. By definition, risk-free debt has a beta of 0. To find your equity beta, write down the formula for your asset beta (firm beta):

➤ *Asset and equity betas*, Formula 8.7, Pg.212.

$$\underbrace{\beta_{Firm}}_{2.0} = \underbrace{\left(\frac{\text{Debt value}}{\text{Firm value}}\right)}_{20\%} \cdot \underbrace{\beta_{Debt}}_{(0)} + \underbrace{\left(\frac{\text{Equity value}}{\text{Firm value}}\right)}_{80\%} \cdot \underbrace{\beta_{Equity}}_{(\beta_{Equity})}$$

Solve this to find that your equity beta is 2.5. This is what you would find on YAHOO! FINANCE. You would not want to base your hurdle rate for your firm's typical average project on the equity beta: Such a mistake would recommend you use a hurdle rate of $\mathscr{E}(r_i) = r_F + [\mathscr{E}(r_M) - r_F] \cdot \beta_i = 4\% + 5\% \cdot 2.5 = 16.5\%$. This would be too high. Instead, you should require your average projects to return $\mathscr{E}(r_i) = 4\% + 5\% \cdot 2.0 = 14\%$.

➤ *Typical, average, and marginal betas*, Sect. 12.3, Pg.343.

If you use comparables, first unlever them.

Conversely, if your project is private but the potential future owners are well-diversified, you may have to find its hurdle rate by looking at public comparables. Let's presume you find a similarly-sized firm with a similar business that YAHOO! FINANCE lists with a beta of 4, or perhaps better yet, the firm's industry. Remember that financial websites always list only the equity beta. The CAPM tells you that the expected rate of return on the equity is $4\% + 5\% \cdot 4 = 24\%$. However, this is not necessarily the hurdle rate for your project. When you look further on YAHOO! FINANCE, you may notice that your comparable is financed with 90% debt and 10% equity. (If the comparable had very little debt, a debt beta of 0 might have been a good assumption, but, unfortunately, in this case it is not.) Corporate debt rarely has good historical return data that would allow you to estimate a debt beta. Consequently, practitioners often estimate the expected rate of return on debt via debt comparables based on the credit rating. Say your comparable's debt is rated BB and say that BB bonds have offered *expected* rates of return of 100 basis points above the Treasury. (This might be 200 basis points *quoted* above the Treasury). With the Treasury standing at 4%, you would estimate the comparable's cost of capital on debt to be 5%. The rest is easy. The expected rate of return on your project should be

➤ *Credit ratings, Sect.* 6.2, *Pg.130*.

$$\mathscr{E}(r_{\text{Project}}) = 90\% \cdot 5\% + 10\% \cdot 24\% = 6.9\%$$

$$= w_{\text{Debt}} \cdot \mathscr{E}(r_{\text{Debt}}) + w_{\text{Equity}} \cdot \mathscr{E}(r_{\text{Equity}})$$

This would make a good hurdle rate estimate for your project.

Q 9.14. According to the CAPM formula, a zero-beta asset should have the same expected rate of return as the risk-free rate. Can a zero-beta asset still have a positive standard deviation? Does it make sense that such a risky asset would not offer a higher rate of return than a risk-free asset in a world in which investors are risk averse?

Q 9.15.A comparable firm (with comparable size and in a comparable business) has a YAHOO! FINANCE–listed equity beta of 2.5 and a debt/asset ratio of 2/3. Assume that the debt is risk free.

1. Estimate the equity beta for your firm if your projects have similar betas, but your firm will carry a debt/asset ratio of 1/3.
2. If the risk-free rate is 3% and the equity premium is 2%, then what should you use as your firm's hurdle rate?
3. What do investors demand as the expected rate of return on the comparable firm's equity and on your own equity?

Q 9.16. You own a stock portfolio that has a market beta of 2.4, but you are getting married to someone who has a portfolio with a market beta of 0.4. You are three times as wealthy as your future significant other. What is the beta of your joint portfolio?

9.5 Is the CAPM the Right Model?

Q: What happens if a stock offers too much or too little expected rate of return? A: Investor stampedes.

Now you know how securities should be priced in a perfect CAPM world, in which investors have good knowledge of the parameters. What would happen if a stock offered more than its appropriate expected rate of return? Investors in the economy would want to buy more of the stock than would be available: Its price would be too low. It would be too good a deal. Investors would immediately flock to it, and because there would not be enough of this stock, investors would bid up its price and thereby lower its expected rate of return. The price of the stock would settle at the correct CAPM expected rate of return. Conversely, what would happen if a stock offered less than its due expected rate of return? Investors would not be willing to hold enough of the stock: The stock's price would be too high, and its price would fall. Neither situation should happen in the real world.

Assets not priced according to the CAPM do not allow you to make money for nothing. However, it could imply good deals.

