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The N-Stage Discount Model and Required Return: A Comment

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Abstract

A number of financial economists have observed that estimates of the market discount rate have a downward bias when dividend timing is ignored. They have done so in academic and utility industry journals as well as in testimony. Most conclude or imply that such a downward bias carries over to the calculation of a regulated utility's required rate of return. This paper demonstrates that in fact the conventional cost of equity calculation, ignoring quarterly compounding and even without adjustment for fractional periods, serves very well as a measure of required return.

Introduction

In a recent issue of *The Financial Review*, Brooks and Helms presented an N-stage dividend discount model [1]. The model is a welcome addition to the analytic tool kit available for estimation of market discount rates.

Nevertheless, I think they make an unwarranted leap from the model to the conclusion that failure to consider quarterly compounding and fractional periods introduces a downward bias in rate of return calculation, and that theirs is "an efficient procedure . . . for estimating the required rate of return" ([1], p. 656). That this presumption may mislead analysts involved in public utility rate proceedings is likely because their illustration involves a regulated electric utility, Commonwealth Edison Company, and their point seems to have relevance for regulatory proceedings.

Brooks and Helms are not alone in their observation. A number of financial economists note that market discount rate estimates are biased downward when div-

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idend timing is ignored. These findings have appeared in academic and utility industry journals as well as in testimony. Academic articles include [1, 2, 7, 8, 12], and examples in the utility literature are [3, 14]. For recent testimony that makes the point, see [9]. The same point is made in Morin's *Utilities' Cost of Capital* [10, pp. 141–142]. Most authors have concluded or implied that such a downward bias carries over to the calculation of the required rate of return.

Linke and Zumwalt [2, 7, 8] are the exceptions. They made it clear that there is a distinction between a utility's market discount rate (k in my notation) and the rate year required return (r) that regulators should allow, and that reconciling the two necessitates a calculation.

I do not dispute the observation that an estimate of the market discount rate has a downward bias when dividend timing is ignored, nor do I find fault with the Linke and Zumwalt market rate to rate year required return adjustment calculations. My intention here is to point out that the conventional cost of equity calculation used in utility rate cases (k^* in my notation), which ignores timing, is (or is easily transformed into) an unbiased estimate of rate year required return (k^{**} in my notation), while the correct market discount rate, if unadjusted, has an upward bias when used to represent required utility return.

Base Case

The market discount rate annually compounded for the year ahead is the rate k that satisfies the equation

$$P_0 = \frac{d1}{(1 + k)^{t1}} + \frac{d2}{(1 + k)^{t2}} + \frac{d3}{(1 + k)^{t3}} + \frac{d4}{(1 + k)^{t4}} + \frac{P_0(1 + \hat{g})}{(1 + k)}, \quad (1)$$

where P_0 is the market price of a stock at time 0; dn is the first, second, third, or fourth dividend expected in the year ahead (quarterly dividends are assumed but the assumption is unimportant); $\hat{g}$ is the expected annual growth rate in stock price; and tn is the fraction of a 365-day year before dividend n is to be received. I consider the ex-dividend date to be the date the dividend is received because it is the date on which the dividend becomes the investor's property, a property that remains the investor's even if the stock is sold prior to the payment date. Brooks and Helms used the actual dividend payment date, which I follow when I use their illustration in this note.

The total dividend expected in the year is $D = d1 + d2 + d3 + d4$, and the conventional cost of equity calculation used in utility rate cases is

$$k^* = (D/P_0) + \hat{g}. \tag{2}$$

If market price (P_0) and book value (B_0) are equal at time 0, and the rate year begins at time 0, then, using k^* as allowed return (r), regulators would approve prices for utility services that provide expected earnings (E):

$$E = k^*P_0. \tag{3}$$

Combining equations (2) and (3),

$$\hat{g} = E - D/P_0. \tag{4}$$

Regulators using this approach will provide an expected cash flow that just satisfies equation (1):

$$P_0 = \frac{d1}{(1+k)^{t1}} + \frac{d2}{(1+k)^{t2}} + \frac{d3}{(1+k)^{t3}} + \frac{d4}{(1+k)^{t4}} + \frac{P_0 + E - D}{(1+k)}. \tag{1a}$$

If regulators set allowed return equal to k (the market discount rate) rather than the smaller k^*, expected cash flow would discount to a current price greater than P_0. The market discount rate (k), when used directly in this way, would produce a required return (r) with an upward bias.

