"Normalized" Risk-Free rate: fiction or science fiction?

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As interest rates on Government Bonds have decreased, some analysts and consultants in Europe and in the US are using what they call "Normalized Risk-Free rate".

We show several inconsistencies and errors in the use of "Normalized Risk-Free rate".

Section 5 is a short case that may be used in class. It contains 26 interesting comments.

JevPSIfi

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[1] I thank very much Professor Salvador Roji for finding errors in a previous version of this document.

1 Valuation of Government bonds. Risk-Free Rate (R_F)

The valuation of companies using discounted cash flows is an extension of the valuation of Government bonds. A Government bond is a piece of paper[2] that details the amount of US$ that will receive its owner and the dates in which the amounts will be received. The amounts promised in the Government bond are called **cash flows** and are the money (cash) that will go from the Government to the pocket of the owner of the bond in the specified days.

The Value of a Government bond (VGB) is the present value of the cash flows promised in the bond (CFgb) using the so called "**risk-free rate**" (R_F):

$$\text{Value of a Government bond} = VGB = PV(CFgb; R_F) \quad (1)$$

The risk-free rate (R_F) is the **required return** to Government bonds.

2 Extension of the valuation of Government bonds to the valuation of companies

The valuation of companies deals with the valuation of the financial debt and with the valuation of the shares (equity). We apply Equation (1) to the valuation of Debt and to the valuation of shares.

2.1 Valuation of the Debt. Cost of Debt (r) and required return to debt (Kd)

The Debt of the company is: several pieces of paper[3] that include the amounts that will receive their owners in specified dates. The amounts promised by the debt are called **Debt cash flows** (CFd) and are interest payments and repayments of debt (∇N).[4]

$$CFd = \text{Interest} + \nabla N \quad (2)$$

As the **Debt cash flows** (CFd) promised by a company are usually riskier[5] than the cash flows promised by the Government bonds (CFgb), the **required return to Debt** (Kd) is usually higher than the risk-free rate (R_F)

$$\text{Required return to debt} = Kd = R_F + RPd \text{ (debt risk premium)} \quad (3)$$

The **debt risk premium** (RPd) depends on the perceived risk on the Debt (expectations of getting less money than the promised Debt cash flows) by every investor. Applying Equation (1) to the Debt of the company, we get:

$$\text{Value of debt} = D = PV(CFd; Kd) \quad (4)$$

2.2 Valuation of the shares. Required return to equity (Ke)

A share of a company is a piece of paper that, contrary to debt, has not dates nor amounts that will receive its owner, the shareholder. We need, first, to estimate the expected cash flows for the owners of the shares in the following

[2] It can also be an electronic record. In that case, we should say: "as if it would be" a piece of paper.
[3] If it is bank debt, the owners of the papers are Banks. If the debt are bonds, notes… the owners of the papers are the persons, the companies and the financial institutions that bought them.
[4] If the company does not repay debt (∇N) but increases its debt (ΔN), Equation (2) would be CFd = Interest - ΔN
[5] The risk of the debt is the probability that the company will not pay some of the promised cash flows. Risk-free debt means that we assume that the company will pay all promised cash flows for sure.

years, named **Equity Cash Flows** (ECF). A usual way of estimating the ECF is to start with the expected Balance Sheets and P&Ls. Equation (5) is the basic accounting identity: assets are equal to liabilities and equity:

Cash + WCR + NFA = N + Ebv (5)

Cash	Debt (**N**) *Bank debt, bonds…*
Working Capital Requirements (**WCR**)	
Net Fixed Assets (**NFA**)	Book value of Equity (**Ebv**) *Shares*

Debt Cash Flow (CFd): money (cash) that goes from the Cash of the company to the pockets of bondholders

Equity Cash Flow (ECF): money (cash) that goes from the Cash of the company to the pockets of shareholders

Equation (6) is the annual change of Equation (5). The increase of the cash of the company before giving anything to the shareholders will be divided between the Equity Cash Flow (**ECF**) and the increase of cash (**ΔCash**) decided by the managers and the main shareholders:

ECF + ΔCash + ΔWCR + ΔNFA = ΔN + ΔEbv (6)

If the increase of the book value of equity (ΔEbv) is due only to the Profit after Tax (PAT) of the year, then[6]:

ECF = PAT - ΔWCR - ΔNFA + ΔN - ΔCash (7)

As the expected equity cash flows (ECF) are riskier than the cash flows promised by the Government bonds (CFgb) and also riskier than the cash flows promised by the Debt of the company (CFd), the **required return to equity (shares)** (Ke) is higher than risk-free rate (R_F) and also higher than the required return to Debt (Kd):

Ke = R_F + RPs (shares risk premium) (8)

The so-called **shares risk premium** (RPs) depends on the estimated (expected) risk of the expected equity cash flows (ECF). Obviously, this parameter depends on the expectations of each investor. Applying Equation (1) to the equity (the shares of the company), we get:

Value of the shares (equity value) = E = PV (ECF; Ke) (9)

With equations (2) to (9) we can value any company.

Many people calculate RPs (shares risk premium) as a product: RPs = β MRP (10)

The MRP (**market risk premium**) is the "shares risk premium" applied to the whole market (or to a portfolio with shares of most of the companies traded in the stock markets). The market risk Premium (MRP) is the answer to

[6] As NFA = GFA (gross fixed assets) – depreciation, equation (7) can be written:
ECF = PAT + depreciation - ΔNOF - ΔGFA + ΔN - ΔCash

the following question: Knowing that your money invested in long-term Government bonds will provide you a return of R_F% almost for sure, which additional return you require to another investment (in a portfolio with shares of most of the companies with shares traded in the financial markets) for feeling compensated for the extra risk that you assume?

