CHAPTER 18

The Valuation of Common Stocks

In Chapter 17 it was noted that one purpose of financial analysis is to identify mispriced securities. Fundamental analysis was mentioned as one approach for conducting a search for such securities. With this approach the security analyst makes estimates of such things as the firm's future earnings and dividends. If these estimates are substantially different from the average estimates of other analysts but are felt to be more accurate, then from the viewpoint of the security analyst, a mispriced security will have been identified. If it is also felt that the market price of the security will adjust to reflect these more accurate estimates, then the security will be expected to have an abnormal rate of return. Accordingly, the analyst will issue either a buy or sell recommendation, depending on the direction of the anticipated price adjustment. Based on the capitalization of income method of valuation, dividend discount models have been frequently used by fundamental analysts as a means of identifying mispriced stocks. This chapter will discuss dividend discount models and how they can be related to models based on price-earnings ratios.

18.1 CAPITALIZATION OF INCOME METHOD OF VALUATION

There are many ways to implement the fundamental analysis approach to identifying mispriced securities. A number of them are either directly or indirectly related to what is sometimes referred to as the **capitalization of income method of valuation.**[1] This method states that the "true" or "intrinsic" value of any asset is based on the cash flows that the investor expects to receive in the future from owning the asset. Because these cash flows are expected in the future, they are adjusted by a **discount rate** to reflect not only the time value of money but also the riskiness of the cash flows.

Algebraically, the intrinsic value of the asset V is equal to the sum of the present values of the expected cash flows:

$$\begin{aligned} V &= \frac{C_1}{(1+k)^1} + \frac{C_2}{(1+k)^2} + \frac{C_3}{(1+k)^3} + \cdots \\ &= \sum_{t=1}^{\infty} \frac{C_t}{(1+k)^t} \end{aligned} \tag{18.1}$$

where C_t denotes the expected cash flow associated with the asset at time t and k is the appropriate discount rate for cash flows of this degree of risk. In this equation the discount rate is assumed to be the same for all periods. Because the symbol ∞ above the summation sign in the equation denotes infinity, all expected cash flows, from immediately after making the investment until infinity, will be discounted at the same rate in determining V.[2]

18.1.1 Net Present Value

For the sake of convenience, let the current moment in time be denoted as zero, or $t = 0$. If the cost of purchasing an asset at $t = 0$ is P, then its **net present value** (NPV) is equal to the difference between its intrinsic value and cost, or:

$$\begin{aligned} \text{NPV} &= V - P \\ &= \left[\sum_{t=1}^{\infty} \frac{C_t}{(1+k)^t}\right] - P. \end{aligned} \tag{18.2}$$

The NPV calculation shown here is conceptually the same as the NPV calculation made for capital budgeting decisions that has long been advocated in introductory finance textbooks. Capital budgeting decisions involve deciding whether or not a given investment project should be undertaken. (For example, should a new machine be purchased?) In making this decision, the focal point is the NPV of the project. Specifically, an investment project is viewed favorably if its NPV is positive, and unfavorably if its NPV is negative. For a simple project involving a cash outflow now (at $t = 0$) and expected cash inflows in the future, a positive NPV means that the present value of all the expected cash inflows is greater than the cost of making the investment. Conversely, a negative NPV means that the present value of all the expected cash inflows is less than the cost of making the investment.

The same views about NPV apply when financial assets (such as a share of common stock), instead of real assets (such as a new machine), are being considered for purchase. That is, a financial asset is viewed favorably and said to be underpriced (or undervalued) if NPV > 0. Conversely, a financial asset is viewed unfavorably and said to be overpriced or (overvalued) if NPV < 0. From Equation (18.2), this is equivalent to stating that a financial asset is underpriced if $V > P$:

$$\sum_{t=1}^{\infty} \frac{C_t}{(1+k)^t} > P. \tag{18.3}$$

Conversely, the asset is overvalued if $V < P$:

$$\sum_{t=1}^{\infty} \frac{C_t}{(1+k)^t} < P.$$

18.1.2 Internal Rate of Return

Another way of making capital budgeting decisions in a manner that is similar to the NPV method involves calculating the **internal rate of return** (IRR) associated with the investment project. With IRR, NPV in Equation (18.2) is set equal to zero and the discount rate becomes the unknown that must be calculated. That is, the IRR for a given investment is the discount rate that makes the NPV of the investment equal to zero. Algebraically, the procedure involves solving the following equation for the internal rate of return k^*:

$$0 = \sum_{t=1}^{\infty} \frac{C_t}{(1+k^*)^t} - P. \tag{18.5}$$

Equivalently, Equation (18.5) can be rewritten as:

$$P = \sum_{t=1}^{\infty} \frac{C_t}{(1+k^*)^t}. \tag{18.6}$$

The decision rule for IRR involves comparing the project's IRR (denoted by k^*) with the required rate of return for an investment of similar risk (denoted by k). Specifically, the investment is viewed favorably if $k^* > k$, and unfavorably if $k^* < k$. As with NPV, the same decision rule applies if either a real asset or a financial asset is being considered for possible investment.[3]

18.1.3 Application to Common Stocks

This chapter is concerned with using the capitalization of income method to determine the intrinsic value of common stocks. Because the cash flows associated with an investment in any particular common stock are the dividends that are expected to be paid throughout the future on the shares purchased, the models suggested by this method of valuation are often known as **dividend discount models** (DDMs).[4] Accordingly, D_t will be used instead of C_t to denote the expected cash flow in period t associated with a particular common stock, resulting in the following restatement of Equation (18.1):

$$\begin{aligned} V &= \frac{D_1}{(1+k)^1} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \cdots \\ &= \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t} \end{aligned} \tag{18.7}$$

Usually the focus of DDMs is on determining the "true" or "intrinsic" value of one share of a particular company's common stock, even if larger size purchases are being contemplated. This is because it is usually assumed that larger size purchases can be made at a cost that is a simple multiple of the cost of one share. (For example, the cost of 1,000 shares is usually assumed to be 1,000 times the cost of one share.) Thus the numerator in DDMs is the cash dividends per share that are expected in the future.

However, there is a complication in using Equation (18.7) to determine the intrinsic value of a share of common stock. In particular, in order to use this equation the investor must forecast *all* future dividends. Because a common stock does not have a fixed lifetime, this suggests that an infinitely long stream of dividends must be forecast. Although this may seem to be an impossible task, with the addition of certain assumptions, the equation can be made tractable (that is, usable).

These assumptions center on dividend growth rates. That is, the dividend per share at any time t can be viewed as being equal to the dividend per share at time $t - 1$ times a dividend growth rate of g_t,

$$D_t = D_{t-1}(1 + g_t) \tag{18.8}$$

or, equivalently:

$$\frac{D_t - D_{t-1}}{D_{t-1}} = g_t. \tag{18.9}$$

For example, if the dividend per share expected at $t = 2$ is \$4 and the dividend per share expected at $t = 3$ is \$4.20, then $g_3 = (\$4.20 - \$4)/\$4 = 5\%$.

The different types of tractable DDMs reflect different sets of assumptions about dividend growth rates, and are presented next. The discussion begins with the simplest case, the zero-growth model.

18.2 THE ZERO-GROWTH MODEL

One assumption that could be made about future dividends is that they will remain at a fixed dollar amount. That is, the dollar amount of dividends per share that were paid over the past year D_0 will also be paid over the next year D_1, and the year after that D_2, and the year after that D_3, and so on—that is,

$$D_0 = D_1 = D_2 = D_3 = \cdots = D_\infty.$$

This is equivalent to assuming that all the dividend growth rates are zero, because if $g_t = 0$, then $D_t = D_{t-1}$ in Equation (18.8). Accordingly, this model is often referred to as the **zero-growth** (or no-growth) **model.**

18.2.1 Net Present Value

The impact of this assumption on Equation (18.7) can be analyzed by noting what happens when D_t is replaced by D_0 in the numerator:

$$V = \sum_{t=1}^{\infty} \frac{D_0}{(1 + k)^t}. \tag{18.10}$$

Fortunately, Equation (18.10) can be simplified by noting that D_0 is a fixed dollar amount, which means that it can be written outside the summation sign:

$$V = D_0 \left[\sum_{t=1}^{\infty} \frac{1}{(1+k)^t} \right]. \qquad (18.11)$$

The next step involves using a property of infinite series from mathematics. If $k > 0$, then it can be shown that:

$$\sum_{t=1}^{\infty} \frac{1}{(1+k)^t} = \frac{1}{k}. \qquad (18.12)$$

Applying this property to Equation (18.11) results in the following formula for the zero-growth model:

$$V = \frac{D_0}{k_0}. \qquad (18.13)$$

Because $D_0 = D_1$, Equation (18.13) is written sometimes as:

$$V = \frac{D_1}{k}. \qquad (18.14)$$

Example

As an example of how this DDM can be used, assume that the Zinc Company is expected to pay cash dividends amounting to \$8 per share into the indefinite future and has a required rate of return of 10%. Using either Equation (18.13) or Equation (18.14), it can be seen that the value of a share of Zinc stock is equal to \$80 (= \$8/.10). With a current stock price of \$65 per share, Equation (18.2) would suggest that the NPV per share is \$15 (= \$80 − \$65). Equivalently, as V = \$80 > P = \$65, the stock is underpriced by \$15 per share and would be a candidate for purchase.

