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The Finite Horizon Expected Return Model

Joseph R. Gordon and Myron J. Gordon

The finite horizon expected return model (FHERM), a new method for estimating the expected return on a share, states that (1) forecasts of abnormal performance have a finite horizon, N, beyond which investors expect a corporation to earn for all future time a return on equity investment equal to the expected return on its shares; and (2) the expected return on a share is the discount rate that equates the share's current price with a dividend expectation for which the dividend in each period from 1 to N is equal to its forecast and the dividend in each period from N + 1 to infinity is equal to the forecast for normalized earnings in Period N + 1. The capital asset pricing model (CAPM) states that the expected return on a share varies with beta and dividend yield, but empirical tests of the CAPM using previous methods for estimating expected return have failed. Empirical evidence strongly supports the joint hypothesis that the FHERM and the CAPM are both true.

Two of the three corecipients of the 1990 Nobel Prize in Economics were Harry Markowitz and William Sharpe. The prize recognized their work in portfolio theory, work that culminated in the capital asset pricing model (CAPM) (see Markowitz 1959, Sharpe 1964, Lintner 1965, and Mossin 1966). The CAPM established that under intuitively attractive assumptions, the expected return (EXR) on a share varies with the share's systematic risk (BETA). The EXR could be a very important capital market statistic because investors would find it useful in choosing among shares and, to the extent that capital markets are perfectly competitive, corporations would find it useful in choosing among investment opportunities.

A considerable body of empirical work during the past 30 years has been devoted to testing the CAPM under the assumption that an average of the realized holding-period returns (ARHPR) on a share over a number of prior time periods is a satisfactory estimate of its EXR. This empirical work has provided little support for the truth of the CAPM. Some of the most painstaking and sophisticated research has not even found a positive correlation between EXR and BETA (see Reinganum 1981, Coggin and Hunter 1985, Lakonishok and Shapiro 1986, and Fama and French 1992).[1] Fama and French summarized their empirical results with the statement: "In short, our tests do not support the most basic prediction of the SLB model [CAPM], that average stock returns are positively related to market βs."

Why has this empirical work, which assumes investors' expectations are simply some average of what was realized in the past, found little or no correlation between EXR and BETA? A plausible explanation is the very high variance of short-term realized holding-period returns. As Black (1993) observed, the averaging process needed to eliminate the noise may leave little information in the average as an estimate of the EXR at any point in time. Regardless, these empirical results force us either to abandon a theorem that contributed significantly to the only Nobel Prize in financial economics or to use a different method for the estimation of EXR.[2]

A share's holding-period return (HPR), by definition, is the sum of its dividend yield (DYD) for the period and its growth rate in price for the period; that is, for any future period,

$$\mathrm{HPR}(T) = \frac{\mathrm{DIV}(T)}{\mathrm{PPS}(T-1)} + \frac{\mathrm{PPS}(T) - \mathrm{PPS}(T-1)}{\mathrm{PPS}(T-1)},\ T > 0, \quad (1)$$

where

DIV(T) = dividend
PPS(T) = end-of-period price
PPS($T-1$) = start-of-period price

The expected return on a share, by definition, is Equation 1 with T equal to 1. PPS(0) is simply the current price, and the accurate estimation of expected DIV(1) is trivial, but the accurate estimation of expected PPS(1) is elusive.

A database available from the Institutional Brokers Estimate System (I/B/E/S International) contains averages of security analyst forecasts of

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earnings per share and of the long-term growth rate in earnings. From the I/B/E/S glossary of terms:

> Long Term Growth Forecasts are received directly from contributing analysts, they are not calculated by I/B/E/S. While different analysts apply different methodologies, the Long Term Growth Forecast generally represents an expected *annual* increase in operating earnings over the company's next full business cycle. In general, these forecasts refer to a period of between three to five years.

Using the I/B/E/S long-term growth forecast as an estimate of a share's expected growth rate in price for the coming period, Harris and Marston (1992) and Gordon (1993) found positive correlation between EXR and BETA. The correlations were very low, however, so either BETA explains very little of the variation in EXR among shares or there is still considerable room for improvement in the estimation of EXR.

The finite horizon expected return model (FHERM) represents a new method for estimating EXR. The motivation for the FHERM is the assumption that investors believe that current forecasts can be used to predict, with acceptable accuracy, abnormal performance up to but not beyond a finite point in the future. The FHERM states that (1) forecasts of abnormal performance have a finite horizon, N, beyond which investors expect a corporation to earn for all future time a return on equity investment equal to the expected return on its shares; and (2) the expected return on a share is the discount rate that equates the share's current price with a dividend expectation, where the dividend in each period from 1 to N is equal to its forecast and the dividend in each period from $N + 1$ to infinity is equal to the forecast for normalized earnings in Period $N + 1$.

