The Absence of a Size Effect Relevant to the Cost of Equity

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Abstract

In this paper, I show that proponents of the size effect are nowhere near their burden of proving that a size effect relevant to the cost of equity exists. I find investors use the CAPM and do not demand compensation for size when setting their required rates of return, which directly contradicts the need to augment or modify the cost of equity with a size premium. I show that small stocks underperform large stocks, which calls into question the basic premise of the size effect. I also find that studies claiming to have found a size effect suffer from at least one of two fatal flaws: the lack of theoretical basis for why a size effect exists and data mining. Consequently, valuation practitioners should abandon the practice of augmenting or modifying their CAPM Cost of Equity with a size premium.

Keywords: size effect, CAPM, cost of capital

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The cost of equity capital is a critical component when valuing a firm. It is used as the rate to discount equity cash flows or it is a component of the weighted-average cost of capital used to discount firm cash flows. The Capital Asset Pricing Model ("CAPM") is typically used when calculating the cost of equity. Standard finance textbooks warn against adding "arbitrary fudge factors" to the discount rate,[1] but practitioners often augment or modify the CAPM Cost of Equity with a size premium reported in publications by Morningstar/Ibbotson and Duff & Phelps.[2] The size premium is thought of as compensation for the outperformance by small stocks relative to large stocks on a risk-adjusted basis. There are studies that document such outperformance,[3] but there are also studies that show the size effect does not exist, the size effect vanished in the 1980s, and the methods, inputs, and/or assumptions in the size effect studies are flawed.[4] The scientific method puts the burden of proof on the party that has claimed to have observed an anomaly, i.e., those finding that a size effect exists.[5] I demonstrate in this article why proponents of the size effect are nowhere close to meeting their burden as it relates to the relation between size and the cost of equity.[6]

The size effect debate is heavily focused on arcane details, but, in this paper, I instead evaluate the existence of a size effect relevant to the cost of equity by keeping an eye on the forest. First, I review the evidence as to whether investors demand compensation for the size effect and I find that they do not. It has been over three decades since the 1981 study by Banz ("Banz 1981") was published,[7] which is the seminal paper on this topic, and numerous studies that find a size effect have also been published since. However, research shows that investors act as if a size effect does not matter in setting their required rate of return. Specifically, equity investors prefer the CAPM over models that include a size proxy, such as the Fama-French model, when setting the rate of return they demand from their investments. This implies either equity investors do not believe a size effect exists or, if they do believe a size effect exists, the impact should not be accounted for in the cost of equity. Regardless, practitioners augmenting or modifying their CAPM Cost of Equity with a size premium are using a cost of equity that is inconsistent with equity investors' actions.

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[1] Richard Brealey, Stewart Myers, and Franklin Allen, *Principles of Corporate Finance, 10th ed*. (McGraw-Hill Irwin, 2011).

[2] Charles Jones, *Investments: Analysis and Management, 11th ed.* (Wiley, 2009).

[3] Rolf Banz, "The Relationship Between Return and Market Value of Common Stocks," *Journal of Financial Economics* 9 (1981): 3-18; Eugene Fama and Kenneth French, "The Cross-Section of Expected Stock Returns," *Journal of Finance* 47(2) (1992): 427-465.

[4] Richard Roll, "A Possible Explanation of the Small Firm Effect," *Journal of Finance* 36(4) (1981): 879-888; Fischer Black, "Beta and Return," *Journal of Portfolio Management* 20 (1993): 8-18; Jonathan Berk, "A Critique of Size-Related Anomalies," *Review of Financial Studies* 8 (1995): 275-286; Dongcheol Kim, "A Re-Examination of Firm Size, Book-To-Market, and Earnings Price in the Cross-Section of Expected Stock Returns," *Journal of Financial and Quantitative Analysis* 32(4) (1997): 463-489; John Cochrane, *Asset Pricing: Revised Edition* (Princeton University Press, 2005); Brealey, Myers, and Allen (2011); Andrew Ang, *Asset Management: A Systematic Approach to Factor Investing* (Oxford University Press, 2014).