The conventional estimate of cost of equity (k^*) is also the correct estimate of required return (r) when price and book values are not equal. To see that it is, consider a payout rate PAY_m. Set it equal to d_m/k^*P_0 at each dividend date n. Then use this payout with the earnings indicated by applying the same required return rate to book value, k^*B_0.

The result is that each term in equation (1) or (1a) is multiplied by the ratio B_0/P_0 so that the dividends and final book value that could be provided and are expected discount to initial book value at the market discount rate, k.

If the rate year does not begin at the same time that the market price is observed, an adjustment in the calculation of k^* (to k^{**}) is required. Because P rises as the time ($t1$) to the first dividend ($d1$) falls, the calculation of k^{**} will vary from k^* with $t1$. The market discount rate (k) will not change.

To get a correct conventional measure of required return, prices must be adjusted to reflect any time difference from $d1$ in market price and rate year book value. For example, if the market price used is the price 30 days before a dividend date, and initial book value for the rate year is 90 days before a dividend date, then the proper price to use in the conventional but adjusted cost of equity calculation, k^{**}, is $P_0/(1 + k)^{60/365}$, and

$$k^{**} = (D/[P_0/(1 + k)^{(60/365)}]) + \hat{g} \tag{5}$$

is the right measure of required return (r). This adjustment is one that staff witnesses for the New York Public Service Commission appear to make in calculating k^* (see [13]). If the market price is 90 days before and the rate year book value 30 days before the dividend date, then $(1 + k)^{-60/365}$ would be used to adjust P_0.

Because the timing difference from dividend dates for market price and rate year book value may result in either an upward or downward adjustment of the same magnitude in the conventional estimate of cost of equity (k^*), omitting the adjustment—failing to use k^{**}—introduces error but not bias. The conventional measure of cost of equity (k^*), a measure that does not consider quarterly compounding and usually fails to consider fractional periods, has no downward bias as an estimate of required return (r). It is, as the Federal Energy Regulatory Commission (FERC) and other regulatory bodies have concluded, a fair measure to use in calculating the allowed return for a utility. The FERC, in its *Generic Determination of Rate of Return on Common Equity for Public Utilities*, embraces the Linke and Zumwalt analysis in Order No. 442 [4], reconsiders it in Order No. 442-A [5], and settles on the required return I develop here in Order No. 461 [6].

First Illustration

For illustration, consider the Brooks and Helms no-growth case for Commonwealth Edison. That this is an illustration simplifying most of the very difficult problems of estimation facing a cost of equity analyst is particularly clear in the case of Commonwealth Edison. Its very complicated situation is described in a November 1990 Salomon Brothers report [11]. On June 9, 1989, Commonwealth Edison stock closed at 37 5/8; the next dividend date is 52 days away on August 1, 1989; the expected dividend on that date is \$0.75, and assumed and expected growth is zero. With this information, k = 8.287 percent, and equation (1) yields

$$37.625 = \frac{0.75}{1.08287^{(54/365)}} + \frac{0.75}{1.08287^{(146/365)}} + \frac{0.75}{1.08287^{(238/365)}} + \frac{0.75}{1.08287^{(327/365)}} + \frac{37.625(1 + 0)}{1.08287},$$

and $k^* = 7.973$ percent or \$37.625.

If market and book values were equal and the rate year began on June 9, 1989, then setting allowed and required return (r) equal to k^* would provide earnings of $k^*P(0.07973 \times 37.625)$, or \$3.00. Because rate case earnings reflect cash flow timing, including dividend payments, as well as short-term interest expense and revenue, the \$3.00 covers the \$0.75 dividends received by investors on May, August, November, and February 1st and maintains book and market value at \$37.625.