The "market risk premium" is also called "*equity premium*", "*equity risk premium*", "*market premium*" and "*risk premium*".

The β (**beta**) is a specific parameter for each company. We know that β=0 corresponds to Government bonds (no risk) and β=1 to an investment with a risk like that of the market. About 80% of the betas used in valuations are in the interval between 0.7 and 1.5.

With the beta and the MRP, Equation (8) becomes equation (11)

$$Ke = R_F + \beta\ MRP \qquad (11)$$

3. Valuation of company AAA

The expected Balance Sheets and P&Ls of company AAA are the following ones (amounts in $ millions):

Balance Sheet ***Year***	***0***	***1***	***2***	**...g = 2%**
Cash	50	51	52.02	
WCR	450	459	468.18	
Gross Fixed Assets (GFA)	1500	1680	1863.60	
- cumul. depreciation		150	303.00	
Net Fixed Assets (NFA)	1500	1530	1560.60	
TOTAL NET ASSETS	**2000**	**2040**	**2080.80**	
Debt (N)	**1000**	1020	1040.40	
Book value of Equity (Ebv)	**1000**	1020	1040.00	
TOTAL Liabilities and Equity	**2000**	**2040**	**2080.80**	

P&L	***1***	***2***	**... g = 2%**
Sales	2400	2448.0	
Cost of sales	1200	1224.0	
Other expenses	810	826.2	
Depreciation	150	153.0	
Interest	25	25.5	
PBT (Profit beforeTaxes)	215	219.3	
Taxes (25%)	53.75	54.83	
PAT (Profit after Taxes)	**161.25**	**164.48**	

The managers of AAA expect that the Balance Sheet (cash, WCR, NFA, N and Ebv) and the P&L will grow annually 2%.

The interest rate of the debt (r) is 2.5%. The interests to be paid in year 1 are \$25 = N r = 1000 x 2.5%. The amount of debt is expected to increase in \$20 million in year 1. Then, in year 1:

(2) CFd = Interest - ΔN = 25 – 20 = 5

The risk-free rate (R_F) of the long-term Government bonds (10 years) is 0.5%. The Financial Manager of AAA considers that a RPd (debt risk premium) of 2% is appropriate for the Debt of AAA. Then:

(3) Kd = R_F + RPd (debt risk premium) = 0.5% + 2% = 2.5%

As the required return to Debt (Kd = 2.5%) is equal to the interest rate paid (r = 2.5%), D = N = 1000. We can calculate D using equation (4):

(4) Value of debt = D = PV (CFd; Kd) = $\frac{5}{1.025} + \frac{5\,(1.02)}{1.025^2} + \frac{5\,(1.02)^2}{1.025^3} \ldots = \frac{5}{0.025-0.02} = 1000$

In company AAA the increase of the book value of equity (ΔEbv) is due only to the Profit after Tax of the year. Applying equation (7) to year 1:

(7) ECF = PAT - ΔWCR - ΔNFA + ΔN - ΔCash = 141.25 = 161.25 – 9 – 30 + 20 - 1

Next table applies equation (7) to years 1 and 2.

	1	2	
PAT (Profit after Tax)	**161.25**	**164.475**	
+ Depreciation	150	153.00	
+ Δ Debt	20	20.40	
- Δ Cash	-1	-1.02	
- Δ WCR	-9	-9.18	
- Investments (ΔGFA)	-180	-183.60	
ECF (Equity Cash Flow)	**141.25**	**144.075**	**… grows 2% annually**

The Financial Manager of AAA considers that 6% is an appropriate shares risk premium (RPs) for the equity (shares) of AAA[7]. Then, the required return to equity (Ke) is:

(8) $Ke = R_F + RPs = 0.5\% + 6\% = 6.5\%$

Now, we can use Equation (9) to calculate the Value of the shares or Equity Value (E) of AAA:

(9) $E = PV(ECF; Ke) = [141.25 / 1.065] + [141.25\ (1.02) / 1.065^2] + \ldots = [141.25 / (0.065 - 0.02)] =$ **3,138.89**

An important difference between the CFd and the ECF. The CFd (5) is the maximum cash flow that the debtholders may get in year 1, whereas the ECF (141.25) is the expected value (approximately , it means that with a 50% probability the ECF in year 1 may be higher than 141.25 and with a 50% probability may be lower than 141.25)

Cash 50 Working Capital Requirements (WCR) 450	Debt (N) 1000	Value of Debt = D = PV (CFd; Kd) =1000
Net Fixed Assets (NFA) 1500	Equity (Ebv) 1000	Value of equity = E = PV (CFac; Ke) = 3138.89

CFd = Interest + ∇N (2) Required return to debt = Kd = R_F + RPd (debt risk premium) (3)

ECF = PAT - ΔWCR - ΔNFA + ΔN - ΔCash (7) Ke = R_F + RPs (shares risk premium) (8)

With equations (2) to (9) we may value any company as we have done in this example. But, as equations (2) to (9) are relatively easy to understand, it is quite common to complicate the valuation with new concepts and new equations.

One of these unnecessary complications is the WACC: the valuation becomes more difficult to understand and acquires a more "scientific", "serious", "intriguing", "impenetrable",… appearance. But these unnecessary complications are the source of many errors.