Is this an arbitrage—a "free money situation"? No. When stocks do not to follow the CAPM formula, buying them is still risky. Yes, some stocks would offer a higher or lower expected rate of return and thus seem to be too good or too bad a deal, attracting too many or too few investors chasing a limited amount of value in this stock—but these stocks would still remain risky investments. No investor could earn risk-free profits. There is no arbitrage here. The market forces working on correcting the (CAPM) mispricing are modest. And remember that there are good reasons why the CAPM may not hold in the first place, too. For example, it relies on many perfect-market assumptions. If investors are taxed or liquidity-constrained (that is, they cannot easily diversify, e.g., because the firm is a startup or family firm) or do not agree on the inputs, then it is quite plausible that some firms or even sectors (such as "value firms" or "growth firms") would offer higher or lower expected rates of return than the CAPM suggests.

What is The Scientific Evidence?

Why use the CAPM?

Unfortunately, in real life, despite its wide use, the evidence in favor of *practical use and application* of the CAPM is either weak or non-existent. If you use the CAPM, you do so based primarily on a belief that it should work, not based on empirical evidence. Say again: the evidence suggests that, even if the CAPM held, input estimates for corporate cash flows that will occur far in the future are usually so imprecise that they render the CAPM practically useless.

Huh? Did you really read me right?

If there is no empirical evidence that CAPM use is justified, then why do we torture you with it? This is a much easier question to answer than how stocks are priced in the real world or what the best estimate of the appropriate hurdle rates for your project is.

The CAPM is based on the important concept of diversification.

Good intuition: The CAPM has impeccable intuition. It is a model that shines through its simplicity and focuses on what *should* matter when owners are many—diversification. It gets executives away from the false notion that many small public investors care about the idiosyncratic risk of projects that the investors can diversify away. It also helps you understand that corporate diversification into a conglomerate is not likely to add value. Your investors can diversify themselves.

They don't need your firm to diversify you for them. And, it explains nicely why stocks should have higher rates of returns than bonds and how to "lever" and "unlever" assets. In general, it is a nice conceptual framework that helps you think about what should matter.

Faith.

Strong Belief: Many instructors and practitioners find the CAPM to be so plausible that they are willing to live with "absence of CAPM evidence." They do not take this absence to mean "evidence of CAPM absence." Thus, they adopt the CAPM based on their prior belief and faith, not based on evidence. Doing this is acceptable as long as you are fully aware that this is really what you are doing. (However, even if you do adopt the CAPM and even if this is not a Rumsfeld-level blunder, you still have to realize that you should greatly shrink your beta and equity-premium inputs for long-term cash flows.)

A crutch

Standin for Expected Cash Flow Default: The CAPM often assigns higher costs of capital to projects that are more likely to fail. If you have not fully adjusted your expected cash flow estimates downwards to adjust for failure (a common human error), the CAPM cost of capital often helps to impose a higher hurdle rate on riskier cash flows.

Important: Everyone expects you to know the CAPM!

Everyone uses it: The CAPM is *the* standard. Exhibit 9.5 shows that 73% of the CFOs reported that they always or almost always use the CAPM. (And use of the CAPM was even more common among large firms and among CFOs with an MBA.) No alternative method was used very often. Consequently, you have no choice but to understand the CAPM model well—*if you will work for a corporation, then the CAPM is the benchmark model that your future employer will likely use and will expect you to understand well*. Again, the CAPM is simply *the* standard. The CAPM is also used as a benchmark by many investors rating their (investment) managers, by government regulatory commissions, by courts in tort cases, and so on. It is literally the dominant, if not the only, widely-used model to estimate the cost of capital. Indeed, there is a whole section on the CFA exam about the CAPM!

There is no generally-used alternative to the CAPM.

Alternatives—please stand up: The famous sociologist Lewin wrote that "there is nothing more practical than a good theory." If not the CAPM, then what else would you use? There are no commonly-accepted alternatives. (A related justification for the CAPM has been that we consider the CAPM like linguists consider Latin—a good language that prepares you well to learn other languages that descended from it. The problem is that the CAPM-descendant models don't work well, either. At best, they are so flexible and slippery that we cannot know whether they work or not. At worst, they or their use has been rejected by the data, too.)

Do you want a bedtime story that "the world is ok" in order to be able to go to sleep?

Be aware that my treatment of the CAPM in an introductory corporate-finance textbook borders on heresy. Most corporate finance text-books make the CAPM their centerpiece. They do this not because the authors believe in it, but because it is dogma that new finance students are too fragile to deserve the hard truth. I am sorry—I wish I could have told you a happy bed-time story about how the world is nice and orderly, too. But it would have been a lie.

Never make the following errors, please.

Now, if you still want to use the CAPM, here is my advice. As a corporate executive, you should always first think hard about when you want to use the CAPM. Think about

Method	Usage Frequency	Usage Recommendation	Explained in
CAPM	(73%)	With Caution	Chapter 9
Historical Average Returns	(39%)	Rarely	Chapter 8
Modified CAPM	(34%)	With Caution	Chapter 9
Backed Out from Gordon Model	(16%)	Occasionally	Chapter 3
Whatever Investors Tell Us	(14%)	Occasionally	Chapter 2

Exhibit 9.5: *CFO Valuation Techniques for the Cost of Capital.* Rarely means “usually no, and often used incorrectly.” Not reported, use of the CAPM is more common among managers with an MBA—and in firms who rely on consultants who in turn use the CAPM. Original Source: John Graham and Campbell Harvey, 2001.