If both the FHERM and the CAPM are true, then the multiple correlation of EXR with BETA and DYD (which becomes significant when the CAPM is adjusted for taxes) should be significantly high within a single, small interval of N that includes the true horizon of investors and, within this interval, the regression coefficients and predicted values should be reasonable and stable. Empirical evidence shows this to be the case—maximum correlation of EXR with BETA and DYD occurs in Year 7, which is very reasonable considering that we used the I/B/E/S average of security analyst forecasts of the long-term growth rate in earnings. Regressions of the seven-year-horizon estimate of EXR on beta, dividend yield, and skewness explain a comparatively large fraction of the variation in EXR among shares and, of course, a much larger fraction of the variation in EXR among portfolios. Furthermore, the risk-free interest rate implicit in the constant term, the price of risk implicit in the coefficient of BETA, and the tax cost of dividends implicit in the coefficient of DYD are reasonable, if not in complete agreement with the CAPM, and are remarkably stable from one quarter to the next.

DERIVATION OF THE FHERM

The derivation of the FHERM starts with the well-known proposition that the expected return on a share is the discount rate that equates the share's current price with its dividend expectation; that is,

$$\text{PPS} = \sum_{T=1}^{\infty} \frac{\text{DIV}(T)}{(1 + \text{EXR})^T}. \qquad (2)$$

On the assumption that the dividend expectation may be represented with its first-period value and one growth rate for all future time, Equation 2 becomes

$$\text{PPS} = \sum_{T=1}^{\infty} \frac{\text{DIV}(1) * (1+\text{GRR})^{T-1}}{(1 + \text{EXR})^T} = \frac{\text{DIV}(1)}{\text{EXR} - \text{GRR}}, \qquad (3)$$

where

PPS = current price per share
DIV(1) = expected dividend per share in Period 1
GRR = expected growth rate in the dividend

Retained earnings are the primary and frequently the sole source of funds for equity investment, and dividends are the primary and frequently the sole means for distributing funds to shareholders. Assuming for the present that retained earnings and dividends are the sole means for realizing their respective objectives, Gordon (1962) showed that the value of EXR that satisfies Equation 3 is

$$\text{EXR} = \text{DYD}(1) + \text{GRR} = \frac{\text{NEPS}(1) * (1 - \text{RTR})}{\text{PPS}} + \text{RTR} * \text{REI}, \qquad (4)$$

where

DYD(1) = DIV(1)/PPS = expected dividend yield in Period 1
NEPS(1) = expected normalized earnings per share in Period 1
RTR = the retention rate
REI = the corporation's return on equity investment

Normalized earnings are what earnings would be without influence from abnormal events.

Equation 4 assumes that a corporation can be expected to earn for all future time an REI that is not necessarily equal to the return investors require on its shares. Under the opposite assumption, that a corporation's REI for all future time must equal its EXR, Equation 4 becomes

$$\text{EXR} = \text{EYD} = \frac{\text{NEPS}(1)}{\text{PPS}}, \tag{5}$$

where EYD is the current earnings yield. It should also be noted that when REI equals EXR, the PPS is independent of dividend policy.[3]

Holt (1962), Brigham and Pappas (1966), and others have argued that a corporation cannot be expected to have an abnormally high or low growth rate forever. Brigham and Pappas established the price of a hypothetical share, given the duration of abnormal growth, the abnormal growth rate, the normal growth rate, and the EXR. Assuming the CAPM is true, however, the FHERM can be used to establish the finite horizon of investors and the expected return of an actual share.

The FHERM states that forecasts of abnormal performance have a finite horizon, N, beyond which investors expect a corporation to earn for all future time a return on equity investment equal to the expected return on its shares; that is,

$$\text{REI}(T) = \text{EXR}, T > N. \tag{6}$$

From Equation 6 and Equations 2 and 5, it follows that the expected return on a share is the discount rate that equates the share's current price with a dividend expectation, where the dividend in each period from 1 to N is equal to its forecast and the dividend in each period from $N + 1$ to infinity is equal to the forecast for normalized earnings in Period $N + 1$; that is,

$$\text{PPS} = \sum_{T=1}^{N} \frac{\text{DIV}(T)}{(1+\text{EXR})^T} + \frac{\text{NEPS}(N+1)}{\text{EXR}(1+\text{EXR})^N}. \tag{7}$$

In addition, given that forecasts of abnormal performance have a finite horizon, N, investors expect dividends, with last year's actual as the base, and normalized earnings, with next year's forecast as the base, both to grow for N periods at the rate forecast for long-term growth in earnings; that is, if DIV(0) equals the dividend in the most recently realized period, then

$$\text{DIV}(T) = \text{DIV}(0) * (1+\text{GRR})^T, 0 \le T \le N, \tag{8}$$

and

$$\text{NEPS}(T+1) = \text{NEPS}(1) * (1+\text{GRR})^T, 0 \le T \le N, \tag{9}$$

and Equation 7 becomes

$$\text{PPS} = \sum_{T=1}^{N} \frac{\text{DIV}(0)*(1+\text{GRR})^T}{(1+\text{EXR})^T} + \frac{\text{NEPS}(1)*(1+\text{GRR})^N}{\text{EXR}*(1+\text{EXR})^N}. \tag{10}$$

Equation 10 is the basis for the empirical work in this study, and it can be solved for EXR given PPS, DIV(0), NEPS(1), GRR, and N. PPS and DIV(0) are realized values, and NEPS(1) and GRR can be estimated from averages of security analyst forecasts of earnings and of the long-term growth rate in earnings. These realized and forecast values are available in a database from I/B/E/S International. But what about N? Analysts do not predict earnings beyond five years, which suggests that any consensus of opinion among investors probably deteriorates quickly after five years. So, a value between 5 and 10 for N is probably reasonable. Note that the value of N suggested by the FHERM is data dependent, which is the way it should be because investors might alter their horizons over time. Also, when N equals zero, EXR is the share's earnings yield, and when N approaches infinity, EXR approaches the share's dividend yield plus its dividend growth rate.