18.2.2 Internal Rate of Return

Equation (18.13) can be reformulated to solve for the IRR on an investment in a zero-growth security. First, the security's current price P is substituted for V, and second, k^* is substituted for k. These changes result in:

$$P = \frac{D_0}{k^*}$$

which can be rewritten as:

$$k^* = \frac{D_0}{P} \qquad (18.15a)$$

$$= \frac{D_1}{P}. \qquad (18.15b)$$

Example

Applying this formula to the stock of Zinc indicates that $k^* = 12.3\%$ (= \$8/\$65). Because the IRR from an investment in Zinc exceeds the required rate of return on Zinc ($12.3\% > 10\%$), this method also indicates that Zinc is underpriced.[5]

18.2.3 Application

The zero-growth model may seem quite restrictive. After all, it seems unreasonable to assume that a given stock will pay a fixed dollar-size dividend forever. Although such a criticism has validity for common stock valuation, there is one particular situation where this model is quite useful.

Specifically, whenever the intrinsic value of a share of high-grade preferred stock is to be determined, the zero-growth DDM will often be appropriate. This is because most preferred stock is nonparticipating, meaning that it pays a fixed dollar-size dividend that will not change as earnings per share change. Furthermore, for high-grade preferred stock these dividends are expected to be paid regularly into the foreseeable future. Why? Because preferred stock does not have a fixed lifetime, and, by restricting the application of the zero growth model to high-grade preferred stocks, the chance of a suspension of dividends is remote.[6]

18.3 THE CONSTANT-GROWTH MODEL

The next type of DDM to be considered is one that assumes that dividends will grow from period to period at the same rate forever, and is therefore known as the **constant growth model.**[7] Specifically, the dividends per share that were paid over the previous year D_0 are expected to grow at a given rate g, so that the dividends expected over the next year D_1 are expected to be equal to $D_0(1 + g)$. Dividends the year after that are again expected to grow by the same rate g, meaning that $D_2 = D_1(1 + g)$. Because $D_1 = D_0(1 + g)$, this is equivalent to assuming that $D_2 = D_0(1 + g)^2$ and, in general:

$$D_t = D_{t-1}(1 + g) \tag{18.16a}$$

$$= D_0(1 + g)^t. \tag{18.16b}$$

18.3.1 Net Present Value

The impact of this assumption on Equation (18.7) can be analyzed by noting what happens when D_t is replaced by $D_0(1 + g)^t$ in the numerator:

$$V = \sum_{t=1}^{\infty} \frac{D_0(1 + g)^t}{(1 + k)^t}. \tag{18.17}$$

Fortunately, Equation (18.17) can be simplified by noting that D_0 is a
lar amount, which means that it can be written outside the summation

$$V = D_0\left[\sum_{t=1}^{\infty} \frac{(1+g)^t}{(1+k)^t}\right].$$

The next step involves using a property of infinite series from mathe
If $k > g$, then it can be shown that:

$$\sum_{t=1}^{\infty} \frac{(1+g)^t}{(1+k)^t} = \frac{1+g}{k-g}. \quad (18.1$$

Substituting Equation (18.19) into Equation (18.18) results in the valuation
mula for the constant-growth model:

$$V = D_0\left(\frac{1+g}{k-g}\right). \quad (18.20)$$

Sometimes Equation (18.20) is rewritten as:

$$V = \frac{D_1}{k-g} \quad (18.21)$$

because $D_1 = D_0(1 + g)$.

Example

As an example of how this DDM can be used, assume that during the past year the Copper Company paid dividends amounting to $1.80 per share. The forecast is that dividends on Copper stock will increase by 5% per year into the indefinite future. Thus dividends over the next year are expected to equal $1.89 [= $1.80 × (1 + .05)]. Using Equation (18.20) and assuming a required rate of return k of 11%, it can be seen that the value of a share of Copper stock is equal to $31.50 [= $1.80 × (1 + .05)/(.11 − .05) = $1.89/(.11 − .05)]. With a current stock price of $40 per share, Equation (18.2) would suggest that the NPV per share is −$8.50 (= $31.50 − $40). Equivalently, as V = $31.50 < P = $40, the stock is overpriced by $8.50 per share and would be a candidate for sale if currently owned.

18.3.2 Internal Rate of Return

Equation (18.20) can be reformulated to solve for the IRR on an investment in a constant-growth security. First, the current price of the security P is substituted for V and then k^* is substituted for k. These changes result in:

$$P = D_0\left(\frac{1+g}{k^*-g}\right). \quad (18.22)$$

which can be rewritten as:

$$k^* = \frac{D_0(1 + g)}{P} + g \tag{18.23a}$$

$$= \frac{D_1}{P} + g. \tag{18.23b}$$

Example

Applying this formula to the stock of Copper indicates that $k^* = 9.72\%$ {= [\$1.80 × (1 + .05)/\$40] + .05 = (\$1.89/\$40) + .05}. Because the required rate of return on Copper exceeds the IRR from an investment in Copper (11% > 9.72%), this method also indicates that Copper is overpriced.

18.3.3 Relationship to the Zero-Growth Model

The zero-growth model of the previous section can be shown to be a special case of the constant-growth model. In particular, if the growth rate g is assumed to be equal to zero, then dividends will be a fixed dollar amount forever, which is the same as saying that there will be zero growth. Letting $g = 0$ in Equations (18.20) and (18.23a) results in two equations that are identical to Equations (18.13) and (18.15a), respectively.

Even though the assumption of constant dividend growth may seem less restrictive than the assumption of zero dividend growth, it may still be viewed as unrealistic in many cases. However, as will be shown next, the constant-growth model is important because it is embedded in the multiple-growth model.

18.4 THE MULTIPLE-GROWTH MODEL

A more general DDM for valuing common stocks is the **multiple-growth model.** With this model, the focus is on a time in the future (denoted by T) after which dividends are expected to grow at a constant rate g. Although the investor is still concerned with forecasting dividends, these dividends do not need to have any specific pattern until this time, after which they will be assumed to have the specific pattern of constant growth. The dividends up until T ($D_1, D_2, D_3, \ldots, D_T$) will be forecast individually by the investor. (The investor also forecasts when this time T will occur.) Thereafter dividends are assumed to grow by a constant rate g that the investor must also forecast, meaning that:

$$D_{T+1} = D_T(1 + g)$$
$$D_{T+2} = D_{T+1}(1 + g) = D_T(1 + g)^2$$
$$D_{T+3} = D_{T+2}(1 + g) = D_T(1 + g)^3$$

...on. Figure 18.1 presents a time line of dividends and growth rates associ-...th the multiple-growth model.

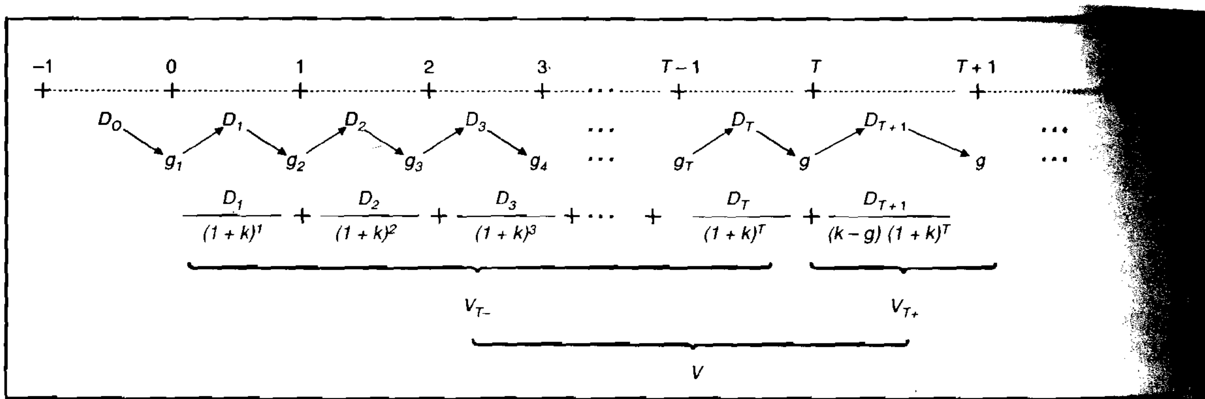

Figure 18.1
Time Line for Multiple-Growth Model

18.4.1 Net Present Value

In determining the value of a share of common stock with the multiple-growth model, the present value of the forecast stream of dividends must be determined. This can be done by dividing the stream into two parts, finding the present value of each part, and then adding these two present values together.

The first part consists of finding the present value of all the forecast dividends that will be paid up to and including time T. Denoting this present value by V_{T-}, it is equal to:

$$V_{T-} = \sum_{t=1}^{T} \frac{D_t}{(1+k)^t}. \tag{18.24}$$

The second part consists of finding the present value of all the forecast dividends that will be paid after time T, and involves the application of the constant-growth model. The application begins by imagining that the investor is not at time zero but is at time T, and has not changed his or her forecast of dividends for the stock. This means that the next period's dividend D_{T+1} and all those thereafter are expected to grow at the rate g. Thus the investor would be viewing the stock as having a constant growth rate, and its value at time T, V_T, could be determined with the constant-growth model of Equation (18.21):

$$V_T = D_{T+1}\left(\frac{1}{k-g}\right). \tag{18.25}$$

One way to view V_T is that it represents a lump sum that is just as desirable as the stream of dividends after T. That is, an investor would find a lump sum of cash equal to V_T, to be received at time T, to be equally desirable as the stream of dividends D_{T+1}, D_{T+2}, D_{T+3}, and so on. Now given that the investor is at time zero, not at time T, the present value at $t = 0$ of the lump sum V_T must be determined. This is done simply by discounting it for T periods at the rate k, resulting in the following formula for finding the present value at time zero for all dividends after T, denoted V_{T+}:

$$V_{T+} = V_T\left[\frac{1}{(1+k)^T}\right] \tag{18.26}$$
$$= \frac{D_{T+1}}{(k-g)(1+k)^T}.$$

Having found the present value of all dividends up to and including time T with Equation (18.24), and the present value of all dividends after time T with Equation (18.26), the value of the stock can be determined by summing up these two amounts:

$$V = V_{T-} + V_{T+}$$
$$= \sum_{t=1}^{T}\frac{D_t}{(1+k)^t} + \frac{D_{T+1}}{(k-g)(1+k)^T}. \tag{18.27}$$

Figure 18.1 illustrates the valuation procedure for the multiple-growth DDM that is given in Equation (18.27).