A N E C D O T E **“Cost of Capital” Expert Witnessing**

When Congress tried to force the “Baby Bells” (the split-up parts of the original AT&T) to open up their local telephone lines to competition, it decreed that the Baby Bells were entitled to a fair return on their infrastructure investment—with fair return to be measured by the CAPM. (The CAPM is either the de facto or legislated standard for measuring the cost of capital in many other regulated industries, too.) The estimated value of the telecommunication infrastructure in the United States is about $10 to $15 billion. A difference in the estimated equity premium of 1% may sound small, but even in as small an industry as local telecommunications, it meant about $100 to $150 million a year—enough to hire hordes of lawyers and valuation consultants opining in court on the appropriate equity premium. Some of my colleagues bought nice houses with the legal fees.

I did not get the call. I lack the ability to keep a straight face while stating that “the equity premium is exactly *x* point *y* percent,” which was an important qualification for being such an expert. In an unrelated case in which I did testify, the opposing expert witness even explicitly criticized my statement that my cost-of-capital estimate was an imprecise range—unlike me, he could provide an exact estimate, and it was 11% per year!

Bradford Cornell, UCLA

whether it is useful for your own cost-of-capital estimates, or whether the CAPM errors seem too large to be useful for your particular needs. Here is what I would definitely warn about:

Don't expect accuracy and don't use it for financial investing.

Accuracy: The CAPM is a poor model if you want precision. If you believe that CAPM expected rates of return should be calculated with any digits after the decimal point, then you are deluded. Please realize that, at best, the CAPM can only offer expected rates of return that are of the “right order of magnitude,” plus or minus a few percentage points perhaps. Actually, if accuracy and precision are important, you are in trouble. We do not have *any* models that can offer it. (Fortunately, it is often less important to be accurate than it is to be *better* estimating value than your competitors. And always remember that valuation is as much an art as it is a science.)

Avoid using the CAPM for financial investment purposes.

Investment purposes: If you are not a corporate executive looking to determine your project hurdle rate, but a financial investor looking for good investments from the universe of financial instruments, with an ability to shift your money around every day, then please do not use the CAPM. Although the CAPM offers the correct intuition that wide diversification needs to be an important part of *any* good investment strategy, there *are* many better investment strategies than just investing in the market index. Some are explained in Section App.9.C (Companion); more will be discussed in an advanced investments course.

➤ *Mean-variance optimization in detail,* *Sect.* App.8.C (Companion), *Pg.*≈*35*.

Please do not confuse the CAPM with the mean-variance framework discussed in the previous chapter. Mean-variance optimization is an asset-selection technique for your individual portfolio, and it works, regardless of whether or not the CAPM holds.

➤ *Corporate Time-Varying Costs of Capital,* *Sect.* 5.5, *Pg.112*.

Long-Term Differences: If you are a corporate executive, be cautious. Look at your cost of capital more holistically. The CAPM has two terms.

The first term is the risk-free rate which applies to all projects, regardless of beta. Fortunately, there is great evidence what you should use. You should use higher costs of capital for cash flows that will occur in the more distant future. And you have a great estimate of the premium that long-term projects need to offer over short-term projects, based on the Treasury yield-curve. You don't even need historical estimates: you can use the prevailing Treasury yield curve. *Use it! It works!*

It is the second term (the beta multiplied by the risk-premium), i.e., your beta risk-adjustment, that is dubious. If your cash flows will occur in many years, be modest. Do not overstate the risk-inputs in the CAPM. Shrink and shrink again.

- As a corporate manager, compare the cost of capital on *your equity* vs. the cost of capital on *your debt* for your long-term cash flows. With an equity premium based on the performance of stocks vs. long-term Treasuries of about 1-2% from 1970 to today, it may not matter much whether your project A has a beta of 0.8 and your project B has a beta of 1.2. The implied cost-of-capital difference between these two projects of under $(1.2-0.8)\cdot 2\% \approx 1\%$/year is already small.
- For long-term cash flows, your best estimate of your equity market-betas should be tilted much more towards 1 than what you think your market-beta is today. Thus, if you fit your historical market-beta to be 0.5 for A and 1.5 for B today, you may well want to use a market-beta shrunk to around 0.9 for A and 1.1 for B if those equity cash flows will occur in 10-20 years. Think about this: A and B would now have a different implied cost of equity capital of $0.2\cdot 2\% \approx 0.4\%$. This is way below your noise-and-uncertainty threshold.

  Asset betas are often even closer—they often give it time-stability, though.

  But let's continue. Say your projects are partly debt-financed, too. Now you need to calculate asset-betas rather than equity betas. Let's say both projects have 50% debt that is almost risk-free. Then your asset beta would be $0.5\cdot 0.0 + 0.5\cdot 0.9 = 0.45$ for A and $0.5\cdot 0.0 + 0.5\cdot 1.1 = 0.55$ for B.

Now you have a project cost of capital difference $(0.55 - 0.45) \cdot 2\% \approx 0.2\%$ between A and B.