Several possible sources of error in this article's use of the FHERM to estimate EXR are the following:

- Investors may not have the same horizon for all corporations.
- Investors may not expect constant abnormal growth of dividends and normalized earnings.
- Investors may not expect the same abnormal growth rate for both dividends and normalized earnings.
- Investors may expect that corporations will issue or repurchase shares rather than using only retained earnings and dividends to finance equity investment and to distribute profits.

Insofar as these sources of error are material, the estimation of EXR and N and the explanation of how EXR varies among shares will be impaired.

TEST OF THE JOINT HYPOTHESIS

For the empirical evidence to be consistent with the joint hypothesis that the FHERM and the CAPM are both true, what must be demonstrated is that when the FHERM is used to estimate EXR, the CAPM explains a large amount of the variation in EXR among shares if and only if the value assigned to N approximates the finite horizon for forecasts of abnormal performance.

Under the strong assumptions that capital markets are perfectly competitive, including the assumption that there are no taxes, the CAPM states

$$\text{EXR} = \text{RFR} + (\text{EXRM} - \text{RFR})\text{BETA}, \qquad (11)$$

where RFR is the risk-free short-term interest rate and EXRM is the expected return on the market portfolio.

Taxes and other market imperfections, however, complicate the explanation of the variation in EXR among shares, and therefore, the regression equation that will be used here is

$$\text{EXR} = \alpha_0 + \alpha_1\text{BETA} + \alpha_2\text{DYD} + \alpha_3\text{SKEW}, \qquad (12)$$

where BETA is as defined earlier, DYD equals DIV(0)/PPS, and SKEW is the right skewness in the share's holding-period returns.

When the CAPM is adjusted for market imperfections, as in Equation 12, the CAPM states that the constant term, α_0, and the beta coefficient, α_1, will be positive, but it does not predict their values. The tax system treats price appreciation more favorably than it does dividends, so the expectation would be to find $\alpha_2 > 0$ (see Litzenberger and Ramaswamy 1982 and Miller and Scholes 1982 for previous research on the relation between EXR and DYD). The popularity of both lotteries and insurance suggests the hypothesis that investors prefer a small probability of a large gain and a large probability of a small loss to the opposite skewness in holding-period returns. So, the expectation would be to find $\alpha_3 < 0$.

Hence, a positive outcome for a test of the CAPM that uses Equation 12 on real data is (1) high multiple correlation, (2) values for α_0, α_1, and α_2 that are positive and statistically significant, and (3) a value for α_3 that is negative if it is statistically significant. The SKEW coefficient turns out not to be statistically significant, and therefore, the proof of the CAPM does not depend on the value of α_3.

Equation 12 does not include among its independent variables sources of risk such as debt–equity ratio or size, because the CAPM claims BETA is a measure of systematic risk from all sources. Insofar as BETA fails to capture completely the influence of each source of risk on EXR, inclusion of source variables would improve the explanation of the variation in EXR among shares. The objective here, however, is to establish how EXR varies with BETA, and the inclusion of variables that represent sources of risk would reduce the level and the significance of the correlation between EXR and BETA, because BETA and the sources of risk are correlated to some degree.[4] Equation 12 includes DYD and SKEW among the independent variables because they are possible nonrisk sources of variation in EXR among shares. Their inclusion should not dilute the relation between EXR and BETA. Rather, it could improve the estimate of the relation between EXR and BETA, and the variation of EXR with DYD and SKEW is also of interest.

We tested many values of *N* to find the "best" one—that is, the value for *N* that results in the best estimate of EXR, the estimate of EXR that best explains how EXR varies with the independent variables of Equation 12. This best value for *N* is considered the consensus among investors of the finite horizon for forecasts of abnormal performance. Is it data mining or sound empirical research to find the consensus among investors in this manner? The latter is true if

- the CAPM test results are positive when and only when the value assigned to *N* is within a small interval that contains the best value for *N*,
- the constant term and the statistically significant coefficients do not change radically among the positive tests of the CAPM,
- the best value for *N* is also a reasonable approximation of the longevity of long-term forecasts by security analysts, and
- these results are obtained over many independent and large samples.

The results do satisfy all these conditions.

ESTIMATION OF VARIABLES

A database for corporations that trade on the NYSE or the Amex, available from the CRSP, was used to identify corporations that were in the S&P 500 at the end of March 1985. A BETA was calculated for each of these corporations on the basis of the monthly holding-period returns during the prior 60 months. This step was repeated every three months until December 1991 so that a value for BETA was obtained for each corporation in 28 quarterly subsets of S&P 500 corporations. The data used to calculate BETA were also used to calculate SKEW for each corporation.