[5] Although there is no evidence that the burden of proof has shifted, valuation practitioners that do not add a size premium often find themselves in the position of having to defend their choice of not adding a size premium. See, for example, Aswath Damodaran, "The Small Cap Premium: Where is the beef?" *Musings on Markets Blog* (April 11, 2015). http://aswathdamodaran.blogspot.com/2015/04/the-small-cap-premium-fact-fiction-and.html, accessed May 18, 2017.

[6] Stephen Carey, *A Beginner's Guide to Scientific Method, 4th ed.* (Wadsworth, Cengage Learning, 2011).

[7] Banz (1981).

Second, I analyze the basic premise underlying the size effect, which is whether small stocks outperform large stocks. Examining the period since the first size effect studies were published in 1981 shows the opposite: small stocks underperformed large stocks. This implies that even on the most basic level, there is no reason to believe a size effect exists. Put differently, the size effect does not even pass the smell test.

Finally, I determine whether the size effect studies suffer from fatal flaws. I reviewed numerous articles that find a size effect and find that all such studies suffer from at least one of two fatal flaws: the lack of a theoretical basis for the size effect and the susceptibility of the results to data mining. Even Banz 1981 and a 1992 study by Fama and French ("Fama-French 1992"),[8] two of the most commonly cited size effect studies, fail on both counts. Without a theoretical basis, we cannot understand why size should matter. Moreover, the lack of a theoretical basis makes it difficult to shake the data mining criticism. Even if a theoretical basis were developed, many articles have shown that making small changes to the assumptions and/or inputs used make the findings of many size effect studies go away. In addition, some studies only find a size effect in limited circumstances, such as during the month of January. These call into question the reliability of the studies that find a size effect and why the size effect needs to be a standard adjustment when estimating the cost of equity.

The remainder of this paper is organized as follows. In Section I, I evaluate whether investors demand compensation for size. I demonstrate in Section II that small stocks do not outperform large stocks. I then describe the two fatal flaws that plague virtually all size effect studies in Section III. Section IV concludes the paper.

I. Investors Do Not Demand Compensation for Size

Whether investors demand compensation for size goes to the heart of whether there is a size effect that is relevant when estimating the cost of equity. If investors do not demand compensation for size, then there is no reason that valuation practitioners should augment or modify their CAPM Cost of Equity with a size premium. Given that it has been over three decades since Banz 1981 and there has been much discussion of the size effect in the academic and practitioner communities since then, we would expect that investors' required rates of return would include a size premium if investors demanded compensation for the size effect. However, the evidence contradicts this claim.

Market-based evidence finds equity investors do not demand compensation for size in their required rates of return. Using data on mutual fund flows, a 2017 study by Berk and van Binsbergen identifies the CAPM as the model that is most consistent with how mutual fund investors set their required rate of return.[9] The authors also find that the additional factors in the Fama-French model, which includes a size proxy, did not add explanatory power.[10]

Survey-based evidence also finds that investors do not demand compensation for size. A 2015 study by Pinto et al. find that 68 percent of the 1,436 investment professionals that responded to their survey use the CAPM. By contrast, the authors find that the Fama-French

[8] Fama and French (1992).

[9] Jonathan Berk and Jules van Binsbergen, "How Do Investors Compute the Discount Rate? They Use the CAPM," *Financial Analysts Journal* 73(2) (2017): 25-32.

[10] In unreported results, using data from Kenneth French's Data Library, I find that over the 1981 to 2016 period the CAPM alone does as well as a two-factor model that consists of the excess market return and size proxy in explaining average portfolio returns. This also suggests that adding a size factor does not add explanatory power.

model was used by less than five percent of respondents.[11] A 2002 study by Graham and Harvey find that over 70 percent of their survey respondents always or almost always use the CAPM, while a multi-factor CAPM only ranked third and was used more infrequently than the simplistic approach of using the firm's average stock return as the cost of equity.[12]

These results imply either one of two things. First, investors do not believe a size effect exists and, therefore, do not demand compensation for it. Second, investors believe a size effect exists, but believe the adjustment for the size effect is not made in the cost of equity. Either way, practitioners that augment or modify their CAPM Cost of Equity with a size premium expose themselves to using a cost of equity that is inconsistent with how investors set their required rate of return.