[7] The financial manager uses equation (10): RPs = β MRP = 1 x 6% = 6%.
The Financial Manager of AAA considers that 6% is an appropriate MRP (market risk premium) and that the appropriate beta for the equity (shares) of AAA is 1. $Ke = R_F + \beta\ MRP = 0.5\% + 1 \times 6\% = 6.5\%$

4. The *free cash flow* (FCF) and the WACC

The **FCF** (*Free cash flow*) that is the hypothetical Equity Cash Flow (ECF) if the company had no Debt:

Free Cash Flow (FCF) = ECF if Debt(N) = 0 (12)

The FCF is equal to the ECF with 3 adjustments: 1) without increase or decrease of Debt; 2) without interest payments; 3) with higher taxes (because there are no interests). The relationship between the FCF and the ECF is:

$FCF = ECF - \Delta N + Int\ (1\text{-}T)$ (13)

We can calculate the FCF using Equations (7) and (12)

$FCF = PAT_u - \Delta WCR - \Delta NFA - \Delta Cash$ (14)

PAT_u is the Profit after Tax of the unlevered company. The subscript "u" means ***unlevered***, that is, without Debt.

We want to value the company using the FCF: we want to get (E + D) calculating the present value of the FCF discounting it at an unknown rate K? This unknown rate K? must accomplish:

E + D = PV (FCF; K?) = PV (ECF; Ke) + PV (CFd; Kd) (15)

K? is "named" **WACC** (*weighted average cost of capital*) and is[8]:

$$WACC = \frac{EKe + D\,Kd(1 - T)}{E + D} \quad (16)$$

What is the parameter 'E' that appears in Equation (16)? It is the same 'E' that appears in Equation (9): the Value of the shares (equity value) (E) obtained in the valuation. Then,

E + D = PV (FCF; WACC) (17)

Example. If company AAA had no Debt, the interests would be zero, the PBT_u (Profit before Tax) in year 1 would be 240, the taxes (25%) 60 and the PAT_u 180. Applying Equation (14) in year 1:

(14) FCF = Profit after Tax_u - ΔWCR - ΔNFA - ΔCash = 180 – 9 – 30 - 1 = 140

Equation (13) provides the same result: $FCF_1 = ECF_1 - \Delta N_1 + Int_1\ (1\text{-}T)$ = 141.25 – 20 + 25 (1 – 0.25) = 140

The WACC is calculated using Equation (16):

(16) $WACC = \frac{E\,Ke\ + D\,Kd\ (1-T)}{E+D} = \frac{3138.89\ x\ 6.5\%\ +1000\ x\ 2.5\%\ (1-25\%)}{3138.89+1000} = 5.38255\%$

Equation (17) provides the same value that we got using (4) and (9):

(17) $E + D = PV\ (FCF; WACC) = \frac{140}{1.0538255} + \frac{140\ (1.02)}{1.0538255^2} + \cdots = \frac{140}{0.0538255-0.02} = 4138.89$

[8] If N = D, that is, if the book value of debt (N) is equal to the value of debt (D). The interests paid are Nr. r is the interest rate (cost) that multiplies the book value N. If the required return to debt (Kd) is equal to the interest rate (r), then N = D. The equation of the WACC when r is different than Kd (and, therefore, D is different than N) is: WACC = (EKe + DKd – NrT) / (E + D)

5. The "normalized Risk-Free Rate" (NoR_F): An invention that does not exist

As interest rates on Government Bonds have decreased, some analysts and consultants in Europe and the US are using what they call "Normalized Risk-Free rates" because they do not like the world as it is: they prefer Figure 2 to Figure 1. The "Normalized Risk-Free rate" is the Risk-Free rate that should exist in a world that these analysts and consultants call "normal", but it is not the world in which we live. That is fiction or science fiction?

We show four inconsistencies: 1) it does not exist: we cannot invest in any financial instrument and get the "Normalized Risk-Free rate" without (or with little) risk; 2) in several cases it is higher than the cost of debt; 3) in several cases it is higher than the required return to debt; 4) most valuations do not adjust the value of Debt accordingly to the "Normalized Risk-Free rate" they use.