How does this expected rate of return difference between A and B compare to your own uncertainty about your projects' relative expected cash flows? Does the CAPM beta risk-adjustment really matter much in light of your uncertainty?

The estimated CAPM cost of capital for long-term cash flows are fragile.

Alternatives

Let me summarize what I believe the data do tell us that is solid enough a rock to build a house on it:

What is solid empirical evidence?

- There definitely is a time-value of money.
- There definitely is a term structure. Long-term cash flows usually require higher costs of capital than short-term cash flows. Your investors can earn higher expected rates of return elsewhere for longer-term commitments, too.
- There definitely is a credit component. Assets with higher probabilities of default have to make up for it with higher promised yields; that is, higher yields when they succeed.
- As a preview to Chapter 10, market imperfections seem to play a role. There seems to be a liquidity premium. Assets that can be quickly liquidated in a market crash are more expensive, and different asset classes seem to have different degrees of liquidity. Because of their collateral, mortgage debt tend to have lower costs of capital than general bonds. Firms with less access to capital markets, such as startups, seem to pay higher costs of capital, although adjusting for default makes this difficult to measure. Investors pay more in personal income tax for interest receipts than they do for capital gains, which makes equities relatively more desirable and reduces their after-tax income. And sentiment and agency considerations seem to play a role in equity trading that is not unimportant. Many of these market imperfections embody some concept of risk, but it is not the market-beta.

➤ *Market Imperfections, Chapter* 10, *Pg.257*.

- After taking into account the premia just mentioned, the remaining equity premium is probably relatively small (1-2%), although we do not know for sure. Our uncertainty is much larger than our certainty about its magnitude. And you need to realize that betas for cash flows far into the future are much closer to 1 than historical regressions would suggest. The "CAPM" beta impact is relatively unimportant.

So what would I do if I was not constrained by my boss? My best alternative cost-of-capital recommendation would start out just like the CAPM: As the first term in a formula, I would recommend that you use the rate of return on bonds of similar maturity as the cash flow that you want to value. Usually, this means that you assign higher costs of capital to cash flows farther in the future. It is only on the second term, the equity risk-adjustment, that I would tinker. Instead of the (shrunk) CAPM market-beta multiplied by the historical equity premium (of 2% or less per annum), I would recommend a more holistic approach.

Use reasonable risk adjustments.

- Take into consideration that projects with high volatility and/or with high leverage are more risky. The equity on these projects probably requires a higher expected rate of return to keep your investors happy. Projects with higher idiosyncratic risk are also the same projects where executives are often the most over-optimistic. (Check again: are you sure your expected cash flows in the NPV numerator are not over-confident?)
- Take into consideration whether you and your owners are well-diversified. If you are not, then you should require higher rates of return on riskier projects. In this case, it is not "beta risk" that matters, but "total risk."
- Take into consideration that your investors may "like" growth firms and are often willing to pay higher prices and thus accept lower average rates of returns for some such projects.

There is little harm if you calculate a (repeatedly-shrunk) CAPM market-beta with a low equity premium (say 2%) to assess whether any other non-CAPM cost-of-capital assessments seems reasonably similar to your CAPM assessment. In this sense, the CAPM can still be a little helpful. Finally, realize that it is in general very difficult to assess over many years whether corporate projects will offer higher or lower average rates of return than the average project in the economy. If you make smart decisions, after your project's initial growth phase is over, would it be reasonable to assume that it will earn similar rates of returns as most other good projects in the economy—not better, not worse?

➤ *Long-Run Excess Profits,* *Sect.* 20.3, *Pg.672*.

What would I do if the boss liked the CAPM?

And if my boss required an approach like the CAPM, what would I do?

- If I ran a large firm with good access to capital markets, I would assume an equity premium of 1-2% per annum and apply this to the equity components of all my long-term cash flows. The exception would be projects for which I would have a strong prior that their market-betas will be very extreme, say, below –1 or greater than 3 (and I would then shrink those betas further to, say, 0 and 1.5, respectively, to account for long-term uncertainty about betas). I would consider long-term corporate debt to have a higher cost of capital than equivalent Treasuries but a lower cost of capital than my own equity—the latter primarily because debt provides a corporate income tax shield (as you will learn in Chapter 17) and not because the equity premium over long-term corporate bonds is high.
- Deviating from the CAPM, if I ran a startup firm, I would assume a cost of capital of 2% to 6% above the *expected* rate of return on my uncollateralized debt. The expected rate of return on my debt could be very high—it could even be in the double digits. (This reflects the fact that more volatile cash flows and firms that struggle with more market imperfections must pay higher costs of capital.) Risk definitely plays a role, but not in the strict CAPM market-beta sense. Alternatively, I would abandon NPV-based models altogether and try to estimate what other similar projects are offering their investors. This is the route we take in Chapter 14.

➤ *Income Taxes and Cost of Capital,* *Chapter* 17, *Pg.545*.

➤ *Comparables,* *Chapter* 14, *Pg.431*.

And I would never use any of my schemes here (or the CAPM) for the pricing of bonds, derivatives, or other extreme kinds of projects.