II. Small Stocks Do Not Outperform Large Stocks

The basic premise of the size effect is that small stocks outperform large stocks. I test this premise by running a simple test of comparing how small stocks perform relative to large stocks on a raw return basis. I use value-weighted size-based decile returns obtained from Kenneth French's Data Library. I use the smallest size-based decile as a proxy for small stocks and the largest size-based decile as a proxy for large stocks. I perform my comparison over the period 1981 to 2016, which is the period after the publication of the first size effect studies. My analysis shows that $100 invested in small stocks would have grown to $3,221 over the period, while the same $100 invested in large stocks would have grown to $3,774. In other words, small stocks underperformed large stocks by 12 percent over the period 1981 to 2016. Since small stocks already underperformed large stocks on a raw return basis, it necessarily follows that small stocks would underperform large stocks even more on a risk-adjusted basis because small stocks are assumed to have higher risk or betas relative to large stocks.[13]

My finding is consistent with many finance textbooks that report the size effect vanishing in the 1980s. For example, one textbook explains: "The small-firm effect completely disappeared in 1980; you can date this as the publication of the first small-firm effect papers or the founding of small-firm mutual funds made diversified portfolios of small stocks available to average investors."[14] Therefore, the fundamentals of the market for small stocks have changed since Banz 1981 and using pre-1980s data is no longer relevant to whether a size effect exists today.

Some recent studies, however, use a different sample period to analyze whether a size effect exists. For example, a 2016 study by Grabowski uses the period from 1990 to 2014 and finds a size effect.[15] Although the author uses post-1981 data, there is no reliable justification for using 1990 as the start date. If I chose an equally arbitrary start date of 2000 or 2005, I will find that small stocks once again underperform large stocks. More importantly, the sample period

[11] Jerald Pinto, Thomas Robinson, and John Stowe, "Equity Valuation: A Survey of Professional Practice," Working Paper (2015) available at SSRN: https://ssrn.com/abstract=2657717, accessed May 22, 2017.
[12] John Graham and Campbell Harvey, "How Do CFOs Make Capital Budgeting and Capital Structure Decisions?" *Journal of Applied Corporate Finance* 15(1) (2002): 8-23.
[13] In unreported results, I find that there is no reliable relation between size and betas. Using the same data from Kenneth French's Data Library, I find the beta of the largest size-based decile is indeed smaller than the beta of the smallest size-based decile. However, the beta of the portfolios in between these two extreme portfolios are in-line or, in most cases, higher than the beta of the smallest size-based decile. Hence, we do not observe a monotonic increase in betas as size decreases, which is inconsistent with the existence of a size effect.
[14] Cochrane (2005).
[15] Grabowski (2016).

cannot be chosen arbitrarily. My 1981 start date is most appropriate for two reasons. First, it includes all periods after the publication of the first size effect studies. Hence, it accounts for all developments that have transpired since then. Second, we need a sufficiently long period to analyze the size effect. Fama and French suggest a period of at least 35 years to be confident with the results, which starting the sample period in 1981 satisfies.[16]

III. Size Effect Studies Suffer from Two Fatal Flaws

Many articles have demonstrated flaws in the size effect studies. For example, some articles criticize the size effect studies for using an improper risk measure or exhibiting an errors-in-variables bias.[17] Many of these issues can, at least in theory, be corrected. I examined numerous articles that find a size effect and find that all such articles suffer from at least one of two fatal flaws: the lack of a theoretical basis for a size effect and the susceptibility of the results to data mining. To elaborate on these two fatal flaws, I use Banz 1981 and Fama-French 1992 as my primary examples. These are two of the most commonly cited articles used to support the existence of a size effect.