Figure 1

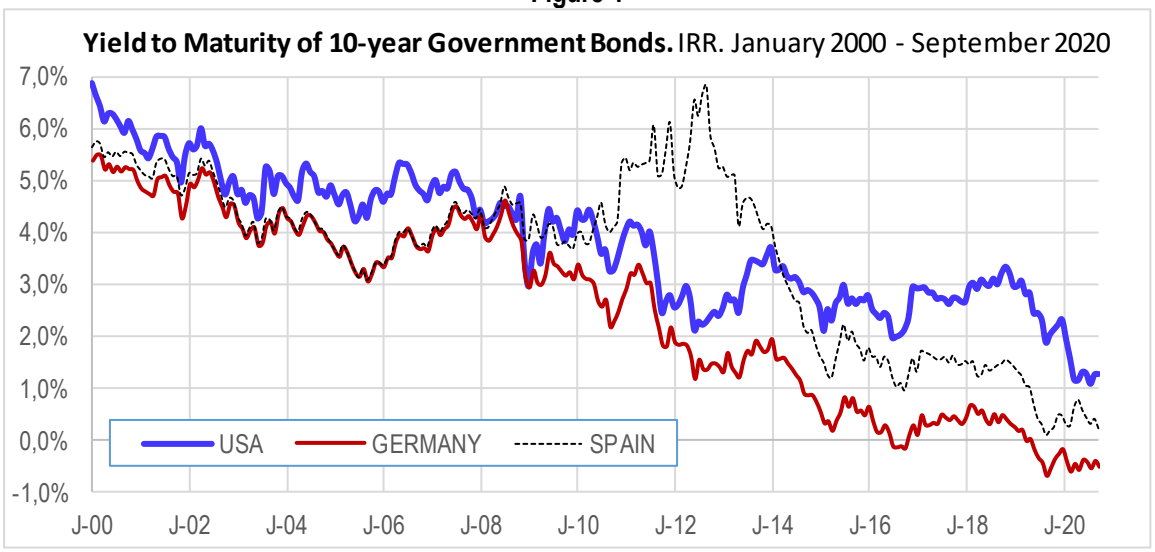
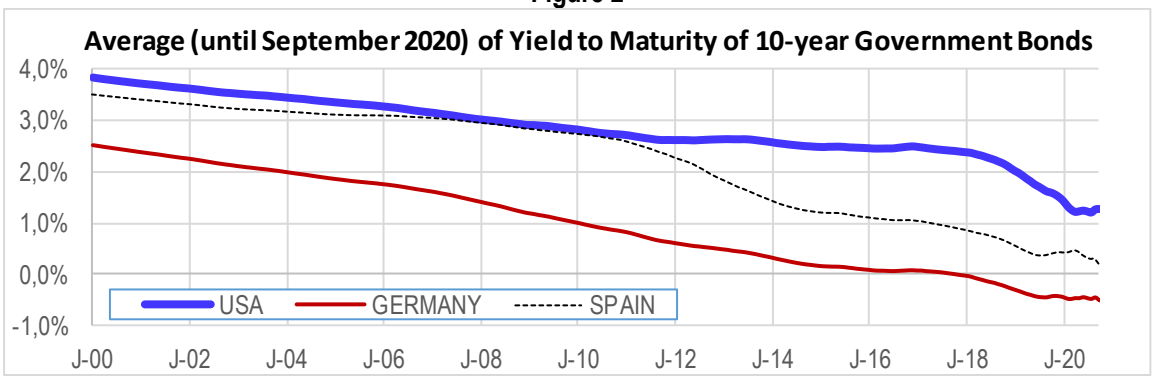


Figure 2



5.1. An attempt to "normalize" the valuation of AAA: A case

Mr. McLane partner of a consultant and financial firm (CFFA), said to the President of AAA: *"you should value your company using a "Normalized Risk-Free rate" (NoR_F) because this "normalization" approach is generally followed by the international financial community".* Mr. McLane also gave table 1 to the President of AAA saying: *"please, give*

this to your financial director. If he does not understand how to calculate the "Normalized Risk-Free rate", my finance people can work with him"

Table 1. Use *"Normalized Risk-Free rate"* instead of *"Risk-Free rate". An advice of CFFA*

In countries whose central banks have adopted monetary policies such us quantitative easing (QE) we do not use Risk-free rate (R_F) for valuation, but the "normalized Risk-Free Rate" (NoR_F)

Quantitative easing (QE) has caused current returns to be exceptionally low due to these public debt purchase measures carried out to stimulate liquidity and credit in these economies (mainly Euro countries, US, and Japan). This fact has led to 10-year sovereign bond yields at historic lows, even below the levels observed in years where they had a better credit rating.

Considering that this situation requires normalization in the long term, at CFFA we consider it appropriate to estimate a risk-free rate level that reflects a sustainable level in the long term, consistent with the inflation estimated in the long term and considered in the projections.

Other alternative approaches start from assuming the spot profitability combined with higher PRMs to reflect long-term returns from the capital market, seeking equivalent results.

We estimate regression models with observations of the real 10-year sovereign debt yields of countries based on the credit rating. With these regressions we infer the normalized profitability in real terms of an individual country. This profitability is translated into nominal terms applicable to the country under analysis using the expectations of the long-term inflation consensus (the same ones used in the financial projections for consistency). The foregoing allows us to obtain a range of nominal returns normalized to 10 years.

Likewise, we have estimated the historical averages of the 10-year returns for the country for the last 5 years and contrasted with the range obtained through regression.

Based on the results of our exercise updated to May 2020, we have concluded with a "normalized Risk-Free Rate" (NoR_F) of 3.00%.

When Mr. Davis, the financial director of AAA, read Table 1, received, and talk to some "valuation specialists" of CFFA. Four days later, Mr. Davis received Table 2.

Table 2. Valuation of the shares of AAA signed by CFFA

Dear Mr. Davis,

After receiving your cash flow expectations and talking to you about the risks of AAA, we propose the following changes to your valuation.

We use a "normalized Risk-Free Rate" (NoR_F) of 3% instead of your R_F = 0,5%.

We use a "normalized Market Risk Premium" (NoMRP) of 3,5% instead of your MRP = 6%.

As we also use a beta of 1, we get the same Ke as you:

$Ke = NoR_F + \beta\ NoMRP = 3\% + 1 \times 3.5\% = 6.5\%$. $Ke = R_F + \beta\ MRP = 0.5\% + 1 \times 6\% = 6.5\%$.

As we use the same expected cash flows and the same Ke, we get an identical valuation as you:

(9) E = PV (ECF; Ke) = [141.25 / 1.065] + [141.25 (1.02) / 1.065^2] + … = [141.25 / (0.065 – 0.02)] = **3,138.89**

The WACC we get is also identical to your WACC: 5.38255%.

We got the same values as you did. We recommend you using normalized parameters because the normalization approach is generally followed by the international financial community. We also recommend you using a "normalized Market Risk Premium" (NoMRP) of 7%. The NoMRP reflects the excess of the expected return of the equity market over the normalized long-term risk-free rate. Our long-term NoMRP estimates are based on our own studies and contrast with recognized empirical sources, maintaining consistency with the NoR_F approach. In this way, we consider a NoMRP applicable to the Valuation Date of 7.0%.