All the other variables were obtained from the I/B/E/S database. For PPS and DIV(0) at the end of each quarter, the values used were the price per share and the annualized quarterly dividend reported for the beginning of April 1985 and every three months thereafter until January 1992. The value used for GRR was the average of the security analysts' forecasts of the long-term growth rate in earnings. The value used for NEPS(1) was derived from the average of the security analysts' forecasts of earnings per share three years hence, EPS(3), as follows:

$$\mathrm{NEPS}(1) = \frac{\mathrm{EPS}(3)}{(1+\mathrm{GRR})^2}. \quad (13)$$

The I/B/E/S forecast for EPS(3) was used to arrive at NEPS(1) instead of actual earnings or the I/B/E/S forecast for EPS(1) in the belief that the Year 3 forecast results in a better estimate of normalized earnings than do the other data.

Some corporations in the S&P 500 were excluded from the sample for the following reasons:

- S&P 500 corporations that were not traded on the NYSE or the Amex were not in the CRSP database.
- Companies were excluded if the computation of BETA was based on fewer than 45 holding-period returns during the prior 60 months.
- Companies were excluded if any data required from the I/B/E/S database were missing.
- Companies were excluded if forecast GRR was based on fewer than three security analyst estimates.
- Companies were excluded if forecast EPS(3) was based on fewer than three analyst estimates, unless forecast EPS(2) was based on at least three analyst estimates, in which case NEPS(1) equals EPS(2)/(1 + GRR).
- Companies were excluded if DYD equals DIV(0)/PPS was in excess of 13 percent, on the grounds that DIV(0) cannot be a normal dividend if it results in such a high dividend yield.
- Companies were excluded if EYD equals NEPS(1)/PPS was below 2 percent, on the grounds that the estimate of normalized earnings cannot be accurate if it results in such a low earnings yield. Note that forecast earnings for some of these companies were negative.

Table 1 presents the number of companies excluded for each reason in each quarter.[5]

EXPLANATION OF RESULTS

Table 2 presents the mean EXR, the adjusted multiple correlation squared (AJR^2), and other regression statistics for Equation 12 in each of the 28 quarters when EXR is based on a horizon of seven years. With few exceptions, the AJR^2s fluctuate in a very narrow range around their mean, which is 0.270. These values of AJR^2 are a striking improvement over the results obtained by Harris and Marston (1992) and by Gordon (1993). In their work, EXR was simply DYD + GRR, which is the value that EXR approaches in Equation 10 as N approaches infinity with BETA as the sole explanatory variable. The AJR^2 obtained here with N equal to 7 is three to five times larger than the values they obtained.[6]

Table 1. Reasons for and Number of Exclusions to 28 S&P 500 End-of-Quarter Samples, 1985–91

	Excluded							
Date	NYSE/Amex Not Traded	BETA #obs < 45	I/B/E/S Missing Data	GRR #ests < 3	NEPS #ests < 3	DYD Value > 13%	EYD Value < 2%	Accepted
1985								
Q1	19	23	10	25	33	1	1	388
Q2	18	22	10	23	8	–	3	416
Q3	21	22	12	28	14	–	3	400
Q4	21	22	7	29	31	–	7	383
1986								
Q1	22	21	8	24	34	–	7	384
Q2	22	19	9	25	1	–	6	418
Q3	24	20	12	26	9	–	7	402
Q4	24	20	13	20	25	–	12	386
1987								
Q1	25	21	12	13	38	2	6	383
Q2	26	22	16	30	1	–	5	400
Q3	25	24	14	30	7	–	6	394
Q4	26	14	13	31	20	2	6	388
1988								
Q1	26	15	21	27	24	–	3	384
Q2	25	17	18	24	3	–	3	410
Q3	26	17	10	22	10	2	4	409
Q4	26	20	10	15	32	–	5	392
1989								
Q1	26	22	10	18	6	–	3	415
Q2	27	23	9	13	3	–	5	420
Q3	28	24	7	16	12	–	4	409
Q4	30	23	5	16	13	–	7	406
1990								
Q1	29	24	6	21	6	1	4	409
Q2	29	23	6	25	6	–	3	408
Q3	30	19	8	20	10	7	5	401
Q4	32	18	7	19	28	1	9	386
1991								
Q1	33	16	9	20	5	–	5	412
Q2	33	17	8	21	1	–	8	412
Q3	33	16	7	22	6	–	10	406
Q4	32	16	8	21	6	–	10	407

Table 2. Statistics from the Regression of EXR on BETA, DYD, and SKEW, with a Seven-Year Horizon and for 28 S&P 500 End-of-Quarter Samples, 1985–91