Example

As an example of how this DDM can be used, assume that during the past year the Magnesium Company paid dividends amounting to \$.75 per share. Over the next year, Magnesium is expected to pay [illegible] [illegible] $D_0)/D_0 = (\$2 - \$.75)/\$.75 = 167\%$. The year after that, dividends are expected to amount to \$3 per share, indicating that $g_2 = (D_2 - D_1)/D_1 = (\$3 - \$2)/\$2 = 50\%$. At this time, the forecast is that dividends will grow by 10% per year indefinitely, indicating that $T = 2$ and $g = 10\%$. Consequently, $D_{T+1} = D_3 = \$3(1 + .10) = \$3.30$. Given a required rate of return on Magnesium shares of 15%, the values of V_{T-} and V_{T+} can be calculated as follows:

$$V_{T-} = \frac{\$2}{(1+.15)^1} + \frac{\$3}{(1+.15)^2}$$
$$= \$4.01$$
$$V_{T+} = \frac{\$3.30}{(.15-.10)(1+.15)^2}$$
$$= \$49.91.$$

ing V_{T-} and V_{T+} results in a value for V of \$4.01 + \$49.91 = \$53.92. With nt stock price of \$55 per share, Magnesium appears to be fairly priced. Magnesium is not significantly mispriced because V and P are nearly of ue.

18.4.2 Internal Rate of Return

The zero-growth and constant-growth models have equations for V that reformulated in order to solve for the IRR on an investment in a stock. U nately, a convenient expression similar to Equations (18.15a), (18.15b), (1 and (18.23b) is not available for the multiple-growth model. This can be noting that the expression for IRR is derived by substituting P for V, and k^* in Equation (18.27):

$$P = \sum_{t=1}^{T} \frac{D_t}{(1 + k^*)^t} + \frac{D_{T+1}}{(k^* - g)(1 + k^*)^T}. \qquad (18.28)$$

This equation cannot be rewritten with k^* isolated on the left-hand side, meaning that a closed-form expression for IRR does not exist for the multiple-growth model.

However, all is not lost. It is still possible to calculate the IRR for an investment in a stock conforming to the multiple-growth model by using an "educated" trial-and-error method. The basis for this method is in the observation that the right-hand side of Equation (18.28) is simply equal to the present value of the dividend stream, where k^* is used as the discount rate. Hence the larger the value of k^*, the smaller the value of the right-hand side of Equation (18.28). The trial-and-error method proceeds by initially using an estimate for k^*. If the resulting value on the right-hand side of Equation (18.28) is larger than P, then a larger estimate of k^* is tried. Conversely, if the resulting value is smaller than P, then a smaller estimate of k^* is tried. Continuing this search process, the investor can hone in on the value of k^* that makes the right-hand side equal P on the left-hand side. Fortunately, it is a relatively simple matter to program a computer to conduct the search for k^* in Equation (18.28). Most spreadsheets include a function that does so automatically.

Example

Applying Equation (18.28) to the Magnesium Company results in:

$$\$55 = \frac{\$2}{(1 + k^*)^1} + \frac{\$3}{(1 + k^*)^2} + \frac{\$3.30}{(k^* - .10)(1 + k^*)^2}. \qquad (18.29)$$

Initially a rate of 14% is used in attempting to solve this equation for k^*. Inserting 14% for k^* in the right-hand side of Equation (18.29) results in a value of \$67.54. Earlier 15% was used in determining V and resulted in a value of \$53.92. This means that k^* must have a value between 14% and 15%, since \$55 is between \$67.54 and \$53.92. If 14.5% is tried next, the resulting value is \$59.97, suggesting that a higher rate should be tried. If 14.8% and 14.9% are subsequently tried, the respective resulting values are \$56.18 and \$55.03. As \$55.03 is the closest to P, the IRR associated with an investment in Magnesium is 14.9%. Given a required return of 15% and an IRR of approximately that amount, the stock of Magnesium appears to be fairly priced.

18.4.3 Relationship to the Constant-Growth Model

The constant-growth model can be shown to be a special case of the multiple-growth model. In particular, if the time when constant growth is assumed to begin is set equal to zero, then:

$$V_{T-} = \sum_{t=1}^{T} \frac{D_t}{(1+k)^t} = 0$$

and

$$V_{T+} = \frac{D_{T+1}}{(k-g)(1+k)^T} = \frac{D_1}{k-g}$$

because $T = 0$ and $(1 + k)^0 = 1$. Given that the multiple-growth model states that $V = V_{T-} + V_{T+}$, it can be seen that setting $T = 0$ results in $V = D_1/(k - g)$, a formula that is equivalent to the formula for the constant-growth model.

18.4.4 Two-Stage and Three-Stage Models

Two dividend discount models that investors sometimes use are the two-stage model and the three-stage model.[8] The two-stage model assumes that a constant growth rate g_1 exists only until some time T, when a different growth rate g_2 is assumed to begin and continue thereafter. The three-stage model assumes that a constant growth rate g_1 exists only until some time T_1, when a second growth rate is assumed to begin and last until a later time T_2, when a third growth rate is assumed to begin and last thereafter. By letting V_{T+} denote the present value of all dividends after the last growth rate has begun and V_{T-} the present value of all the preceding dividends, it can be seen that these models are just special cases of the multiple-growth model.

In applying the capitalization of income method of valuation to common stocks, it might seem appropriate to assume that the stock will be sold at some point in the future. In this case the expected cash flows would consist of the dividends up to that point as well as the expected selling price. Because dividends after the selling date would be ignored, the use of a dividend discount model may seem to be improper. However, as will be shown next, this is not so.

18.5 VALUATION BASED ON A FINITE HOLDING PERIOD

capitalization of income method of valuation involves discounting all divi- that are expected throughout the future. Because the simplified models growth, constant growth, and multiple growth are based on this method, involve a future stream of dividends. Upon reflection it may seem that dels are relevant only for an investor who plans to hold a stock forever, uch an investor would expect to receive this stream of future dividends.

But what about an investor who plans to sell the stock in a year? In this situation, the cash flows that the investor expects to receive from purchasing a share of the stock are equal to the dividend expected to be paid one year from now (for ease of exposition, it is assumed that common stocks pay dividends annually) and the expected selling price of the stock. Thus it would seem appropriate to determine the intrinsic value of the stock to the investor by discounting these two cash flows at the required rate of return as follows:

$$V = \frac{D_1 + P_1}{1 + k} = \frac{D_1}{1 + k} + \frac{P_1}{1 + k} \tag{18.30}$$

where D_1 and P_1 are the expected dividend and selling price at $t = 1$, respectively.

In order to use Equation (18.30), the expected price of the stock at $t = 1$ must be estimated. The simplest approach assumes that the selling price will be based on the dividends that are expected to be paid after the selling date. Thus the expected selling price at $t = 1$ is:

$$P_1 = \frac{D_2}{(1 + k)^1} + \frac{D_3}{(1 + k)^2} + \frac{D_4}{(1 + k)^3} + \cdots = \sum_{t=2}^{\infty} \frac{D_t}{(1 + k)^{t-1}}. \tag{18.31}$$

Substituting Equation (18.31) for P_1 in the right-hand side of Equation (18.30) results in:

$$\begin{aligned} V &= \frac{D_1}{1 + k} + \left[\frac{D_2}{(1 + k)^1} + \frac{D_3}{(1 + k)^2} + \frac{D_4}{(1 + k)^3} + \cdots\right]\left(\frac{1}{1 + k}\right) \\ &= \frac{D_1}{(1 + k)^1} + \frac{D_2}{(1 + k)^2} + \frac{D_3}{(1 + k)^3} + \frac{D_4}{(1 + k)^4} + \cdots \\ &= \sum_{t=1}^{\infty} \frac{D_t}{(1 + k)^t} \end{aligned}$$

which is exactly the same as Equation (18.7). Thus valuing a share of common stock by discounting its dividends up to some point in the future and its expected selling price at that time is equivalent to valuing stock by discounting all future dividends. Simply stated, the two are equivalent because the expected selling price is itself based on dividends to be paid after the selling date. Thus Equation (18.7), as well as the zero-growth, constant-growth, and multiple-growth models that are based on it, is appropriate for determining the intrinsic value of a share of common stock regardless of the length of the investor's planned holding period.

Example

As an example, reconsider the common stock of the Copper Company. Over the past year it was noted that Copper paid dividends of $1.80 per share, with the forecast that the dividends would grow by 5% per year forever. This means that dividends over the next two years (D_1 and D_2) are forecast to be \$1.89 [= \$1.80 × (1 + .05)] and \$1.985 [= \$1.89 × (1 + .05)], respectively. If the investor plans to sell the stock after one year, the selling price could be estimated by noting that at $t = 1$, the forecast of dividends for the forthcoming year would be D_2, or \$1.985. Thus the anticipated selling price at $t = 1$, denoted P_1, would be equal to \$33.08 [= \$1.985/(.11 − .05)]. Accordingly, the intrinsic value of Copper to such an investor would equal the present value of the expected cash flows, which are D_1 = \$1.89 and P_1 = \$33.08. Using Equation (18.30) and assuming a required rate of 11%, this value is equal to \$31.50 [= (\$1.89 + \$33.08)/(1 + .11)]. Note that this is the same amount that was calculated earlier when all the dividends from now to infinity were discounted using the constant-growth model: $V = D_1/(k - g)$ = \$1.89/(.11 − .05) = \$31.50.