Suppose, however, that the rate year begins on January 1, 1990. The book value estimated for that date is \$32.68, according to Value Line, and the next dividend is 31 rather than 52 days away. Adjusting price for the difference in dividend timing and calculating a conventional but adjusted required return

$$\begin{aligned} k^{**} &= (D/[P_0/(1 + k)^{(-21/365)}]) + \hat{g} \\ &= (3/[37.625/1.08287^{(-21/365)}]) + 0 \\ &= 7.9370\%. \end{aligned}$$

Earnings based on an allowed and required return (k^{**}) of 7.9370 percent and a book value ($32.68) would be set at $2.594 and, with payout at 25 percent of earnings each quarter ($d_1/k^{**}P_0$), would be associated with a $0.6485 dividend on February 1, 1990 and on subsequent dividend dates.

The present value of the expected dividend flow and the unchanged or zero growth end-of-year 1990 book value is the January 1, 1990 book value:

$$B_0 = \frac{0.6485}{(1.08287)^{(31/365)}} + \frac{0.6485}{(1.08287)^{(120/365)}} + \frac{0.6485}{(1.08287)^{(212/365)}} + \frac{0.6485}{(1.08287)^{(304/365)}} + \frac{32.68(1 + 0)}{(1.08287)}.$$

$$B_0 = 32.68.$$

In other words, the allowed return set equal to the conventional but adjusted cost of equity estimate (k^{**}) provides earnings and dividends sufficient to support book value at the market discount rate. In this illustration, the conventional but adjusted cost of equity calculation (k^{**}) provides the correct estimate of the required rate of return.

Second Illustration

My first illustration has assumed no growth and full payout of dividends. A second illustration with dividend growth and fractional payout may be more useful. Linke and Zumwalt [7, pp. 16–17] provided the material for that illustration. A stock with dividend due one quarter away is now selling at $8.2294, which is also its book value. The dividend expected is $0.25 at the end of the current and the following quarter and $0.265 in each of the four following quarters. Price, like dividends, is expected to increase 6 percent from one year to the next, so that one year from now price is expected to be 8.2294 × 1.06, or $8.7232. The rate year begins with the first dividend one quarter away, so k^* and k^{**} are equal.

The market discount rate (k) is 19.375:

$$8.2294 = \frac{0.25}{1.19375^{0.25}} + \frac{0.25}{1.19375^{0.50}} + \frac{0.265}{1.19375^{0.75}} + \frac{0.265}{1.19375} + \frac{8.7232}{1.19375}.$$

The conventional cost of equity (k^*) is

$$\begin{aligned} k^* &= D/P_0 + \hat{g} \\ &= 1.03/8.2294 + 0.06 \\ &= 18.516\%. \end{aligned}$$

The conventional cost of equity (k^*) is less than the market discount rate (k), but as a measure of required return (r), it is still correct. The earnings it provides, $k^*P_0 = 0.18516 \times 8.2294 = 1.524$, are just sufficient to cover dividends and support book and market value growth of 6 percent:

	Q1	Q2	Q3	Q4	Year
Book, Start of Q	8.2294	8.3604	8.4914	8.6074	
Earnings	0.381	0.381	0.381	0.381	1.524
Dividend	0.25	0.25	0.265	0.265	1.03
Book, End of Q	8.3604	8.4914	8.6074	8.7234	

This illustration is obvious. The point may be less clear, but more interesting, if book value varies from market price, and rate year timing differs from market timing. The results remain the same, however: While the conventional cost of equity may have a downward bias as an estimate of the market discount rate, it is a correct and unbiased estimate of a utility's required return.

Conclusion

Although many analysts have concluded that required return has a downward bias if it is calculated ignoring quarterly compounding and fractional periods, it would be surprising if they were correct. Too many rate cases have come and gone, and too many utilities have survived and sustained market prices above book, to make downward bias in the conventional calculation of required return a likely reality.

Brooks and Helms and the other authors are correct when they say that the conventional cost of equity calculation is a downward-biased estimate of the market discount rate. They are not correct, however, in concluding that it has a bias as a measure of required return. As a measure of required return, the conventional cost of equity calculation (k^*), ignoring quarterly compounding and even without adjustment for fractional periods, serves very well.

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