5.2. Comments about the Valuation of the shares of AAA using a "normalized Risk-Free Rate" (NoR_F)

When Mr. Davis, the financial director of AAA, read Table 1, listened to some "valuation specialists" of CFFA, and reviewed Table 2 sent the email in Table 3 to Mr. McLane (copying to his President).

Table 3. Email of Mr. Davis to Mr. McLane

Dear Mr. McLane,

I admit that the normalization that you proposed to my President may have an "intriguing", "impenetrable" and even "scientific" or "serious" appearance… but only for people that have no idea of finance and valuation.

The way your company explains the "normalization" process is unclear, unnecessary and is the source of important errors.

You say (Table 1) that *"Other alternative approaches start from assuming the spot profitability combined with higher PRMs to reflect long-term returns from the capital market, seeking equivalent results."* Then, why do you complicate the valuation?

As you know, I teach a course on valuation at the local Business School. I gave my MBA students Table 1 and Table 2 (the valuation of AAA that your "valuation specialists" provided me free of charge. My only question was "comments?"

Comments from my MBAs:

1. I did a WhatsApp survey. Just a few of my old colleagues in valuation use "NoR_F" and they use different figures: 4%, 3.75%, 3.5%, 3%, 2.9%, 2.3%, 1.5%... instead of the Internal Rate of Return of the 10 year Government Bond that is 0.5%.
2. The "NoR_F" is nothing. CFFA says (Table 1) that they use the "*expectations of the long-term inflation consensus*" to calculate it. There are many "*expectations of the long-term inflation*" and no one is a good predictor of inflation.
3. If some employee of CFFA knew how to calculate well the "*expectations of the long-term inflation*", he would be billionaire and, of course, he would not work for CFFA.

4. With the "recipe" of Table 1, I can get any number from 0% to 5%. What does a NoR_F = 3.00% mean?
5. I cannot invest in any financial instrument that gives me during the following 10 years the "normalized Risk-Free Rate" (NoR_F) of 3%. Then, the "normalized Risk-Free Rate" (NoR_F) is useless for valuation. Perhaps it has some use for presentations to philosophers, lawyers, romantic literature professors…
6. Today, the market price of a Government Bond that promises 5$ per year from year 1 to year 9 and 1005$ on year 10 is 1000$. The Internal rate of return (IRR) of that bond is 0.5%. If the "normalized Risk-Free Rate" (NoR_F) had any meaning, it should be the appropriate discount rate for valuing 10 year Government Bonds. If you value this Government Bond with the "normalized Risk-Free Rate" (NoR_F = 3%), the value is 786.74$. What is "value of 786.74$", but market price of 1000$?
7. CFFA could have some credibility only if Mr. McLane and his valuation staff are short of 10-year Government Bonds. If they believe (and are consistent with) their Table 1, they should sell many Government bonds at 1000$ that have a value of only 786.74$. For doing so, they may short-sell 10-year Government Bonds or sell futures on 10-year Government Bonds.
8. I agree with professor Damodaran: *"Cherry-picking historical data is fundamentally dangerous for many reasons. In this case, the same government controls that may have affected yields on bonds also extended to great segments of the economy and thus indirectly affected both risk and return on equities. To hold equity returns constant while altering the bond rate strikes me as unreal".*
9. Professor Ibbotson says: "*it is dangerous to adjust the data itself, because every period is special in its own way*". The "normalized Risk-Free Rate" (NoR_F) does not exist. It is nothing.
10. If the valuation date is today's date, then you should apply the current risk-free rate, and not a normalized risk-free rate.
11. I read: "*FED Chairman Bernanke instituted QE in 2008. QE is designed to push down the notes' yields (and interest rates tied to US Treasury rates). QE should increase the excess reserves of the banks, and the prices of the financial assets*". Is it a reason to use a non-existant normalized risk-free rate?
12. The risk-free rate is not just a number in the discount-rate calculation, but an alternative return that I can get with the minimum risk. Now I can buy 10-year Government Bonds and I get 0.5%. But I cannot buy anything to get the "normalized Risk-Free Rate" of 3%.
13. Who sells "normalized bonds" that yield 3% today? I would like to buy two hundred.
14. I see the "normalized Risk-Free Rate" (NoR_F) of 3.00% in table 1. I suppose that this 3% exists also in some spreadsheet of CFFA. But it does not exist in any financial market.
15. The "normalized Risk-Free Rate" (3%) can be considered as a bad and wrong assumption. There is a huge difference between a bad and wrong assumption (NoR_F =3.00%) and reality (R_F =0.5%).
16. Damodaran comments on it very well: *"if you do your valuation, using a normalized risk free rate of 3% (instead of the actual risk free rate of 0.5%), and decide that stocks are overvalued, I wish you the very best of luck putting your money in that normalized treasury bond, since it exists only in your estimation".*
17. The definition I have of R_F in my textbook *(Rate of return available on a risk-free security. The rate of return available on a risk-free security, or risk-free rate, reflects the minimum return that investors expect to receive from their*

investment. Analysts and valuators often use the yield on long-term U.S. Treasury bonds as a proxy for the risk-free rate) leaves no room for the unnecessary invention of a "normalized Risk-Free Rate".