Am I the only professor who recommends against using the CAPM? No. Eugene Fama, perhaps the most famous active finance professor alive and partly responsible for the original spread of the CAPM, nowadays strongly recommends against the combined use of NPV models with asset-pricing models like the CAPM, where you use the CAPM expected rate of return as your cost of capital in an NPV calculation. Such use means you divide one uncertain number by another. This practice combines your errors and uncertainty about expected cash flows in the numerator with your errors and uncertainty about expected returns in the denominator. Yikes!

NPV or Comparables? Eugene Fama thinks Comparables are better.

Conclusion

IMPORTANT

- The CAPM is the benchmark model in the real world. Most corporations use it.
- Everyone will expect you to understand the CAPM. Regardless of whether the model holds or not, you have to know it.
- The empirical evidence suggests that the CAPM is not a great model for predicting expected rates of return.
- The first CAPM term (that long-term projects have to offer higher expected rates of return) seems to hold better than the second CAPM term (the risk adjustment).
- For cash flows many years into the future, you must realize (a) that market-betas revert back towards 1 and (b) that the equity premium is low.
- The CAPM never offers great accuracy.
- Mean-variance optimization (Section 8.2) works even if the CAPM does not.

Q 9.17. Does the empirical evidence suggest that the CAPM is correct?

Q 9.18. If the CAPM is wrong, why do you need to learn it?

Q 9.19. Is the CAPM likely to be more accurate for a project where the beta is very high, one where it is very low, or one where it is zero?

Q 9.20. To value an ordinarily risky project, that is, a project with a beta in the vicinity of about 1, what is the relative contribution of your personal uncertainty (lack of knowledge) in (a) the risk-free rate, (b) the equity premium, (c) the beta, and (d) the expected cash flows? Consider both long-term and short-term investments. Where are the trouble spots?

Summary

This chapter covered the following major points:

- The CAPM provides an "opportunity cost of capital" for investors, which corporations can use as the cost of capital in the NPV formula. The CAPM formula is

  $$\mathscr{E}(r_i) = r_F + [\mathscr{E}(r_M) - r_F] \cdot \beta_i$$

  Thus, there are three inputs: the risk-free rate of return (r_F), the expected rate of return on the market ($\mathscr{E}(r_M)$), and the project's or firm's market beta (β_i). Only the latter is project-specific.
- The line plotting expected rates of return against market beta is called the security market line (SML).
- The CAPM provides an expected rate of return, consisting of the time premium and the risk premium. It ignores the default premium. In the NPV formula, the default risk and default premium work through the expected cash flow in the numerator, not through the expected rate of return (cost of capital) in the denominator.
- For r_F, you should use bonds that match the timing of your project's cash flows. Thus, cash flows farther in the future often require higher opportunity costs of capital. Even if you do not believe the CAPM, term adjustment is important.
- The expected rate of return on the market is a critical CAPM input if market beta is high—but it is difficult to guess. There are many guesstimation methods, but no one really knows which one is best. Reasonable estimates for the equity premium ($\mathscr{E}(r_M) - r_F$) can range from about 1% to 8% per annum, although 2% seems most reasonable to me for cash flows more than a few years into the future.
- There are a number of methods to estimate market beta. Many users rely on industry betas and not on firms' own historical betas as estimates of future market betas, *and* they shrink them towards 1. When your cash flows are farther in the future, you have to shrink your beta estimates even more drastically towards 1.
- Never believe the CAPM blindly. Its estimates are poor. Use it more like a "general direction" estimate than like an "accurate guide" estimate.
- Even though its estimate are poor, understand the CAPM well. Everyone will expect you to.
- The chapter appendix discusses certainty equivalence and CAPM alternatives (such as the APT and the Fama-French-Momentum model). You must use the certainty equivalence form of the CAPM when projects are purchased or sold for prices other than their fair market values. It is also often the only method if only underlying cash flows rather than value estimates are available.

This negative perspective on the CAPM is so uncommon in a textbook (but not among the experts actually studying the models) that it is important that you don't misunderstand it. Let's end this chapter with a FAQ:

- **Q**: Should riskier projects not have to promise higher rates of return?

  A: Riskier projects have to promise a higher rate of return, i.e., offer higher default premiums. This is not the same as higher risk premiums in the CAPM sense. In NPV applications, make sure to reflect the default risk in the expected cash flow numerator. Riskier projects need to pay off a lot more when they succeed, just to make up for the fact that they fail more often.

- **Q**: Should riskier long-term cash flows not require higher expected rates of return?

  A: Long-term projects command term premiums. Thus, in NPV applications, you should usually use higher required costs of capital for more distant cash flows. You can but do not need the CAPM for this. The U.S. Treasury Yield Curve gives you a working first estimate about how much extra premium long-term cash flows should require.

- **Q**: Should riskier stocks and cash flows have higher expected discount rates?

  A: Maybe, but be careful. First, make it modest. Don't be too overconfident in your ability to judge equity risks. If you can judge the risks well, make sure your estimates first flow into your expected cash flows in the NPV numerator. Second, don't be too wedded to the CAPM for an extra "risk-premium kicker." Instead, combine your cost-of-capital estimate with judgment-based and other risk measures, such as volatility (especially if your owners are not fully diversified).