18.6 MODELS BASED ON PRICE-EARNINGS RATIOS

Despite the inherent sensibility of DDMs, many security analysts use a much simpler procedure to value common stocks. First, a stock's earnings per share over the forthcoming year E_1 are estimated, and then the analyst (or someone else) specifies a "normal" **price-earnings ratio** for the stock. The product of these two numbers gives the estimated future price P_1. Together with estimated dividends D_1 to be paid during the period and the current price P, the estimated return on the stock over the period can be determined:

$$\text{Expected return} = \frac{(P_1 - P) + D_1}{P} \tag{18.32}$$

where $P_1 = (P_1/E_1) \times E_1$.

Some security analysts expand this procedure, estimating earnings per share and price-earnings ratios for optimistic, most likely, and pessimistic scenarios to produce a rudimentary probability distribution of a security's return. Other analysts determine whether a stock is underpriced or overpriced by comparing the stock's actual price-earnings ratio with its "normal" price-earnings ratio, as will be shown next.[10]

In order to make this comparison, Equation (18.7) must be rearranged and some new variables introduced. To begin, it should be noted that earnings per share E_t are related to dividends per share D_t by the firm's **payout ratio** p_t,

$$D_t = p_t E_t. \tag{18.33}$$

...more, if an analyst has forecast earnings-per-share and payout ratios, ...he has implicitly forecast dividends.

...ion (18.33) can be used to restate the various DDMs where the focus is ...ing what the stock's price-earnings ratio should be instead of on esti... intrinsic value of the stock. In order to do so, $p_t E_t$ is substituted for D_t

in the right-hand side of Equation (18.7), resulting in a general formula for determining a stock's intrinsic value that involves discounting earnings:

$$V = \frac{D_1}{(1+k)^1} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \cdots$$
$$= \frac{p_1E_1}{(1+k)^1} + \frac{p_2E_2}{(1+k)^2} + \frac{p_3E_3}{(1+k)^3} + \cdots$$
$$= \sum_{t=1}^{\infty} \frac{p_tE_t}{(1+k)^t}. \tag{18.34}$$

Earlier it was noted that dividends in adjacent time periods could be viewed as being "linked" to each other by a dividend growth rate g_t. Similarly, earnings per share in any year t can be "linked" to earnings per share in the previous year $t - 1$ by a growth rate in earnings per share, g_{et},

$$E_t = E_{t-1}(1 + g_{et}). \tag{18.35}$$

This implies that

$$E_1 = E_0(1 + g_{e1})$$
$$E_2 = E_1(1 + g_{e2}) = E_0(1 + g_{e1})(1 + g_{e2})$$
$$E_3 = E_2(1 + g_{e3}) = E_0(1 + g_{e1})(1 + g_{e2})(1 + g_{e3})$$

and so on, where E_0 is the actual level of earnings per share over the past year, E_1 is the expected level of earnings per share over the forthcoming year, E_2 is the expected level of earnings per share for the year after E_1, and E_3 is the expected level of earnings per share for the year after E_2.

These equations relating expected future earnings per share to E_0 can be substituted into Equation (18.34), resulting in:

$$V = \frac{p_1[E_0(1 + g_{e1})]}{(1+k)^1} + \frac{p_2[E_0(1 + g_{e1})(1 + g_{e2})]}{(1+k)^2}$$
$$+ \frac{p_3[E_0(1 + g_{e1})(1 + g_{e2})(1 + g_{e3})]}{(1+k)^3} + \cdots. \tag{18.36}$$

As V is the intrinsic value of a share of stock, it represents what the stock would be selling for if it were fairly priced. It follows that V/E_0 represents what the price-earnings ratio would be if the stock were fairly priced, and is sometimes referred to as the stock's "normal" price-earnings ratio. Dividing both sides of Equation (18.36) by E_0 and simplifying results in the formula for determining the "normal" price-earnings ratio:

$$\frac{V}{E_0} = \frac{p_1(1 + g_{e1})}{(1+k)^1} + \frac{p_2(1 + g_{e1})(1 + g_{e2})}{(1+k)^2}$$
$$+ \frac{p_3(1 + g_{e1})(1 + g_{e2})(1 + g_{e3})}{(1+k)^3} + \cdots. \tag{18.37}$$

This shows that, other things being equal, a stock's "normal" price-earnings ratio will be higher:

The *greater* the expected payout ratios $(p_1, p_2, p_3, \ldots)$,
The *greater* the expected growth rates in earnings per share $(g_{e1}, g_{e2}, g_{e3}, \ldots)$,
The *smaller* the required rate of return (k).

The qualifying phrase "other things being equal" should not be overlooked. For example, a firm cannot increase the value of its shares by simply making greater payouts. This will increase $p_1, p_2, p_3, \ldots$, but will decrease the expected growth rates in earnings per share $g_{e1}, g_{e2}, g_{e3}, \ldots$. Assuming that the firm's investment policy is not altered, the effects of the reduced growth in its earnings per share will just offset the effects of the increased payouts, leaving its share value unchanged.

Earlier it was noted that a stock was viewed as underpriced if $V > P$ and overpriced if $V < P$. Because dividing both sides of an inequality by a positive constant will not change the direction of the inequality, such a division can be done here to the two inequalities involving V and P, where the positive constant is E_0. The result is that a stock can be viewed as being underpriced if $V/E_0 > P/E_0$ and overpriced if $V/E_0 < P/E_0$. Thus a stock will be underpriced if its "normal" price-earnings ratio is greater than its actual price-earnings ratio, and overpriced if its "normal" price-earnings ratio is less than its actual price-earnings ratio.

Unfortunately, Equation (18.37) is intractable, meaning that it cannot be used to estimate the "normal" price-earnings ratio for any stock. However, simplifying assumptions can be made that result in tractable formulas for estimating "normal" price-earnings ratios. These assumptions, along with the formulas, parallel those made previously regarding dividends and are discussed next.

18.6.1 The Zero-Growth Model

The zero-growth model assumed that dividends per share remained at a fixed dollar amount forever. This is most likely if earnings per share remain at a fixed dollar amount forever, with the firm maintaining a 100% payout ratio. Why 100%? Because if a lesser amount were assumed to be paid out, it would mean that the firm was retaining part of its earnings. These retained earnings would be put to some use, and would thus be expected to increase future earnings and hence dividends per share.

Accordingly, the zero-growth model can be interpreted as assuming $p_t = 1$ for all time periods and $E_0 = E_1 = E_2 = E_3$ and so on. This means that $D_0 = E_0$ and $D_1 = E_1 = D_2 = E_2$ and so on, allowing valuation Equation (18.13) to be restated as:

$$V = \frac{E_0}{k}. \tag{18.38}$$

Dividing Equation (18.38) by E_0 results in the formula for the "normal" price-earnings ratio for a stock having zero growth:

$$\frac{V}{E_0} = \frac{1}{k}. \tag{18.39}$$

Example

Earlier it was assumed that the Zinc Company was a zero-growth firm idends of $8 per share, selling for $65 a share, and having a required turn of 10%. Because Zinc is a zero-growth company, it will be assum has a 100% payout ratio which, in turn, means that E_0 = $8. At this po tion (18.38) can be used to note that a "normal" price-earnings ratio for 1/.10 = 10. As Zinc has an actual price-earnings ratio of $65/$8 = 8.1, cause $V/E_0 = 10 > P/E_0 = 8.1$, it can be seen that Zinc stock is underpric

18.6.2 The Constant-Growth Model

Earlier it was noted that dividends in adjacent time periods could be viewed being connected to each other by a dividend growth rate g_t. Similarly, it noted that earnings per share can be connected by an earnings growth rate g_e. The constant-growth model assumes that the growth rate in dividends per share will be the same throughout the future. An equivalent assumption is that earnings per share will grow at a constant rate g_e throughout the future, with the payout ratio remaining at a constant level p. This means that:

$$E_1 = E_0(1 + g_e) = E_0(1 + g_e)^1$$
$$E_2 = E_1(1 + g_e) = E_0(1 + g_e)(1 + g_e) = E_0(1 + g_e)^2$$
$$E_3 = E_2(1 + g_e) = E_0(1 + g_e)(1 + g_e)(1 + g_e) = E_0(1 + g_e)^3$$

and so on. In general, earnings in year t can be connected to E_0 as follows:

$$E_t = E_0(1 + g_e)^t. \tag{18.40}$$

Substituting Equation (18.40) into the numerator of Equation (18.34) and recognizing that $p_t = p$ results in:

$$V = \sum_{t=1}^{\infty} \frac{pE_0(1 + g_e)^t}{(1 + k)^t}$$
$$= pE_0\left[\sum_{t=1}^{\infty} \frac{(1 + g_e)^t}{(1 + k)^t}\right]. \tag{18.41}$$

The same mathematical property of infinite series given in Equation (18.19) can be applied to Equation (18.41), resulting in:

$$V = pE_0\left(\frac{1 + g_e}{k - g_e}\right). \tag{18.42}$$

It can be noted that the earnings-based constant-growth model has a numerator that is identical to the numerator of the dividend-based constant-growth model, because $pE_0 = D_0$. Furthermore, the denominators of the two models are identical. Both assertions require that the growth rates in earnings and dividends be the same (that is, $g_e = g$). Examination of the assumptions of the models reveals that these growth rates must be equal. This can be seen by recalling that constant earnings growth means:

$$E_t = E_{t-1}(1 + g_e).$$

Now when both sides of this equation are multiplied by the constant payout ratio, the result is:

$$pE_t = pE_{t-1}(1 + g_e).$$

Because $pE_t = D_t$ and $pE_{t-1} = D_{t-1}$, this equation reduces to:

$$D_t = D_{t-1}(1 + g_e)$$

which indicates that dividends in any period $t - 1$ will grow by the earnings growth rate, g_e. Because the dividend-based constant-growth model assumed that dividends in any period $t - 1$ would grow by the dividend growth rate g, it can be seen that the two growth rates must be equal for the two models to be equivalent.