			BETA		DYD		SKEW		
Date	EXR Value	Constant Term	Coefficient	*t*-Statistic	Coefficient	*t*-Statistic	Coefficient	*t*-Statistic	Adjusted R^2
1985									
Q1	12.98%	0.1042	0.0068	2.69	0.50	10.60	–0.0020	–1.02	0.240
Q2	11.96	0.0955	0.0056	2.44	0.53	11.04	–0.0004	–0.22	0.244
Q3	12.09	0.0930	0.0063	2.60	0.59	12.84	–0.0006	–0.28	0.312
Q4	11.05	0.0829	0.0070	2.82	0.60	12.64	0.0018	0.88	0.309
1986									
Q1	10.42	0.0792	0.0092	3.98	0.50	11.41	0.0018	0.96	0.257
Q2	10.57	0.0726	0.0174	7.32	0.47	10.46	–0.0007	–0.38	0.229
Q3	10.26	0.0685	0.0186	6.75	0.47	9.11	–0.0009	–0.46	0.193
Q4	9.60	0.0653	0.0157	5.42	0.47	8.27	0.0034	1.59	0.162
1987									
Q1	10.17	0.0705	0.0131	4.10	0.57	10.08	0.0042	1.81	0.213
Q2	9.46	0.0569	0.0193	6.91	0.62	12.36	0.0005	0.28	0.284
Q3	9.81	0.0608	0.0180	6.70	0.61	12.20	0.0030	1.61	0.279
Q4	11.42	0.0690	0.0238	6.20	0.58	11.16	–0.0021	–0.93	0.253
1988									
Q1	11.07	0.0694	0.0222	6.07	0.59	12.29	–0.0053	–3.11	0.311
Q2	11.36	0.0818	0.0160	4.72	0.46	9.77	–0.0034	–1.84	0.207
Q3	11.32	0.0716	0.0237	6.92	0.53	10.59	–0.0050	–2.69	0.246
Q4	11.13	0.0746	0.0196	5.60	0.50	9.77	–0.0022	–1.23	0.202
1989									
Q1	11.20	0.0682	0.0246	7.65	0.56	11.98	–0.0038	–2.27	0.275
Q2	10.58	0.0638	0.0246	7.33	0.54	10.25	–0.0043	–2.48	0.223
Q3	10.65	0.0616	0.0258	8.75	0.55	12.15	–0.0022	–1.42	0.285
Q4	10.71	0.0565	0.0296	9.15	0.58	11.82	–0.0010	–0.60	0.282
1990									
Q1	11.24	0.0578	0.0303	8.83	0.65	14.31	–0.0026	–1.52	0.349
Q2	10.89	0.0519	0.0312	8.99	0.69	15.89	–0.0003	–0.15	0.392
Q3	12.71	0.0470	0.0482	12.01	0.67	14.79	–0.0056	–2.48	0.435
Q4	11.62	0.0653	0.0288	9.19	0.53	12.86	–0.0054	–3.22	0.349
1991									
Q1	10.17	0.0654	0.0194	7.52	0.50	12.13	–0.0042	–3.21	0.287
Q2	10.13	0.0700	0.0157	5.47	0.48	10.24	–0.0041	–2.88	0.215
Q3	10.11	0.0660	0.0188	7.53	0.49	10.91	–0.0042	–3.13	0.256
Q4	9.53	0.0589	0.0183	7.23	0.57	11.86	–0.0021	–1.52	0.267
Mean	10.86	0.0696	0.0199	6.46	0.55	11.56	–0.0017	–1.07	0.270

Turning to the coefficients of the independent variables at N equals 7 in Table 2, the coefficients of BETA are all positive and have t-statistics greater than 2. Both the coefficients and the t-statistics rise sharply over the first six quarters, and thereafter, with few exceptions fluctuate in fairly narrow ranges around their means, 0.02 and 6.46, respectively. The DYD coefficients and t-statistics fluctuate in narrow ranges around their means, 0.55 and 11.56, respectively. SKEW does not do as well. Its coefficients do not become consistently negative until the last 17 quarters, and even then its t-statistics are not always above 2.

Table 3 presents, for each of six EXRs ($N = 0$, 5, 7, 10, 20, and $N \to \infty$) and for each of the independent variables of Equation 12, the mean of the 28 sample means and the mean of the 28 sample standard deviations. Note the FHERM states that N is finite; however, under the version of the FHERM used here and stated in Equation 10, as N approaches infinity, EXR approaches DYD + GRR. Also note that the EXR for corporations paying no dividend is undefined when using Equation 3 or Equation 4, but it is defined for all finite values of N when using Equation 10. As N rises from zero to infinity, the mean EXRs rise by decreasing amounts from 8.47 percent to 14.70 percent; at N equals 7, the mean EXR is equal to 10.86 percent. For comparison, Harris and Marston (1992) reported a mean EXR equal to 16.31 percent, and Ibbotson Associates (1996) reported a mean of the monthly realized holding-period returns on the S&P 500 equal to 11.7 percent based on the years 1926 to 1994 and 10.6 percent based on the years 1965 to 1994.