18. I have access to valuation reports that use NoR_F and NoMRP. The pairs (NoR_F; NoMRP) that I have found are: (1.50%; 6.50%), (2.90%; 6.00%), (3.10%; 8.00%), (3.50%; 5.00%), (3.75%; 4.25%), (4.00%; 4.00%), (3.50%; 4.00%), (2.30%; 7.50%), (2.30%; 5.50%), (2.20%; 5.60%). Great dispersion!!! No one has any sense, but obviously, the ones that use (3.10%; 8.00%) and (2.30%; 7.50%) are even less credible and "out of this world". The pair proposed by CFFA (3.00%; 7.00%) is unacceptable.
19. The way CFFA justifies their NoMRP ("*Our long-term NoMRP estimates are based on our own studies and contrast with recognized empirical sources, maintaining consistency with the NoRF approach. In this way, we consider a NoMRP applicable to the Valuation Date of 7.0%")* may result in any number from -2% to 9%.
20. There is a published survey about the Risk-Free Rate (R_F) and the Market Risk Premium (MRP) used "to calculate the required return to equity (Ke). The answers of more than 500 finance and economics professors, analysts and managers of companies in March 2020 were, on average, R_F = 1.3% and MRP = 6.3%. It seems that CFFA should revise their "*own studies and contrast with recognized empirical sources*", *"normalization in the long term", " inflation estimated in the long term", "regression models"* and *"normalized profitability in real terms".*

5.3. The most important error due to using a "normalized Risk-Free Rate" (NoR_F)

Two days later, Mr. Davis sent another email (Table 4) to Mr. McLane.

Table 4. Email of Mr. Davis to Mr. McLane

Dear Mr. McLane,

Following my previous Email, I must tell you and your team a very important error due to using the "normalization".

1. It is not true your statement *"The WACC we get is also identical to your WACC: 5.38255%"* (Table 2).

2. In my valuation, R_F = 0.5%, r (interest rate paid) = 2,5%, Kd (required return to Debt) = 2.5%, and CFd_1 (Cash Flow for the debt) = 5$. Value of debt = D_0 = PV (CFd; Kd) = 5 / (2.5% - 2%) = 1000$ = N_0 (Debt book value).

3. In your valuation, NoR_F = 3.0%, r = 2,5%, and CFd_1 = 5$. But Kd cannot be smaller than NoR_F. because the debt of AAA is not risk-free. If we assume that Kd = 5%, then D_0 = 5 / (5% - 2%) = 166,67$, much smaller than N_0 = 1000$.

4. As D is different than N, the equation for the WACC is: WACC = (EKe + DKd – NrT) / (E + D) = 6.23529%

5. Even if you (wrongly) assume in your valuation that the debt of AAA is risk-free and Kd = NoR_F = 3.0%, then D_0 = 5 / (3% - 2%) = 500$, much smaller than N_0 = 1000$. And WACC = 5,84733%.

6. How may you explain to my banks and my bondholders that the value of their debt with AAA is much smaller than the book value?

6. Other errors in valuations due to using unnecessary complications

"*WACC: definition, misconceptions and errors*" (http://ssrn.com/abstract=1620871) lists seven errors due to not remembering the definition of WACC. Some of the errors are found in many valuations.

"*119 common errors in company valuations*" (http://ssrn.com/abstract=1025424) contains a collection and classification of errors seen in company valuations performed by financial analysts, investment banks and financial consultants.

"The Most Common Error in Valuations using WACC"(https://ssrn.com/abstract=3512739) presents a real valuation performed by a well-known investment bank, with two common errors and with two very different values for the equity of a firm:

a) €6,9 million calculating the Present Value of expected free cash flows (FCF) discounted with the WACC rate and then, subtracting the value of debt;

b) €4,2 million calculating the Present Value of expected equity cash flows (ECF) discounted with the Ke rate (required return to equity).

"A Wrong Valuation Using WACC and the Right Solution" (https://ssrn.com/abstract=3574850) identifies the two main errors of the valuation of the investment bank and calculate the "right solution": €5,9 million. The paper also contains 210 answers and 56 comments from readers

7. Conclusion

As interest rates on Government Bonds have decreased, some analysts and consultants in Europe and in the US are using what they call "Normalized Risk-Free rate" because they do not like the world as it is. The "Normalized Risk-Free rate" is the Risk-Free rate that should exist in a world that the user usually calls "normal", but not in the world in which we live. Is it fiction or science fiction?

We show several inconsistencies in the use of "Normalized Risk-Free rate". 1) It does not exist because we cannot invest in any financial instrument and get the "Normalized Risk-Free rate" without (or with little) risk. 2) In several cases, it is higher than the cost of debt. 3) Often, it is higher than the required return to debt. 4) Most valuations do not adjust the value of Debt accordingly to the "Normalized Risk-Free rate" they use. 5) The WACC calculated by all "Normalized Risk-Free rate" is wrong.

Company valuation using discounted cash flows is based on the valuation of the Government bonds: it consists of applying the procedure used to value the Government bonds to the debt and shares of a company. This is very easy to understand (sections 1, 2 and 3).

Section 5 is a short case that may be used in class. It contains 26 comments that allow us to conclude that "normalization" is an unacceptable invention (arbitrary, makes no sense, fiction). It is also the source of important valuation errors.