Preview of the Chapter Appendix in the Companion

In the Appendix

The appendix to this chapter explains

- the "certainty equivalence value" (CEV) which allows you to use the CAPM for projects that you are not buying at the appropriate equilibrium price. For example, you need the CEV to work out how to value an inheritance that will be higher if your business fails. (Being free today does not mean that there is no value to such a promise.)
- how to use the CEV formula to estimate the value of a project for which you have historical cash flows, but no market value information.
- how the CAPM is derived from the fact that the optimal portfolio is always the combination of two portfolios, one of which may be the risk-free asset.
- what the CAPM alternatives are and how to use them. The first alternative is the APT (arbitrage pricing theory) and its relative, the Intertemporal CAPM. The

second alternative are Fama-French value and momentum models. These seem to predict better than any alternatives, but are less grounded in theory (or, you may say, reason) than the former.

Keywords

Answers

Q 9.1 Yes, the perfect market is an assumption underlying the CAPM. In addition,

1. Investors are rational utility maximizers.
2. Investors care only about overall portfolio mean rate of return and risk at one given point in time.
3. All parameters are known (not discussed until later in the chapter).
4. All assets are traded. Every investor can purchase every asset.

Q 9.2 With $r_F = 4\%$ and $\mathcal{E}(r_M) = 7\%$, the cost of capital for a project with a beta of 3 is $\mathcal{E}(r) = r_F + [\mathcal{E}(r_M) - r_F] \cdot \beta_i = 4\% + (7\% - 4\%) \cdot 3 = 13\%$.

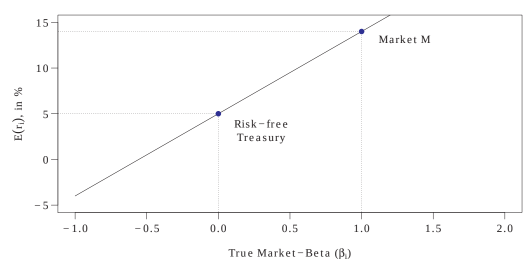
Q 9.3 With $r_F = 4\%$ and $\mathcal{E}(r_M) = 12\%$, the cost of capital for a project with a beta of 3 is $\mathcal{E}(r) = r_F + [\mathcal{E}(r_M) - r_F] \cdot \beta_i = 4\% + (12\% - 4\%) \cdot 3 = 28\%$.

Q 9.4 With $r_F = 4\%$ and $\mathcal{E}(r_M) = 12\%$, the cost of capital for a project with a beta of –3 is $\mathcal{E}(r) = r_F + [\mathcal{E}(r_M) - r_F] \cdot \beta_i = 4\% + (12\% - 4\%) \cdot (-3) = -20\%$. Yes, it does make sense that a project can offer a negative expected rate of return. Such projects are such great investments that you would be willing to expect losses on them, just because of the great insurance that they are offering.

Q 9.5 No—the real-world SML is based on historical data and not true expectations. It would be a scatterplot of historical risk and reward points. If the CAPM holds, a straight, upward-sloping line would fit them best.

Q 9.6 Write down the CAPM formula and solve $\mathcal{E}(r_i) = r_F + [\mathcal{E}(r_M) - r_F] \cdot \beta_i = 4\% + (7\% - 4\%) \cdot \beta_i = 5\%$. Therefore, $\beta_i = 1/3$. Note that we are ignoring the promised rate of return.

Q 9.7 The security market line is



Q 9.8 The equity premium, $\mathcal{E}(r_M) - r_F$, is the premium that the market expects to offer on the risky market above and beyond what it offers on Treasuries.

Q 9.9 It does not matter what you choose as the per-unit payoff of the bond. If you choose \$100, you expect it to return \$99.

1. Thus, the price of the bond is $PV = \$99/(1 + [3\% + 5\% \cdot 0.2]) \approx \$95.19$.
2. Therefore, the promised rate of return on the bond is $\$100/\$95.19 - 1 \approx 5.05\%$.
3. The risk-free rate is 3%, so this is the time premium (which contains any inflation premium). The (expected) risk premium is 1%. The remaining 1.05% is the default premium.

Q 9.10 The cost needs to be discounted with the current interest rate. Because payment is up-front, this cost is \$30,000 now! The appropriate expected rate of return for cash flows (of your earnings) is $3\% + 5\% \cdot 1.5 = 10.5\%$. You can now use the annuity formula to determine the PV if you graduate:

$$\frac{\$5{,}000}{10.5\%} \cdot \left[1 - \left(\frac{1}{1+10.5\%}\right)^{40}\right] \approx \$47{,}619 \cdot 98.2\%$$
$$\approx \$46{,}741.46$$

With 90% probability, you will do so, which means that the appropriate risk-adjusted and discounted cash flow is about \$42,067.32. The NPV of your education is therefore about \$12,067.32.