In Table 3, the 28 standard deviations of EXR fluctuate in a very narrow range around their mean of 2.12 percent at N equals 7. Hence, practically all of the 400 or so corporations in each quarter have

Table 3. Average Mean and Average Standard Deviation of EXR, BETA, DYD, and SKEW, with Six Horizons and for 28 S&P 500 End-of-Quarter Samples, 1985–91

	EXR Horizon								
Average	0 Years	5 Years	7 Years	10 Years	20 Years	$\to \infty$	BETA	DYD	SKEW
Mean	8.47%	10.28%	10.86%	11.59%	13.06%	14.70%	1.11	3.14%	0.229
Standard deviation	3.00	2.24	2.12	2.08	2.35	3.21	0.35	2.14	0.508

values of EXR that are equal to 10.86 percent plus or minus 4.24 percent. The means of the standard deviations and hence the range of variation in EXR among shares is considerably higher than 2.12 percent at N equals zero and as N approaches infinity. With regard to the independent variables, the mean of the BETA means is above 1, perhaps because equal weights and not value weights were used, and the mean of the SKEW standard deviations is quite large.

Table 4 presents summary numbers for the important regression statistics obtained for Equation 12 when EXR is based on N equal to 0, 5, 7, 10, and 20 and N approaching infinity. The table shows the mean and standard deviation of the indicated statistic over the 28 quarterly regressions. The AJR^{2}s rise from 0.180 at N equals zero to 0.270 at N equals 7, and they fall sharply to 0.061 as N approaches infinity. Over the interval $5 \leq N \leq 10$, the AJR^{2}s change little. The BETA coefficients change little as N rises from zero to infinity, but their t-statistics are materially larger in the interval $5 \leq N \leq 10$ than at N equals zero or as N approaches infinity. The DYD coefficients decline continuously to zero as N rises from zero to infinity, but their t-statistics reach a maximum at N equals 7. The SKEW coefficients do not perform as well as the other coefficients. The mean of their t-statistics is not significantly different from zero at N equals zero, and it only becomes statistically significant when N is greater than 12.

There are two reasons for the striking improvement in these results over those reported in Harris and Marston (1992) and in Gordon (1993). One is the recognition that forecasts of growth have a finite horizon, and the other is the addition of dividend yield as an independent variable. Either one alone, however, does not materially improve the results, as can be seen from the simple correlations in Table 5 and the AJR^{2}s in Table 4. Note that the average simple correlation between EXR and BETA is 0.182 as N approaches infinity, which is about what Harris and Marston and Gordon obtained, and it decreases to 0.056 at N equals 7. Note also that the average simple correlation between EXR and DYD as N approaches infinity is negative and that the addition of DYD as an independent variable causes the AJR^2 as N approaches infinity to increase only slightly, from $(0.182)^2$ equals 0.033 to 0.061. The two innovations combined, however, result in the high AJR^2 of 0.270 at N equals 7. Why? Because BETA and DYD have a very high negative correlation; the average at N equals 7 is –0.449.

The high simple correlation between EXR and DYD when N is small may be attributed to the correlation between EYD and DYD and to the fact that EXR becomes EYD when N becomes zero. The average simple correlation between EXR and DYD, however, rises from 0.366 to 0.421 as N rises from zero to 7, and the average t-statistic for the DYD coefficient is maximized at N equals 7. Hence, the partial correlation between EXR and DYD seems to be partly, if not entirely, the result of the tax advantage of capital gains over dividends.

Table 4. Mean and Standard Deviation of Statistics from the Regression of EXR on BETA, DYD, and SKEW, with Six Horizons and for 28 S&P 500 End-of-Quarter Samples, 1985–91

	Horizon					
Statistic	0 Years	5 Years	7 Years	10 Years	20 Years	$\rightarrow \infty$
Adjusted R^2						
Mean	0.180	0.261	0.270	0.254	0.153	0.061
Standard deviation	0.046	0.054	0.061	0.069	0.061	0.022
Constant term						
Mean	0.0428	0.0629	0.0696	0.0780	0.0968	0.1287
BETA coefficient						
Mean	0.0191	0.0197	0.0199	0.0201	0.0199	0.0180
BETA t-statistic						
Mean	3.957	5.947	6.460	6.641	5.521	3.544
Standard deviation	2.015	2.278	2.320	2.244	1.713	1.164
DYD coefficient						
Mean	0.644	0.576	0.549	0.512	0.405	–0.000
DYD t-statistic						
Mean	9.010	11.392	11.563	10.881	7.176	0.101
Standard deviation	1.405	1.496	1.686	1.889	1.779	1.399
SKEW coefficient						
Mean	0.0017	–0.0008	–0.0017	–0.0027	–0.0048	–0.0082
SKEW t-statistic						
Mean	0.619	–0.535	–0.068	–1.674	–2.344	–2.712
Standard deviation	0.971	1.387	1.559	1.679	1.545	1.167

Table 5. Simple Correlations among EXR, BETA, and DYD, with Three Horizons and for 28 S&P 500 End-of-Quarter Samples, 1985–91