A. Comments from readers

1) I enjoyed reading the student comments. I agree that the normalized risk-free rate concept is not appropriate.
2) Normalized Risk-Free Rates have no meaning to me. I guess they are smoothed.
3) I believe that working with a 'normalized' risk-free rate is an over-simplification at best.
4) Analysts should be careful in reading between the lines in terms of the assumptions behind this 'normalization'.
5) Your research is marvelous on the subject of "Normalized" Risk-Free rate.
6) I agree completely with your paper. It is NOT a good idea to use a normalized govt bond rate. The arguments you present are excellent.
7) You and Damodaran are a breath of fresh air in the stale, stagnant and questionable theories espoused by most of the community of business evaluators.
8) The risk-free rate is the risk-free rate, not something else you might conjure up, I agree.
9) I don't agree with the "normalizing" approach suggested by many analysts and consultants, but up to now I didn't dispose of convincing arguments for contrasting it.
10) I add my total agreement with the great dangers implied by the use of WACC. By far I prefer starting with the unlevered case (in my opinion the most important cost of capital figure is the unlevered one) and introducing the effect of financial leverage just in a second step.
11) Totally agreed the concept of a "normalised" risk free rate is untenable and in fact illogical. If anything, it seems to be an attempt to make amends for the consistent very low or negative rates policies of Western central banks, which do not work in the real world in the way the central bankers seem to think it will, and making up for that an analyst comes up with a normalised rate. This is illogical and not, as you say, real world.
    A normalised risk-free rate of 3% is actually a sub-investment grade bond yield.
    Better for central banks to realise that continued very low or negative rates is just storing up trouble for the future, not helping things to return to "normal".
12) I would synthesize the problem in the following way:
    a) you have to choose between two different worlds that you cannot mix: a real one and a fictional one;
    b) If you choose the fictional one, then you should change ALL the parameters accordingly: even the cost of debt, the market risk premium, the inflation rate should be all imagined according to such fictional world (that, by the way, you can imagine only assuming that it is a perfect reply of the "normal past"): undoubtedly a very big bet!
13) Your paper is really convincing about inconsistencies and errors in the use of "Normalized Risk Free rates". I shall use Section 5 in class.
14) I have after 25 years in finance in the city converted to external lecturer in finance and I teach valuation. I would argue that you should use historic risk free rates where f.ex T bills are 4%.... What do you think?
15) There can be no such thing as a normalised risk-free rate. It is either pure or it is not. If it is not, it cannot serve its true purpose, but reverts instead to being a mere benchmark rate.
16) Normalised interest rates is useful only if all interest rates and all prices are normalized. Normalisation refers to an ideal world, in which the trend is shown clearly and to my belief normalization is recommended either for teaching purposes or for a power point presentation to the Board of Directors of a large corporation.
17) I just can add that I can see Risk-Free rate as an angular coefficient on the cartesian axes (2D).
18) Normalized Risk-Free rate describe the real world where we live. It is applied because of manipulation of interest rates of Government Bonds by Central Banks. In fact, the negative interest rate just means that you will have a certainty of losses and it scientifically does not mean a reduction of risk of losses.
19) We currently use an adjusted risk free rate – some would say "normalised" but we don't use the adjusted rate in the hope we will return to some "normal" market in an historical context. A couple of points to the thinking behind our approach:
    1. We don't see any reliable model that assesses "current" market risk premiums – which we believe are inversely correlated with the risk free rate. AS such, rather than arbitrarily adjusting MRP, we adjust risk free and apply it to a longer term view of MRP. We see the MRP models to be too volatile
    2. Our risk free rate is based on an NPV approach, where we take the spot risk free and weight this on an NPV basis with the forecast risk free at the end of the current risk free bond term (for Australia, that is 10 years)
    3. Our cost of debt is calculated with reference to the adjusted risk free, applying a consistent methodology
    4. A lot of our DCF based work is modelled to an equity level so the short term cost of debt advantages are modelled into the cash flows

20) It may not be an ideal solution but it addresses a number of the issues you raised, still references the rate that you can achieve and see in the market today and recognises MRP is a longer term premium and may not reflect current market dynamics.
21) The risk-free rate is the risk-free rate, not something else you might conjure up, I agree.
22) Regarding the 'normalised' risk-free rate (NRfR) I am not sympathizing with the use of it. Despite evidences, still, some of my colleagues do use it. I believe the reason why is that they have a subscription to the Cost of Capital Navigator, and the NRfR would be one of the alternatives suggested by the website (usually the first one). Somehow, they transfer the reputation from D&Ph system of data, information & intelligence for valuers (that is rock solid widely), to this particular case, in which D&Ph could be wrong (well, I think they might).
23) I am happy for this article and I will forward it to some of my colleagues that I know for sure that use NRfR.
24) Thanks for the case study, I will use it in my valuation master class.
25) Normalization introduces by definition a perspective basis and hence bias and therefore makes this concept largely inapplicable as a principle based use of capm or any other finance metric.
26) The derivation of the normalization adjustment in itself creates complexity which does not add in equal proportion, robustness to the math (or disproportionately increases complexity relative to accuracy).
27) Your articles are stimulating. I recommend them to my students.
28) I am in total agreement with you. In these abnormal circumstances there is nothing normal.
29) As usual, your article is a howler! I loved it. It's even more pertinent today when you read the Times of London article that the Bank of England is starting to ask it's client banks "if interest rates go to 0.001% would that be OK for your computer systems? Could you handle rates as low as 0.000001%? "oh, no, don't worry – we're not really planning on doing this, no, no, no, nooooo. We're just asking the question as a hypothetical right now. And don't worry, rates aren't going to go negative."
30) The valuation profession is becoming so encrusted with archaic models and assumptions its absolutely amazing. What I've found in my 35 years of practice is that you can jump up and down and point out the folly of these positions, but it's like the Chinese education system. If you get them at an early, impressionable age you can teach anything. It becomes like showing a flat-earther that the world is round: "Ha! You can prove it all you want, I still don't believe you."
31) I believe that McKinsey (Koller et al) may use some variation of the "normalized risk-free rate" concept in their approach to valuation, for the same reasons stated in your paper. At least they did in the early days of QE (2010); not sure if that has changed or not.
32) If the DCF method corresponds to a fair value as defined by IFRS, we should apply data close to spot data.
33) I particularly like this SSRN article…the standard assumptions around NoRf conclusions are producing some very strange results by analysts—so any wisdom on the subject is helpful.
34) The normalized rfr is obviously wrong in many ways.
35) Indeed, there is no basis to build NoRf without adopting a number of irrational assumptions.
36) I took the liberty to forward your paper to my son who studies finance.
37) I need to tell you this. Still after 30 years I recall one of your sessions when you asked the class our opinion about the cases. Many students started requesting cases from Nike, Volkswagen etc. After listening to this for maybe 5 minutes you went to the board and added 5 or 6 digits to all the number and then you stated "now it is Coca Cola". I hope nearly all got the point.
38) I found the McKinsey approach somewhat suspect (although understandable) 10 years ago, but it has not aged well. It is one thing to average out the noise inherent in volatile short-term market yields; it is quite another to "normalize" current market rates which have resulted from fundamental macro or other changes (eg QE). Your comments captured well the various flaws and inconsistencies in the McKinsey approach.
39) Normalised interest rates are pure fiction.