Q 9.11 Use the 1-year Treasury rate for the 1-year project, especially if the 1-year project produces most of its cash flows at the end of the year. If it produces constant cash flows throughout the year, a 6-month Treasury rate might be more appropriate. Because the 10-year project could have a duration of cash flows much shorter than 10 years, depending on use, you might choose a risk-free Treasury rate that is between 5 and 10 years. Of course, it would be even better if you match the individual project cash flows with individual Treasuries.

Q 9.12 The duration of this cash flow is around, or a little under, 5 years. Thus, a 5-year zero-coupon U.S. Treasury would be a reasonably good guess. You should not be using a 30-day or 30-year Treasury. A 10-year zero-coupon Treasury would be a better match for a project that yields cash only once at the end of 10 years. That is, for our project that has cash flows each year for 10 years, the 10-year Treasury as a benchmark would have too much of its payments as principal repayment at the end of its 10-year term.

Q 9.13 An estimate between 1% and 8% per year is reasonable. Anything below 0% and above 10% would seem unreasonable to me. For reasoning, please see the different methods in the chapter.

Q 9.14 Yes, a zero-beta asset can still have its own idiosyncratic risk. And, yes, it is perfectly kosher for a zero-beta asset to offer the same expected rate of return as the risk-free asset. The reason is that investors hold gazillions of assets, so the idiosyncratic risk of the zero-beta asset will just diversify away.

Q 9.15 This is an asset beta versus equity beta question. Because the debt is almost risk free, we can use $\beta_{\text{Debt}} \approx 0$.

1. First, compute an unlevered asset beta for your comparable with its debt-to-asset ratio of 2 to 3. This is $\beta_{\text{Asset}} = w_{\text{Debt}} \cdot \beta_{\text{Debt}} + w_{\text{Equity}} \cdot \beta_{\text{Equity}} = (2/3) \cdot 0 + (1/3) \cdot 2.5 \approx 0.833$. Next, assume that your project has the same asset beta, but a smaller debt-to-asset ratio of 1 to 3, and compute your own equity beta: $\beta_{\text{Asset}} = w_{\text{Debt}} \cdot \beta_{\text{Debt}} + w_{\text{Equity}} \cdot \beta_{\text{Equity}} \Rightarrow 0.833 \approx (1/3) \cdot 0 + (2/3) \cdot \beta_{\text{Equity}} \Rightarrow \beta_{\text{Equity}} = 1.25$.
2. With an asset beta of 0.83, your firm's asset hurdle rate should be $\mathscr{E}(r_i) = 3\% + 2\% \cdot 0.83 \approx 4.7\%$.
3. Your comparable's equity expected rate of return would be $\mathscr{E}(r_{\text{Comps Equity}}) = 3\% + 2\% \cdot 2.5 = 8\%$. Your own equity's expected rate of return would be $\mathscr{E}(r_{\text{Your Equity}}) = 3\% + 2\% \cdot 1.25 = 5.5\%$

Q 9.16 Your combined happy-marriage beta would be $\beta_{\text{Combined}} = (3/4) \cdot 2.4 + (1/4) \cdot 0.4 = 1.9$.

Q 9.17 No, the empirical evidence suggests that the CAPM does not hold. The most important violation seems to be that value firms had market betas that were low, yet average returns that were high. The opposite was the case for growth firms.

Q 9.18 Even though the CAPM is empirically rejected, it remains the benchmark model that everyone uses in the real world. Moreover, even if you do not trust the CAPM itself, at the very least it suggests that covariance with the market could be an important factor.

Q 9.19 The CAPM should work very well if beta is about 0. The reason is that you do not even need to guess the equity premium if this is so.

Q 9.20 For short-term investments, the expected cash flows are most critical to estimate well (see Section 4.1 on Page 64). In this case, the trouble spot (d) is really all that matters. For long-term projects, the cost of capital becomes relatively more important to get right, too. The market betas and risk-free rates are usually relatively low maintenance (though not trouble free), having only modest degrees of uncertainty. The equity premium will be the most important problem factor in the cost-of-capital estimation. Thus, the trouble spots for long-term projects are (b) and (d).

End of Chapter Problems

Q 9.21. What are the assumptions underlying the CAPM? Are the perfect market assumptions among them? Are there more?

Q 9.22. If the CAPM holds, then what should you do as the manager if you cannot find projects that meet the hurdle rate suggested by the CAPM?

Q 9.23. In a perfect world and in the absence of externalities, should you take only the projects with the highest NPV?

Q 9.24. Write down the CAPM formula. Which are economy-wide inputs, and which are project-specific inputs?

Q 9.25. The risk-free rate is 6%. The expected rate of return on the stock market is 8%. What is the appropriate cost of capital for a project that has a beta of 2?

Q 9.26. The risk-free rate is 6%. The expected rate of return on the stock market is 10%. What is the appropriate cost of capital for a project that has a beta of –2? Does this make economic sense?

Q 9.27. Draw the SML if the true expected rate of return on the market is 6% per annum and the risk-free rate is 2% per annum. How would the figure look if you were not sure about the expected rate of return on the market?