A. Lack of a theoretical basis

The most fundamental flaw of the size effect studies is the lack of a theoretical basis for finding a size effect. In fact, Banz 1981 concludes by admitting "[t]here is no theoretical foundation for such an effect." As for Fama-French 1992, an article by Fischer Black observes: "Fama and French also give no reasons for a relation between size and expected return."[18] Put differently, the size effect is merely an artifact of the data. There have been articles that attempt to come up with a theoretical basis for the size effect, but, to the best of my knowledge, these theories have yet to obtain empirical confirmation beyond the simulated results provided.[19]

Banz 1981 asserts that size may be a proxy for one or more unknown factors that are correlated with size. This assertion has been disputed by some studies,[20] but some articles have claimed to identify specific factors. The most commonly associated factor with size is illiquidity.[21] However, these studies on illiquidity appear to confuse the impact of illiquidity on the firm's price with the impact of illiquidity on the firm's value. For example, one author of a size effect study defines liquidity as "the speed at which a large quantity of a security can be traded with a minimal impact on the price and with the lowest transaction costs."[22] When market

[16] Eugene Fama and Kenneth French, "Q&A: Small Stocks for the Long Run," *Fama/French Forum* (January 23, 2012). https://famafrench.dimensional.com/questions-answers/qa-small-stocks-for-the-long-run.aspx, accessed May 21, 2017.
[17] Roll (1981); (Kim 1997).
[18] Black (1993).
[19] Jonathan Berk, Richard Green, and Vasant Naik, "Optimal Investment, Growth Options, and Security Returns," *Journal of Finance* 54(5) (1999): 1553-1607; Joao Gomes, Leonid Kogan, and Lu Zhang, "Equilibrium Cross Section of Returns," *Journal of Political Economy* 111(4) (2003): 693-732; Murray Carlson, Adlai Fisher, and Ron Giammarino, "Corporate Investment and Asset Price Dynamics: Implications for the Cross-Section of Returns," *Journal of Finance* 59(6) (2004): 2577-2603.
[20] Berk (1995).
[21] Yakov Amihud and Haim Mendelson, "The Effects of Beta, Bid-Ask Spread, Residual Risk, and Size on Stock Returns," *Journal of Finance* 44(2) (1989): 479-486; Roger Ibbotson, Zhiwu Chen, Daniel Kim, and Wendy Hu, "Liquidity as an Investment Style," *Financial Analysts Journal* 69(3) (2013): 30-44.
[22] Grabowski (2016).

frictions exist, such as the existence of costs and constraints to trading due to illiquidity, the price of a security could deviate from the value of the security.[23] This is particularly true when the large block trade has no signaling effect about the value of the firm but the trade is brought about by an investor-specific need (e.g., the investor needs the funds to buy a yacht). Consistent with this, studies have modeled the impact of illiquidity as a temporary effect that is uncorrelated with fundamental value.[24] To the extent that the block trade is driven by value-relevant information, the first-best and more appropriate adjustment would be to modify the expected cash flows for the impact of that information.

B. Data mining

One indicator of data mining is that the result does not stem from theory.[25] Hence, given the above discussion, the results of size effect studies are especially susceptible to data mining. Regardless, even if a credible theory does emerge, the volume of articles that question the robustness of many size effect studies still make data mining a primary and fatal concern. For example, the results of many size effect studies are not robust to small changes to its inputs and assumptions. Let us take the choice of sample period as an example. Even Banz 1981 admits that the size effect is not very stable through time and an analysis of the ten year sub-periods in his sample of NYSE firms from 1926 to 1975 shows substantial differences in the magnitude of the size factor coefficient. As for Fama-French 1992, they use a sample period that overlaps with Banz 1981 but their results for the size effect go away when only the post-Banz data is used.[26] In fact, a 2012 study by Fama and French confirmed the lack of a size effect when using a more recent sample period.[27]

Another sign of data mining is that the size effect only exists in specific situations. I discuss two examples here. First, recall that the size effect studies initially grouped firms into deciles or ten size-based portfolios.[28] As it became harder to find a size effect when grouping stocks into deciles, more recent size effect studies have begun grouping stocks into 25 size-based portfolios.[29] However, finding results by looking long and hard at the data to find patterns, such as finding an effect only in a limited subsample, is another indication of data mining.[30] Therefore, the need to use portfolios of smaller and smaller firms implies that the size premium is not robust and, therefore, should not be considered a standard adjustment when estimating the cost of equity.