Equation (18.42) can be restated by dividing each side by E_0, resulting in the following formula for determining the "normal" price-earnings ratio for a stock with constant growth:

$$\frac{V}{E_0} = p\left(\frac{1 + g_e}{k + g_e}\right). \tag{18.43}$$

Example

Earlier it was assumed that the Copper Company had paid dividends of $1.80 per share over the past year, with a forecast that dividends would grow by 5% per year forever. Furthermore, it was assumed that the required rate of return on Copper was 11%, and the current stock price was $40 per share. Now assuming that E_0 was $2.70, it can be seen that the payout ratio was equal to 66⅔% (= $1.80/$2.70). This means that the "normal" price-earnings ratio for Copper, according to Equation (18.43), is equal to 11.7 [= .6667 × (1 + .05) /(.11 − .05)]. Because this is less than Copper's actual price-earnings ratio of 14.8 (= $40/$2.70), it follows that the stock of Copper Company is overpriced.

18.6.3 The Multiple-Growth Model

Earlier it was noted that the most general DDM is the multiple-growth model, where dividends are allowed to grow at varying rates until some point in time T, after which they are assumed to grow at a constant rate. In this situation the present value of all the dividends is found by adding the present value of all dividends up to and including T, denoted by V_{T-}, and the present value of all dividends after T, denoted by V_{T+}:

$$\begin{aligned} V &= V_{T-} + V_{T+} \\ &= \sum_{t=1}^{T} \frac{D_t}{(1 + k)^t} + \frac{D_{T+1}}{(k - g)(1 + k)^T}. \end{aligned} \tag{18.27}$$

In general, earnings per share in any period t can be expressed as [illegible] equal to E_0 times the product of all the earnings growth rates from time [illegible] time t:

$$E_t = E_0(1 + g_{e1})(1 + g_{e2}) \cdots (1 + g_{et}). \tag{18.44}$$

Because dividends per share in any period t are equal to the payout ratio for [illegible] period times the earnings per share, it follows from Equation (18.44) that:

$$\begin{aligned} D_t &= p_t E_t \\ &= p_t E_0(1 + g_{e1})(1 + g_{e2}) \cdots (1 + g_{et}). \end{aligned} \tag{18.45}$$

Replacing the numerator in Equation (18.37) with the right-hand side of Equation (18.45) and then dividing both sides by E_0 gives the following formula for determining a stock's "normal" price-earnings ratio with the multiple-growth model:

$$\begin{aligned} \frac{V}{E_0} &= \frac{p_1(1 + g_{e1})}{(1 + k)^1} + \frac{p_2(1 + g_{e1})(1 + g_{e2})}{(1 + k)^2} + \cdots \\ &\quad + \frac{p_T(1 + g_{e1})(1 + g_{e2}) \cdots (1 + g_{eT})}{(1 + k)^T} \\ &\quad + \frac{p(1 + g_{e1})(1 + g_{e2}) \cdots (1 + g_{eT})(1 + g)}{(k - g)(1 + k)^T}. \end{aligned} \tag{18.46}$$

Example

Consider the Magnesium Company again. Its share price is currently \$55, and per share earnings and dividends over the past year were \$3 and \$.75, respectively. For the next two years, forecast earnings and dividends, along with the earnings growth rates and payout ratios, are:

$$\begin{array}{llll} D_1 = \$2.00 & E_1 = \$5.00 & g_{e1} = 67\% & p_1 = 40\% \\ D_2 = \$3.00 & E_2 = \$6.00 & g_{e2} = 20\% & p_2 = 50\%. \end{array}$$

Constant growth in dividends and earnings of 10% per year is forecast to begin at $T = 2$, which means that $D_3 = \$3.30$, $E_3 = \$6.60$, $g = 10\%$, and $p = 50\%$.

Given a required return of 15%, Equation (18.46) can be used as follows to estimate a "normal" price-earnings ratio for Magnesium:

$$\begin{aligned} \frac{V}{E_0} &= \frac{.40(1 + .67)}{(1 + .15)^1} + \frac{.50(1 + .67)(1 + .20)}{(1 + .15)^2} + \frac{.50(1 + .67)(1 + .20)(1 + .10)}{(.15 - .10)(1 + .15)^2} \\ &= .58 + .76 + 16.67 \\ &= 18.01. \end{aligned}$$

Because the actual price-earnings ratio of 18.33 (= \$55/\$3) is close to the "normal" ratio of 18.01, the stock of the Magnesium Company can be viewed as fairly priced.

18.7 SOURCES OF EARNINGS GROWTH

So far no explanation has been given as to why earnings or dividends will be expected to grow in the future. One way of providing such an explanation uses the constant-growth model. Assuming that no new capital is obtained externally and no shares are repurchased (meaning that the number of shares outstanding does not increase or decrease), the portion of earnings not paid to stockholders as dividends will be used to pay for the firm's new investments. Given that p_t denotes the payout ratio in year t, then $(1 - p_t)$ will be equal to the portion of earnings not paid out, known as the **retention ratio.** Furthermore, the firm's new investments, stated on a per-share basis and denoted by I_t, will be:

$$I_t = (1 - p_t)E_t. \tag{18.47}$$

If these new investments have an average return on equity of r_t in period t and every year thereafter, they will add $r_t I_t$ to earnings per share in year $t + 1$ and every year thereafter. If all previous investments also produce perpetual earnings at a constant rate of return, next year's earnings will equal this year's earnings plus the new earnings resulting from this year's new investments:

$$\begin{aligned} E_{t+1} &= E_t + r_t I_t \\ &= E_t + r_t(1 - p_t)E_t \\ &= E_t[1 + r_t(1 - p_t)]. \end{aligned} \tag{18.48}$$

Because it was shown earlier that the growth rate in earnings per share is:

$$E_t = E_{t-1}(1 + g_{et}) \tag{18.35}$$

it follows that:

$$E_{t+1} = E_t(1 + g_{et+1}). \tag{18.49}$$

A comparison of Equations (18.48) and (18.49) indicates that:

$$g_{et+1} = r_t(1 - p_t). \tag{18.50}$$

If the growth rate in earnings per share g_{et+1} is to be constant over time, then the average return on equity for new investments r_t and the payout ratio p_t must also be constant over time. In this situation Equation (18.50) can be simplified by removing the time subscripts:

$$g_e = r(1 - p). \tag{18.51a}$$

Because the growth rate in dividends per share g is equal to the growth rate in earnings per share g_e, this equation can be rewritten as:

$$g = r(1 - p). \tag{18.51b}$$

From this equation it can be seen that the growth rate g depends on (1) the proportion of earnings that is retained $1 - p$, and (2) the average return on equity on the earnings that are retained r.

The constant-growth valuation formula given in Equation (18.20) can be modified by replacing g with the expression on the right-hand side of Equation (18.51b), resulting in:

$$
\begin{aligned}
V &= D_0\left(\frac{1+g}{k-g}\right) \\
&= D_0\left[\frac{1+r(1-p)}{k-r(1-p)}\right]. \\
&= D_1\left[\frac{1}{k-r(1-p)}\right].
\end{aligned}
$$

Under these assumptions, a stock's value (and hence its price) should be greater, the greater its average return on equity for new investments, other things being equal.

Example

Continuing with the Copper Company, recall that $E_0 = \$2.70$ and $p = 66\frac{2}{3}\%$. This means that $33\frac{1}{3}\%$ of earnings per share over the past year were retained and reinvested, an amount equal to \$.90 (= .3333 × \$2.70). The earnings per share in the forthcoming year E_1 are expected to be \$2.835 [= \$2.70 × (1 + .05)] because the growth rate g for Copper is 5%.

The source of the increase in earnings per share of \$.135 (= \$2.835 − \$2.70) is the \$.90 per share that was reinvested at $t = 0$. The average return on equity for new investments r is 15%, because \$.135/\$.90 = 15%. That is, the reinvested earnings of \$.90 per share can be viewed as having generated an annual increase in earnings per share of \$.135. This increase will occur not only at $t = 1$, but also at $t = 2$, $t = 3$, and so on. Equivalently, a \$.90 investment at $t = 0$ will generate a perpetual annual cash inflow of \$.135 beginning at $t = 1$.

Expected dividends at $t = 1$ can be calculated by multiplying the expected payout ratio p of $66\frac{2}{3}\%$ times the expected earnings per share E_1 of \$2.835, or .6667 × \$2.835 = \$1.89. It can also be calculated by multiplying 1 plus the growth rate g of 5% times the past amount of dividends per share D_0 of \$1.80, or 1.05 × \$1.80 = \$1.89.

It can be seen that the growth rate in dividends per share of 5% is equal to the product of the retention rate ($33\frac{1}{3}\%$) and the average return on equity for new investments (15%), an amount equal to 5% (= .3333 × .15).

Two years from now ($t = 2$), earnings per share are anticipated to be \$2.977 [= \$2.835 × (1 + .05)], a further increase of \$.142 (= \$2.977 − \$2.835) that is due to the retention and reinvestment of \$.945 (= .3333 × \$2.835) per share at $t = 1$. This expected increase in earnings per share of \$.142 is the result of earning (15%) on the reinvestment (\$.945), because .15 × \$.945 = \$.142.

The expected earnings per share at $t = 2$ can be viewed as having three components. The first is the earnings attributable to the assets held at $t = 0$, an amount equal to \$2.70. The second is the earnings attributable to the reinvestment of \$.90 at $t = 0$, earning \$.135. The third is the earnings attributable to the reinvestment of \$.945 at $t = 1$, earning \$.142. These three components, when summed, can be seen to equal $E_2 = \$2.977$ (= \$2.70 + \$.135 + \$.142).