	EXR and BETA			EXR and DYD			
Date	0 Years	7 years	$\to \infty$	0 Years	7 Years	$\to \infty$	BETA and DYD
1985							
Q1	–0.196	–0.122	0.187	0.457	0.474	–0.112	–0.475
Q2	–0.181	–0.131	0.231	0.420	0.482	–0.092	–0.463
Q3	–0.177	–0.130	0.167	0.447	0.548	0.024	–0.416
Q4	–0.188	–0.133	0.173	0.491	0.541	–0.062	–0.448
1986							
Q1	–0.088	–0.047	0.164	0.327	0.473	–0.021	–0.437
Q2	0.058	0.144	0.219	0.289	0.359	–0.091	–0.402
Q3	0.073	0.139	0.227	0.308	0.316	–0.136	–0.429
Q4	0.026	0.103	0.232	0.343	0.302	–0.169	–0.426
1987							
Q1	–0.052	–0.021	0.144	0.428	0.414	–0.099	–0.455
Q2	0.018	0.074	0.182	0.439	0.444	–0.096	–0.431
Q3	–0.016	0.062	0.234	0.425	0.436	–0.109	–0.452
Q4	–0.032	0.029	0.148	0.334	0.422	0.027	–0.491
1988							
Q1	–0.051	–0.033	0.103	0.392	0.478	0.028	–0.525
Q2	–0.050	–0.041	0.096	0.315	0.396	–0.011	–0.544
Q3	0.054	0.053	0.101	0.269	0.377	–0.008	–0.533
Q4	0.007	0.007	0.117	0.246	0.368	–0.016	–0.551
1989							
Q1	0.063	0.043	0.068	0.330	0.405	–0.056	–0.549
Q2	0.049	0.084	0.152	0.341	0.338	–0.169	–0.534
Q3	0.048	0.121	0.208	0.362	0.386	–0.159	–0.504
Q4	0.109	0.175	0.198	0.331	0.364	–0.115	–0.459
1990							
Q1	0.085	0.127	0.161	0.394	0.472	–0.016	–0.408
Q2	0.060	0.109	0.203	0.425	0.520	–0.023	–0.405
Q3	0.247	0.345	0.296	0.360	0.473	0.102	–0.204
Q4	0.107	0.236	0.261	0.295	0.433	0.030	–0.293
1991							
Q1	–0.009	0.115	0.219	0.400	0.416	–0.142	–0.404
Q2	–0.054	0.052	0.195	0.371	0.379	–0.110	–0.425
Q3	0.007	0.132	0.222	0.344	0.371	–0.129	–0.417
Q4	–0.014	0.065	0.184	0.367	0.411	–0.127	–0.488
Mean	–0.003	0.056	0.182	0.366	0.421	–0.066	–0.449

AGREEMENT WITH THE TAX-ADJUSTED CAPM

How closely do the empirical values for the constant term, the BETA coefficient, and the DYD coefficient in Equation 12 obtained with N equal to 7 agree with those predicted by the tax-adjusted CAPM? When the favorable treatment of capital gains under the personal income tax is recognized, the expression for EXR becomes

$$\text{EXR} = \gamma_0 + \gamma_1 \text{BETA} + \gamma_2 \text{DYD}. \tag{14}$$

Unfortunately, the coefficients of Equation 14 depend in a complicated way on how marginal tax rates vary with income and on the distribution of income among investors. All that can be said is that γ_0, γ_1, and γ_2 are all positive under the tax-adjusted CAPM.

Under the simplifying assumptions that dividends and interest are taxed at a uniform rate regardless of income and that capital gains, both realized and unrealized, are tax-free, it can be shown that

$$\gamma_0 = \text{RFR}(1 - \text{TRP}),$$
$$\gamma_1 = \text{EXRM} - \text{RFR} - (\text{DYDM} - \text{RFR})\,\text{TRP},$$

and

$$\gamma_2 = \text{TRP}$$

(see Elton and Gruber 1991). Here, TRP equals the tax rate on interest and dividend payments, DYDM equals the dividend yield on the market portfolio, and RFR and EXRM are as defined earlier.

These values of γ may be compared with the values of α_0, α_1, and α_2 in Table 4 at N equals 7. The skewness term may be ignored because it is comparatively unimportant. The marginal tax rate on the highest level of personal income was 50 percent from 1985 to 1986 and 31 percent from 1987 to 1991. Realized capital gains during the same years were taxed at varying rates, including the same rates as other income, and unrealized capital gains were tax free of course. Setting TRP equal to 0.31 and ignoring the taxation of realized capital gains,

$$\gamma_2 = 0.31 < 0.55 = \alpha_2.$$

Using the average rate of 6.69 percent on 90-day Treasury bills over the sample period for RFR,

$$\gamma_0 = 6.69\%*(1 - 0.31)$$
$$= 0.0462 < 0.0696 = \alpha_0.$$

Finally, with EXRM and DYDM equal to the average sample values from Table 3 at N equals 7,

$$\gamma_1 = 10.86\% - 6.69\% - (3.14\% - 6.69\%)0.31$$
$$= 0.0527 > 0.0199 = \alpha_1 .$$

Therefore, the constant term and the coefficients obtained for Equation 12 depart materially from the values predicted by the tax-adjusted CAPM under the tax assumptions stated above. Differences should be expected, however, as a result of some combination of (1) error in the tax assumptions stated above, (2) error in the estimation of the independent variables for Equation 12, and (3) departures in capital markets from the assumptions of the CAPM and the FHERM. Consequently, because the empirical values α_0, α_1, and α_2 are within an order of magnitude of their predicted values, γ_0, γ_1, and γ_2, this exercise has demonstrated that the constant term and the coefficients obtained for Equation 12 are reasonable.