Q 9.28. A junk bond with a beta of 0.4 will default with 20% probability. If it does, investors receive only 60% of what is due to them. The risk-free rate is 3% per annum and the risk premium is 5% per annum. What is the price of this bond, its promised rate of return, and its expected rate of return?

Q 9.29. What would it take for a bond to have a larger risk premium than default premium?

Q 9.30. A corporate zero-bond promises 7% in one year. Its market beta is 0.3. The equity premium is 4%; the equivalent Treasury rate is 3%. What is the appropriate bond price today?

Q 9.31. Explain the basic schools of thought when it comes to equity premium estimation.

Q 9.32. If you do not want to estimate the equity premium, what are your alternatives to finding a cost-of-capital estimate?

Q 9.33. Explain in 200 words or less: What are reasonable guesstimates for the market risk premium and why?

Q 9.34. Should you use the same risk-free rate of return both as the CAPM formula intercept and in the equity premium calculation, or should you assume an equity premium that is independent of investment horizon?

Q 9.35. Should a negative-beta asset offer a higher or a lower expected rate of return than the risk-free asset? Does this make sense?

Q 9.36. An unlevered firm has an asset market beta of 1.5. The risk-free rate is 3%. The equity premium is 4%.

1. What is the firm's cost of capital?
2. The firm refinances itself. It repurchases half of its stock with debt that it issues. Assume that this debt is risk free. What is the equity beta of the levered firm?
3. According to the CAPM, what rate of return does the firm have to offer to its *creditors*?
4. According to the CAPM, what rate of return does the firm have to offer to its *levered equity holders*?
5. Has the firm's weighted average cost of capital changed?

Q 9.37. Consider the following historical rate of return series:

Year	IBM	S&P 500	Year	IBM	S&P 500
1991	–0.175	0.263	2001	0.430	–0.130
1992	–0.400	0.045	2002	–0.355	–0.234
1993	0.156	0.071	2003	0.205	0.264
1994	0.322	–0.015	2004	0.072	0.090
1995	0.257	0.341	2005	–0.158	0.030
1996	0.676	0.203	2006	0.198	0.136
1997	0.393	0.310	2007	0.129	0.035
1998	0.775	0.267	2008	–0.208	–0.385
1999	0.175	0.195	2009	0.586	0.235
2000	–0.208	–0.101	2010	0.143	0.128

Assume that IBM had so little debt that it was practically risk-free.

1. What was IBM's equity beta over this sample period?
2. If IBM had a debt-equity ratio of 70%, what was its asset beta? (Hint: To determine a D/A ratio, make up an example in which a firm has a 70% D/E ratio.)
3. How important is the 1992 observation to your beta estimate?

4. If HP is similar to IBM in its business but has a debt-equity ratio of 10%, what would you expect HP's levered equity beta to be? (Hint: Use the same leverage conversion trick.)

Q 9.38. Look up betas on YAHOO! FINANCE today, and compare them to those in Exhibit 8.6 on Page 209.

1. How does the beta of Intel today compare to its earlier estimate from May 2008? Was its beta stable (over time)?
2. How does the beta of AMD today compare to its earlier estimate from May 2008? Was its beta stable?
3. AMD is a much smaller firm than Intel. How do their betas compare?

Q 9.39. A comparable firm (in a comparable business) has an equity beta of 2.5 and a debt-equity ratio of 2. The debt is almost risk free. Estimate the beta for your equity if projects have constant betas, but your firm will carry a debt-equity ratio of 1/2. (Hint: To translate a debt-equity ratio into a debt-asset ratio, make up an example.)

Q 9.40. A Fortune 100 firm is financed with $15 billion in debt and $5 billion in equity. Its historical equity beta has been 2. If the firm were to increase its leverage from $15 billion to $18 billion and use the cash to repurchase shares, what would you expect its levered equity beta to be?

Q 9.41. The prevailing risk-free rate is 5% per annum. A competitor to your own firm, though publicly traded, has been using an overall project cost of capital of 12% per annum. The competitor is financed by 1/3 debt and 2/3 equity. This firm has had an estimated equity beta of 1.5. What is it using as its equity premium estimate?

Q 9.42. Apply the CAPM. Assume the risk-free rate of return is the current yield on 5-year bonds. Assume that the market's expected rate of return is 3% per year above this. Download 5 years of daily rate of return data on four funds: NAESX, VLACX, VUVLX, and VWUSX.

- What were the historical average rates of return?
- What were the historical market betas?
- What were the historical market betas, adjusted (shrunk) toward 1 by averaging with 1?
- How do these estimates compare to the market beta estimates of the financial website from which you downloaded the data?
- Does it appear as if these funds followed a CAPM-like relationship?

Q 9.43. Draw some possible security markets relations that would not be consistent with the CAPM. The x axis would be the true market beta, the y axis would be the true expected rate of return.

Q 9.44. Does the empirical evidence suggest that the CAPM is correct?

Q 9.45. Why do you need to understand the CAPM?

Q 9.46. Under what circumstances is the CAPM a good model to use? What are the main arguments in favor of using it? When is it not a good model?

Q 9.47. If you use the CAPM, explain for what kinds of projects it is important to get accurate equity-premium estimates.