Dividends at $t = 2$ are expected to be 5% larger than at $t = 1$, or \$1.985 (= 1.05 × \$1.89) per share. This amount corresponds to the amount calculated by multiplying the payout ratio times the expected earnings per share at $t = 2$, or \$1.985 (= .6667 × \$2.977). Figure 18.2 summarizes the example.

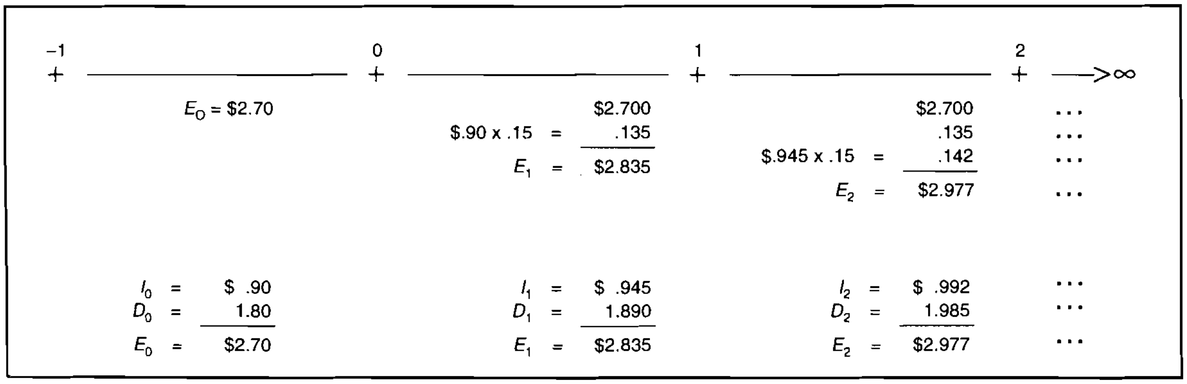

Figure 18.2
Growth in Earnings for Copper Company

18.8 A THREE-STAGE DDM

As this chapter's Institutional Issues discusses, the three-stage DDM is the most widely applied form of the general multiple-growth DDM. Consider analyzing the *ABC* Company.

18.8.1 Making Forecasts

Over the past year, *ABC* has had earnings per share of $1.67 and dividends per share of $.40. After carefully studying ABC, the security analyst has made the following forecasts of earnings per share and dividends per share for the next five years:

$E_1 = \$2.67$	$E_2 = \$4.00$	$E_3 = \$6.00$	$E_4 = \$8.00$	$E_5 = \$10.00$
$D_1 = \$\ .60$	$D_2 = \$1.60$	$D_3 = \$2.40$	$D_4 = \$3.20$	$D_5 = \$\ 5.00.$

These forecasts imply the following payout ratios and earnings-per-share growth rates:

$p_1 = 22\%$	$p_2 = 40\%$	$p_3 = 40\%$	$p_4 = 40\%$	$p_5 = 50\%$
$g_{e1} = 60\%$	$g_{e2} = 50\%$	$g_{e3} = 50\%$	$g_{e4} = 33\%$	$g_{e5} = 25\%.$

Furthermore, the analyst believes that *ABC* will enter a transition stage at the end of the fifth year (that is, the sixth year will be the first year of the transition stage), and that the transition stage will last three years. Earnings per share and payout ratio for year 6 are forecast to be $E_6 = \$11.90$ and $p_6 = 55\%$. {Thus g_{e6} = 19% [= ($11.90 − $10.00)/$10.00] and $D_6 = \$6.55$ (= .55 × $11.90)}.

The last stage, known as the maturity stage, is forecast to have an earnings-per-share growth rate of 4% and a payout ratio of 70%. Now it was shown in equation (18.51b) that with the constant-growth model, $g = r(1 - p)$, where r is the average return on equity for new investment and p is the payout ratio. Given

Applying Dividend Discount Models

Over the last 30 years, dividend discount models (DDMs) have achieved broad acceptance among professional common stock investors. Although few investment managers rely solely on DDMs to select stocks, many have integrated DDMs into their security valuation procedures.

The reasons for the popularity of DDMs are twofold. First, DDMs are based on a simple, widely understood concept: The fair value of any security should equal the discounted value of the cash flows expected to be produced by that security. Second, the basic inputs for DDMs are standard outputs for many large investment management firms—that is, these firms employ security analysts who are responsible for projecting corporate earnings.

Valuing common stocks with a DDM technically requires an estimate of future dividends over an infinite time horizon. Given that accurately forecasting dividends three years from today, let alone 20 years in the future, is a difficult proposition, how do investment firms actually go about implementing DDMs?

One approach is to use constant or two-stage dividend growth models, as described in the text. However, although such models are relatively easy to apply, institutional investors typically view the assumed dividend growth assumptions as overly simplistic. Instead, these investors generally prefer three-stage models, believing that they provide the best combination of realism and ease of application.

Whereas many variations of the three-stage DDM exist, in general, the model is based on the assumption that companies evolve through three stages during their lifetimes. (Figure 18.3 portrays these stages.)

1. **Growth stage:** Characterized by rapidly expanding sales, high profit margins, and abnormally high growth in earnings per share. Because of highly profitable expected investment opportunities, the payout ratio is low. Competitors are attracted by the unusually high earnings, leading to a decline in the growth rate.
2. **Transition stage:** In later years, increased competition reduces profit margins and earnings growth slows. With fewer new investment opportunities, the company begins to pay out a larger percentage of earnings.

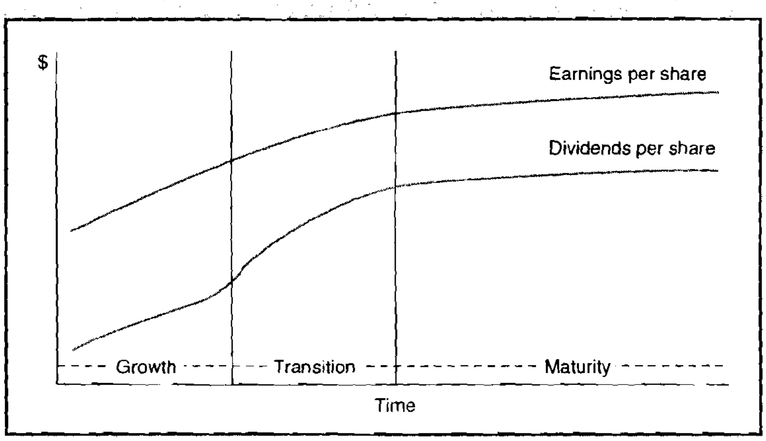

Figure 18.3
The Three Stages of the Multiple-Growth Model
Source: Adapted from Carmine J. Grigoli, "Demystifying Dividend Discount Models," *Merrill Lynch Quantitative Research*, April 1982.

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3. **Maturity (steady-state) stage:** Eventually the company reaches a position where its new investment opportunities offer, on average, only slightly attractive returns on equity. At that time its earnings growth rate, payout ratio, and return on equity stabilize for the remainder of its life.

The forecasting process of the three-stage DDM involves specifying earnings and dividend growth rates in each of the three stages. Although one cannot expect a security analyst to be omniscient in his or her growth forecast for a particular company, one can hope that the forecast pattern of growth—in terms of magnitude and duration—resembles that actually realized by the company, particularly in the short run.

Investment firms attempt to structure their DDMs to make maximum use of their analysts' forecasting capabilities. Thus the models emphasize specific forecasts in the near term, when it is realistic to expect security analysts to project earnings and dividends more accurately. Conversely, the models emphasize more general forecasts over the longer term, when distinctions between companies' growth rates become less discernible. Typically, analysts are required to supply the following for their assigned companies:

1. expected annual earnings and dividends for the next several years;
2. after these specific annual forecasts end, earnings growth and the payout ratio forecasts until the end of the growth stage;
3. the number of years until the transition stage is reached;
4. the duration (in years) of the transition stage—that is, once abnormally high growth ends, the number of years until the maturity stage is reached.

Most three-stage DDMs assume that during the transition stage, earnings growth declines and payout ratios rise linearly to the maturity-stage steady-state levels. (For example, if the transition stage is ten years long, earnings growth at the maturity stage is 5% per year, and earnings growth at the end of the growth stage is 25%, then earnings growth will decline 2% in each year of the transition stage.) Finally, most three-stage DDMs make standard assumptions that all companies in the maturity stage have the same growth rates, payout ratios, and return on equity.

With analysts' inputs, plus an appropriate required rate of return for each security, all the necessary information for the three-stage DDM is available. The last step involves merely calculating the discounted value of the estimated dividends to determine the stock's "fair" value.

The seeming simplicity of the three-stage DDM should not lead one to believe that it is without its implementation problems. Investment firms must strive to achieve consistency across their analysts' forecasts. The long-term nature of the estimates involved, the substantial training required to make even short-term earnings forecasts accurately, and the coordination of a number of analysts covering many companies severely complicate the problem. Considerable discipline is required if the DDM valuations generated by a firm's analysts are to be sufficiently comparable and reliable to guide investment decisions. Despite these complexities, if successfully implemented, DDMs can combine the creative insights of security analysts with the rigor and discipline of quantitative investment techniques.

that the maturity stage has constant growth, this equation can be reformulated and used to determine r:

$$r = g/(1 - p).$$

Thus r for *ABC* has an implied value of 13.33% [= 4%/(100% − 70%)], which is assumed to be consistent with the long-run growth forecasts for similar companies.