CONCLUSION

The finite horizon expected return model is a new method for estimating the EXR on a share. The FHERM states that

- forecasts of abnormal performance have a finite horizon, *N*, beyond which investors expect a corporation to earn for all future time a return on equity investment equal to the expected return on its shares; and
- the expected return on a share is the discount rate that equates the share's current price with a dividend expectation, where the dividend in each period from 1 to *N* is equal to its forecast and the dividend in each period from *N* + 1 to infinity is equal to the forecast for normalized earnings in Period *N* + 1.

Other methods of estimating EXR have been criticized because they do not recognize that investment decisions are primarily based on current forecasts and that those forecasts have a limited horizon. Those maxims are the very foundation of the FHERM, however, and hence, the FHERM is, at least intuitively, a very accurate description of investors' expected return.

The evidence strongly supports the joint hypothesis that the FHERM and the CAPM are both true. For the estimation of EXR, a version of the FHERM modified for I/B/E/S forecasts was used—it assumes that investors also expect dividends, with last year's actual as the base, and normalized earnings, with next year's forecast as the base, both to grow for *N* years at the rate forecast for long-term growth in earnings.

For the explanation of how EXR varies among shares, a version of the CAPM modified for market imperfections was used—it assumes that EXR depends not only on beta but also on dividend yield and skewness. Multiple tests in which each test used a different value for the horizon, *N*, produced the proof—a single, small interval of high correlation, with its maximum at *N* equals 7. Hence, the consensus among investors is that the future has a finite horizon of approximately seven years. Over all shares and all periods, the mean value of EXR equal to 10.86 percent at *N* equals 7 is more reasonable for the sample years than either the mean value of EXR equal to 8.47 percent at *N* equals zero, which represents the earnings yield model and forecasts of normal performance only, or the mean value of EXR equal to 14.70 percent as *N* approaches infinity, which represents the dividend growth model and forecasts of eternal abnormal performance.

Compared with previous efforts at estimating the expected return on a share and at explaining its variation among shares—especially the empirical work in which average realized return is used as the estimate of EXR and shares are grouped in portfolios—the FHERM performed exceptionally well. Consequently, the FHERM has promising potential to be the basis for further research on the estimation of EXR and on the explanation of how EXR varies among shares. Possibly, with this research, EXR will realize its potential to be a very important capital market statistic.[7]

NOTES

1. There are exceptions to this conclusion. For instance, Kothari, Shanken, and Sloan (1995) found significant positive correlation between average realized return and beta using annual data with the shares grouped in portfolios. Nevertheless, they found "virtually no relation between beta and average return over the relatively short post-1962 period." See also Jagannathan and Wang (1996), who tested a multifactor model in which betas were obtained not only for the market portfolio but also for human capital, measured by the growth rate in employment income, and for the yield spread between high- and low-grade bonds. They found the coefficient on the market portfolio to be negative but barely different from zero, while the betas on the other two risk variables were significantly positive. Kan and Zhang (1996) showed that "useless factors" are found to be useful under this two-pass methodology.
2. The estimates of BETA seem fairly good and have become widely used. High-BETA stocks rise and fall with the market more than low-BETA stocks and are, therefore, said to be riskier.
3. Note that under either of two conditions, the price of a share is independent of the corporation's dividend policy. One is

the condition used here and stated above that REI equals EXR. The other condition is the Miller–Modigliani theorem, under which a distinction is drawn between retained earnings and the sale of shares as a source of equity capital (and between dividends and repurchase as a means of distribution) and one is a perfect substitute for the other. When retained earnings are the sole source of equity funds, dividends are the sole means of distributions, and capital structure is given, dividend policy is investment policy.

4. The limitations of BETA as a measure of risk from all sources have been investigated extensively by others, and further examination will not be attempted here.
5. The reasons for excluding firms and the number excluded for each reason are presented so that our empirical results can be verified by replication and the comparative consequences of different bases for exclusion can be established. The first three bases for exclusion are beyond our control, the next two bases for exclusion represent a judgment call on our part that we consider sound, and the last two bases for exclusion result in only a few exclusions that are quite justifiable.
6. The contrast between this article's results and those obtained using average realized returns as the estimate of EXR are far more striking. As noted earlier, results obtained using average realized returns find either (1) no correlation at all between EXR and BETA, (2) no correlation for long subperiods between EXR and BETA, or (3) betas for other variables such as human capital are far more important than the beta for the market portfolio. Furthermore, statistical work using average realized returns has been based on shares being grouped in portfolios, so that the methodology does not estimate the EXR on an individual actual share, precisely what is needed to use the CAPM. This article's AJR^2 at N equals 7 would have been much higher if the shares had been grouped in portfolios.
7. The authors benefited from comments on earlier drafts by Don Brean, Raymond Kan, and Alan White. We gratefully acknowledge the contribution of I/B/E/S International in providing forecast data for earnings per share and growth rate, available through the Institutional Brokers Estimate System. These data have been provided as part of a broad academic program to encourage earnings expectation research.

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