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[23] Ananth Madhavan, "Market Microstructure: A Survey," *Journal of Financial Markets* 3 (2000): 205-258.
[24] Jack Bao, Jun Pan, and Jiang Wang, "The Illiquidity of Corporate Bonds," *Journal of Finance* 66(3) (2011): 911-946.
[25] Andrew Lo and A. Craig MacKinlay, "Data-Snooping Biases in Tests of Financial Asset Pricing Models," *Review of Financial Studies* 3(3) (1990): 431-467; Black (1993); Brealey, Myers, and Allen (2011).
[26] Black (1993); Andrew Ang and Joseph Chen, "CAPM over the long run: 1926-2011," *Journal of Empirical Finance* 14 (2007): 1-40.
[27] Eugene Fama and Kenneth French, "Size, Value, and Momentum in International Stock Returns," *Journal of Financial Economics* 105 (2012): 457-472.
[28] Marc Reinganum, "Abnormal Returns in Small Firm Portfolios," *Financial Analysts Journal* 37(2) (1981): 52-56; Carlson et al (2004).
[29] Grabowski (2016).
[30] Brealey, Myers, and Allen (2011)

The second example is that a size effect is found only in January,[31] during which half of the effect is observed during the first five trading days of January.[32] The most common rationale provided for this so-called January Effect is that investors sell stocks with capital losses at the end of the tax year, which is December in most cases, to offset taxable income during that same tax year. As it relates to small stocks, the argument is that small stocks are likely candidates for tax-loss selling because their high volatility would lead to higher probabilities of larger capital losses.[33] The January Effect is then observed when the small stocks' price rebounds back to its fundamental value in the beginning of the new tax year when the selling pressure has been alleviated. Consequently, all else equal, the January Effect has no impact on the value of the firm and what is observed is a temporary price effect due to factors unrelated to the firm's fundamental value (i.e., the investors' tax strategy).

IV. Conclusion

The scientific method puts the burden of proof on the party that has claimed to have observed the anomaly. My review of the evidence and analysis strongly suggests the proponents of the size effect are nowhere close to meeting their burden. I find that investors use the CAPM and do not demand compensation for size when setting their required rate of return, which directly contradicts the need to augment or modify the CAPM Cost of Equity with a size premium. I show that small stocks do not outperform large stocks, which calls into question the very premise of a size effect. I also find that studies finding a size effect suffer from the twin fatal flaws of lacking a theoretical basis and data mining, which are very difficult, if not impossible, to overcome. Given the above, practitioners should abandon the practice of augmenting or modifying the CAPM Cost of Equity with a size premium.

[31] Jeffrey Jaffe, Donald Keim, and Randolph Westerfield, "Earnings Yields, Market Values, and Stock Returns," *Journal of Finance* 44(1) (1989): 135-148; Christopher Lamoureux and Gary Sanger, "Firm Size and Turn-of-the-Year Effects in the OTC/NASDAQ Market," *Journal of Finance* 44(5) (1989): 1219-1245; Carroll et al. (1992).
[32] Donald Keim, "The CAPM and Equity Return Regularities," *Financial Analysts Journal* 42(3) (1986): 19-34.
[33] Philip Brown, Donald Keim, Allan Kleidon, and Terry Marsh, "Stock Return Seasonalities and the Tax-Loss Selling Hypothesis," *Journal of Financial Economics* 12 (1983): 105-127.