At this point there are only two missing pieces of information that are needed to determine the value of *ABC*—the earnings-per-share growth rates and the payout ratios for the transition stage. Taking earnings per share fir[illegible] forecast that $g_{e6} = 19\%$ and $g_{e9} = 4\%$. One method of determining h[illegible] "decay" to 4% is to note that there are three years between the sixth [illegible] years, and 15% between 19% and 4%. A "linear decay" rate would be d[illegible] by noting that 15%/3 years = 5% per year. This rate of 5% would be [illegible] from 19% to get g_{e7}, resulting in 14% (= 19% − 5%). Then it would be [illegible] ed from 14% to get g_{e8}, resulting in 9% (= 14% − 5%). Finally, as a check [illegible] be noted that 4% (= 9% − 5%) is the value that was forecast for g_{e9}.

A similar procedure can be used to determine how the payout ratio [illegible] in year 6 will grow to 70% in year 9. The "linear growth" rate will be (7[illegible] 55%)/3 years = 15%/3 years = 5% per year, indicating that $p_7 = 60\%$ (= [illegible] + 5%) and $p_8 = 65\%$ (= 60% + 5%). Again a check indicates that 70% [illegible] 65% + 5%) is the value that was forecast for p_9.

With these forecasts of earnings-per-share growth rates and payout ratios [illegible] hand, forecasts of dividends per share can now be made:

$$
\begin{aligned}
D_7 &= p_7 E_7 \\
&= p_7 E_6 (1 + g_{e7}) \\
&= .60 \times \$11.90 \times (1 + .14) \\
&= .60 \times \$13.57 \\
&= \$8.14 \\
D_8 &= p_8 E_8 \\
&= p_8 E_6 (1 + g_{e7})(1 + g_{e8}) \\
&= .65 \times \$11.90 \times (1 + .14) \times (1 + .09) \\
&= .65 \times \$14.79 \\
&= \$9.61 \\
D_9 &= p_9 E_9 \\
&= p_9 E_6 (1 + g_{e7})(1 + g_{e8})(1 + g_{e9}) \\
&= .70 \times \$11.90 \times (1 + .14) \times (1 + .09) \times (1 + .04) \\
&= .70 \times \$15.38 \\
&= \$10.76.
\end{aligned}
$$

18.8.2 Estimating the Intrinsic Value

Given a required rate of return on *ABC* of 12.4%, all the necessary inputs for the multiple-growth model have been determined. Hence it is now possible to estimate *ABC*'s intrinsic (or fair) value. To begin, it can be seen that $T = 8$, indicating that V_{T-} involves determining the present value of D_1 through D_8,

$$
\begin{aligned}
V_{T-} &= \left[\frac{\$.60}{(1 + .124)^1}\right] + \left[\frac{\$1.60}{(1 + .124)^2}\right] + \left[\frac{\$2.40}{(1 + .124)^3}\right] \\
&\quad + \left[\frac{\$3.20}{(1 + .124)^4}\right] + \left[\frac{\$5.00}{(1 + .124)^5}\right] + \left[\frac{\$6.55}{(1 + .124)^6}\right] \\
&\quad + \left[\frac{\$8.14}{(1 + .124)^7}\right] + \left[\frac{\$9.61}{(1 + .124)^8}\right] \\
&= \$18.89.
\end{aligned}
$$

Then V_{T+} can be determined using D_9:

$$V_{T+} = \frac{\$10.76}{(.124 - .04)(1 + .124)^8}$$
$$= \$50.28.$$

Combining V_{T-} and V_{T+} results in the intrinsic value of *ABC:*

$$V = V_{T-} + V_{T+}$$
$$= \$18.89 + \$50.28$$
$$= \$69.17.$$

Given a current market price for *ABC* of \$50, it can be seen that its stock is underpriced by \$19.17 (= \$69.17 − \$50) per share. Equivalently, it can be noted that the actual price-earnings ratio for *ABC* is 29.9 (= \$50/\$1.67) but that a "normal" price-earnings ratio would be higher, equal to 41.4 (= \$69.17/\$1.67), again indicating that *ABC* is underpriced.

18.8.3 Implied Returns

As shown with the previous example, once the analyst has made certain forecasts, it is relatively straightforward to determine a company's expected dividends for each year up through the first year of the maturity stage. Then the present value of these predicted dividends can be calculated for a given required rate of return. However, many investment firms use a computerized trial-and-error procedure to determine the discount rate that equates the present value of the stock's expected dividends with its current price. Sometimes this long-run internal rate of return is referred to as the security's **implied return.** In the case of *ABC*, its implied return is 14.8%.

18.8.4 The Security Market Line

After implied returns have been estimated for a number of stocks, the associated beta for each stock can be estimated. Then for all the stocks analyzed, this information can be plotted on a graph that has implied returns on the vertical axis and estimated betas on the horizontal axis.

At this point there are alternative methods for estimating the security market line (SML).[11] One method involves determining a line of best fit for this graph by using a statistical procedure known as simple regression (as discussed in Chapter 17). That is, the values of an intercept term and a slope term are determined from the data, thereby indicating the location of the straight line that best describes the relationship between implied returns and betas.[12]

Figure 18.4 provides an example of the estimated SML. In this case the SML has been determined to have an intercept of 8% and a slope of 4%, indicating that, in general, securities with higher betas are expected to have higher implied returns in the forthcoming period. Depending on the sizes of the implied returns, such lines can have steeper or flatter slopes, or even negative slopes.

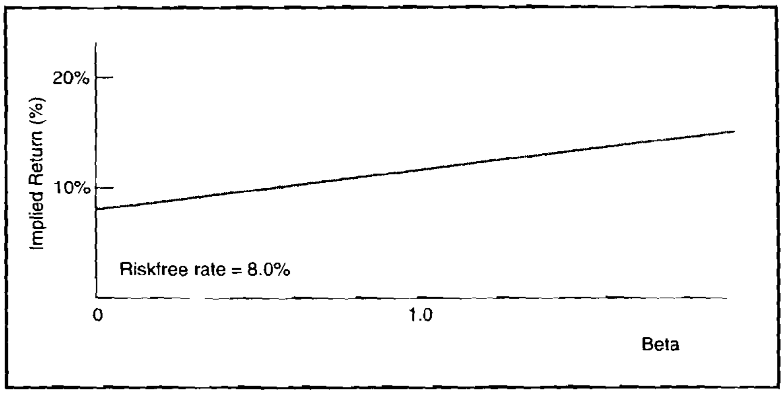

Figure 18.4
A Security Market Line Estimated from Implied Returns

The second method of estimating the SML involves calculating the implied return for a portfolio of common stocks. This is done by taking a value-weighted average of the implied returns of the stocks in the portfolio, with the resulting return being an estimate of the implied return on the market portfolio. Given this return and a beta of 1, the "market" portfolio can be plotted on a graph having implied returns on the vertical axis and betas on the horizontal axis. Next the riskfree rate, having a beta of 0, can be plotted on the same graph. Finally, the SML is determined by simply connecting these two points with a straight line.

Either of these SMLs can be used to determine the required return on a stock. However, they will most likely result in different numbers, as the two lines will most likely have different intercepts and slopes. For example, note that in the first method the SML may not go through the riskfree rate, whereas the second method forces the SML to go through this rate.

18.8.5 Required Returns and Alphas

Once a security's beta has been estimated, its required return can be determined from the estimated SML. For example, the equation for the SML shown in Figure 18.4 is:

$$k_i = 8 + 4\beta_i.$$

Thus if *ABC* has an estimated beta of 1.1, then it would have a required return equal to 12.4% [$= 8 + (4 \times 1.1)$].

Once the required return on a stock has been determined, the difference between the stock's implied return (from the DDM) and this required return can be calculated. This difference is then viewed as an estimate of the stock's *alpha* and represents "... the degree to which a stock is mispriced. Positive alphas indicate undervalued securities and negative alphas indicate overvalued securities."[13] In the case of *ABC*, its implied and required returns were 14.8% and 12.4%, respectively. Thus its estimated alpha would be 2.4% (= 14.8% − 12.4%). Because this is a positive number, *ABC* can be viewed as being underpriced.

18.8.6 The Implied Return on the Stock Market

Another product of this analysis is that the implied return for a portfolio of stocks can be compared with the expected return on bonds. (The latter is typically represented by the current yield-to-maturity on long-term Treasury bonds.) Specifically, the difference between stock and bond returns can be used as an input for recommendations concerning asset allocation between stocks and bonds. That is, it can be used to form recommendations regarding what percent of an investor's money should go into stocks and what percent should go into bonds. For example, the greater the implied return on stocks relative to bonds, the larger the percentage of the investor's money that should be placed in common stocks.

18.9 DIVIDEND DISCOUNT MODELS AND EXPECTED RETURNS

The procedures described here are similar to those employed by a number of brokerage firms and portfolio managers.[14] A security's implied return, obtained from a DDM, is often treated as an expected return, which in turn can be divided into two components—the security's required return and alpha.

However, the expected return on a stock over a given holding period may differ from its DDM-based implied rate k^*. A simple set of examples will indicate why this difference can exist.

Assume that a security analyst predicts that a stock will pay a dividend of \$1.10 per year forever. On the other hand, the consensus opinion of "the market" (most other investors) is that the dividend will equal \$1.00 per year forever. This suggests that the analyst's prediction is a deviant or nonconsensus one.

Assume that both the analyst and other investors agree that the required rate of return for a stock of this type is 10%. Using the formula for the zero-growth model, the value of the stock is $D_1/.10 = 10D_1$, meaning that the stock should sell for ten times its expected dividend. Because other investors expect to receive \$1.00 per year, the stock has a current price P of \$10 per share. The analyst feels that the stock has a value of $\$1.10/.10 = \$11$ and thus feels that it is underpriced by $\$11 - \$10 = \$1$ per share.