<u>In Spanish</u>

1) Estoy totalmente de acuerdo contigo sobre el NoR_F. De hecho, digo a los alumnos que utilicen en la valoración el YTM o TIR del bono alemán a 10 años.
2) Estoy totalmente de acuerdo con tu critica. Una tasa libre de riesgo normalizada es no aceptar la realidad que estamos atravesando hoy y cualquier calculo que se realice va a ser muy subjetivo. Esa tasa no sabemos qué representa.
3) No te tuve como profesor pero estoy de acuerdo contigo. Aunque sé que es un error garrafal utilizar la Tasa sin riesgo normalizada, yo la utilizo en mis informes porque mis dos jefes son auténticos fans.
4) Mi conclusión es que no hay necesidad de usar, como tú dices claramente, la "ciencia ficción".
5) La tabla 1 parece una parrafada de "un vendedor de crecepelo calvo".
6) Mi director es un fenómeno haciendo presentaciones y hablando de las previsiones del PIB. Nos obliga a utilizar a todos la misma tasa normalizada. Dice que así parece más robusta la valoración.
7) De acuerdo completamente con tu artículo.

8) Mi socio es un gran vendedor. Sabe mucho de contabilidad pero muy poco de finanzas (aunque él cree que lo sabe todo). Le gusta que utilicemos tasas normalizadas porque, según él, es lo moderno.
9) No entiendo la utilización de tipos medios, no le veo el sentido.
10) Aunque rechazas la aplicación de una tasa libre de riesgo ajustada, puede que no estés considerando todo el análisis que se hace para llegar a su aplicación (en estos momentos, generalizada en España no solo por las consultoras sino por bancos de inversión y empresas de toda índole en sus valoraciones internas y tests de deterioro).
Nosotros hacemos directamente el análisis del Riesgo total de mercado (el Ke de mercado), y es ahí donde nos posicionamos. Para ello hacemos diferentes análisis de rentabilidades de cotizadas, tanto históricos como prospectivos (en base a estimaciones de dividendos o beneficios). Lo hacemos en términos reales y nominales para aislar el efecto de la inflación. Ahora nuestra posición es que para España el RM total debe estar entre el 9,5% y el 10% nominal, con inflaciones a largo plazo entre el 1,7-1,8%.
Y después, queremos ver sobre qué parte de este RM total hay que aplicar la beta para ajustar el riesgo sectorial. Si hiciéramos lo que dice tu papel, y asumiendo que el bono a 10 años está en casi el 0%, la beta se aplicaría sobre todo el RM, y creemos que se estaría exagerando las diferencias de riesgo sectorial. Por eso lo normalizamos.
Espero te ofrezca algo más de luz sobre nuestras razones para hacerlo. No es porque estemos locos o no sepamos. Esta decisión se tomó cuando el bono español estaba en 2011 en el 7%, es decir, por las razones contrarias (podíamos infraponderar el riesgo sectorial).
11) Estoy de acuerdo con lo absurdo de la búsqueda de una "normalización histórica" en los cálculos del DCF.
12) La normalización es ridícula. Pero algunos quieren que estos temas parezcan más "intelectuales", cuando no es más que cuestión de hacer sumas, restas, (y algunas divisiones).
13) Yo no creo que el uso de un Risk Free normalizado sea porque el mundo real no le guste al que lo usa, mi interpretación es que hay un demandante de bonos del estado que altera su precio, y por tanto su yield. Yo no creo que sea precio de mercado un mercado donde hay un monstruo que puede imprimir trillones de EUROS o USD, que su objetivo no es ganar dinero sino instrumentar objetivos de política monetaria, y que compra bonos con fines muy distintos al del resto de demandantes. Eso quiebra la dinámica de formación de precios, y un bono del estado ya no es un referente de lo que yo pagaría por prestarle dinero a un estado en función de sus riesgos políticos, financieros y demás. La evidencia es que España emite letras a corto con interés negativo, en un contexto de deuda pública por encima del 110% y un déficit público cercano al 14% este año. Eso no es racional. Por tanto, utilizar un sinsentido para calcular el Ke utilizando el CAPM puede ser otro sinsentido.
14) Creo que los que normalizan confunden tasa esperada, requerida e histórica.
15) No me gusta el término "normal" y menos las tasas "normalizadas.

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