18.9.1 Rate of Convergence of Investors' Predictions

this situation the implied return according to the analyst is $\$1.10/\$10 = 11\%$. analyst buys a share now with a plan to sell it a year later, what rate of re- might the analyst expect to earn? The answer depends on what assumption regarding the *rate of convergence of investors' predictions*—that is, the an- depends on the expected market reaction to the mispricing that the analyst currently exists.

cases shown in Table 18.1 are based on an assumption that the analyst is that his or her forecast of future dividends is correct. That is, in all of the analyst expects that at the end of the year, the stock will pay the dividend of \$1.10.

TABLE 18.1
ALPHA AND THE CONVERGENCE OF PREDICTIONS

	Expected Amount of Convergence		
	0% (A)	100% (B)	50% (C)
Dividend predictions D_2			
Consensus of other investors	1.00	1.10	1.05
Analyst	1.10	1.10	1.10
Expected stock price P_1	10.00	11.00	10.50
Expected return:			
Dividence yield D_1/P	11%	11%	11%
Capital gain $(P_1 - P)/P$	0	10	5
Total expected return	11%	21%	16%
Less required return	10	10	10
Alpha	1%	11%	6%

Note: P_1 is equal to the consensus dividend prediction at t = 1 divided by the required return of 10%. The example assumes that the current stock price P is \$10, and dividends are forecast by the consensus at t = 0 to remain constant at \$1.00 per share, whereas the analyst forecasts the dividends at t = 0 to remain constant at \$1.10 per share.

No Convergence

In column (A), it is assumed that other investors will regard the higher dividend as a fluke and steadfastly refuse to alter their projections of subsequent dividends from their initial estimate of \$1.00. As a result, the security's price at $t = 1$ can be expected to remain at \$10 (= \$1.00/.10). In this case the analyst's total return is expected to be 11% (= \$1.10/\$10), which will be attributed entirely to dividends as no capital gains are expected.

The 11% expected return can also be viewed as consisting of the required return of 10% plus an alpha of 1% that is equal to the portion of the dividend unanticipated by other investors, \$.10/\$10. Accordingly, if it is assumed that there will be no convergence of predictions, the expected return would be set at the implied rate of 11% and the alpha would be set at 1%.

Complete Convergence

Column (B) shows a very different situation. Here it is assumed that the other investors will recognize their error and completely revise their predictions. At the end of the year, it is expected that they too will predict future dividends of \$1.10 per year thereafter; thus the stock is expected to be selling for \$11 (= \$1.10/.10) at $t = 1$. Under these conditions, the analyst can expect to achieve a total return of 21% by selling the stock at the end of the year for \$11, obtaining 11% (= \$1.10/\$10) in dividend yield and 10% (= \$1/\$10) in capital gains.

The 10% expected capital gains result directly from the expected repricing of the security because of the complete convergence of predictions. In this case the fruits of the analyst's superior prediction are expected to be obtained all in one year. Instead of 1% "extra" per year forever, as in column (A), the analyst expects to obtain 1% (= \$.10/\$10) in extra dividend yield plus 10% (= \$1/\$10) in capital gains this year. By continuing to hold the stock in subsequent years, the analyst would expect to earn only the required return of 10% over those years. Accordingly, the expected return is 21% and the alpha is 11% when it is assumed that there is complete convergence of predictions.

Partial Convergence

Column (C) shows an intermediate case. Here the predictions of the other investors are expected to converge only halfway toward those of the analyst (that is, from \$1.00 to \$1.05 instead of to \$1.10). Total return in the first year is expected to be 16%, consisting of 11% (= \$1.10/\$10) in dividend yield plus 5% (= \$.50/\$10) in capital gains.

Since the stock is expected to be selling for \$10.50 (= \$1.05/.10) at $t = 1$, the analyst will still feel that it is underpriced at $t = 1$ because it will have an intrinsic value of \$11 (= \$1.10/.10) at that time. To obtain the remainder of the "extra return" owing to this underpricing, the stock would have to be held past $t = 1$. Accordingly, the expected return would be set at 16% and the alpha would be set at 6% when it is assumed that there is halfway convergence of predictions.

In general, a security's expected return and alpha will be larger, the faster the assumed rate of convergence of predictions.[15] Many investors use the implied rate (that is, the internal rate of return k^*) as a surrogate for a relatively short-term (for example, one year) expected return, as in column (A). In doing so, they are assuming that the dividend forecast is completely accurate, but that there is no convergence. Alternatively, investors could assume that there is some degree of convergence, thereby raising their estimate of the security's expected return. Indeed, investors could further alter their estimate of the security's expected return by assuming that the security analyst's deviant prediction is less than perfectly accurate, as will be seen next.[16]

18.9.2 Predicted versus Actual Returns

An alternative approach does not simply use outputs from a model "as is," but *adjusts* them, based on relationships between previous predictions and actual outcomes. Panels (a) and (b) of Figure 18.5 provide examples.

Each point in Figure 18.5(a) plots a *predicted return* on the stock market as a whole (on the horizontal axis) and the subsequent *actual return* for that period (on the vertical axis). The line of best fit (determined by simple regression) through the points indicates the general relationship between prediction and outcome. If the current prediction is 14%, history suggests that an estimate of 10% would be superior.

Each point in Figure 18.5(b) plots a predicted alpha value for a security (on the horizontal axis) and the subsequent "abnormal return" for that period (on the vertical axis). Such a diagram can be made for a given security, or for all the securities that a particular analyst makes predictions about, or for all the securities that the investment firm makes predictions about. Again a line of best fit can be drawn through the points. In this case, if the current prediction of a security's

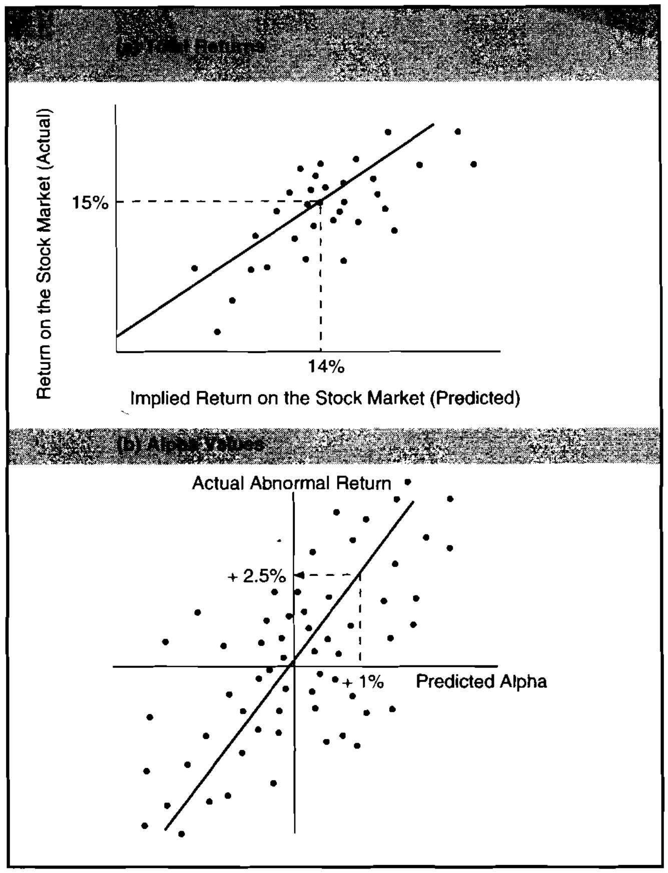

Figure 18.5
Adjusting Predictions

alpha is +1%, this relationship suggests that an "adjusted" estimate of +2.5% would be superior.

An important by-product of this type of analysis is the measure of correlation between predicted and actual outcomes, indicating the nearness of the points to the line. This **information coefficient** (IC) can serve as a measure of predictive accuracy. If it is too small to be significantly different from zero in a statistical sense, the value of the predictions is subject to considerable question.[17]

18.10 SUMMARY

1. The capitalization of income method of valuation states that the intrinsic value of any asset is equal to the sum of the discounted cash flows investors expect to receive from that asset.

2. Dividend discount models (DDMs) are a specific application of the capitalization of income method of valuation to common stocks.
3. To use a DDM, the investor must implicitly or explicitly supply a forecast of all future dividends expected to be generated by a security.
4. Investors typically make certain simplifying assumptions about the growth of common stock dividends. For example, a common stock's dividends may be assumed to exhibit zero growth or growth at a constant rate. More complex assumptions may allow for multiple growth rates over time.
5. Instead of applying DDMs, many security analysts use a simpler method of security valuation that involves estimating a stock's "normal" price-earnings ratio and comparing it with the stock's actual price-earnings ratio.
6. The growth rate in a firm's earnings and dividends depends on its earnings retention rate and its average return on equity for new investments.
7. Determining whether a security is mispriced using a DDM can be done in one of two ways. First, the discounted value of expected dividends can be compared with the stock's current price. Second, the discount rate that equates the stock's current price to the present value of forecast dividends can be compared with the required return for stocks of similar risk.
8. The rate of return that an analyst with accurate non-consensus dividend forecasts can expect to earn depends on the rate of convergence of other investors' predictions to the predictions of the analyst.

QUESTIONS AND PROBLEMS

1. Consider five annual cash flows (the first occurring one year from today):

Year	Cash Flow
1	\$5
2	\$6
3	\$7
4	\$8
5	\$9

Given a discount rate of 10%, what is the present value of this stream of cash flows?

2. Alta Cohen is considering buying a machine to produce baseballs. The machine costs \$10,000. With the machine, Alta expects to produce and sell 1,000 baseballs per year for \$3 per baseball, net of all costs. The machine's life is five years (with no salvage value). Based on these assumptions and an 8% discount rate, what is the net present value of Alta's investment?

[illegible] Hub Collins has invested in a project that promised to pay \$100, \$200, and \$300, respectively, at the end of the next three years. If Hub paid \$513.04 for this investment, what is the project's internal rate of return?

[illegible]ton Products currently pays a dividend of \$4 per share on its common [illegible]ck.