

## ACTUARIAL ANALYSIS

Actuarial science is a discipline that applies various statistical methods to assess risk probabilities and other related functions. Actuaries often study human mortality. The results from historical mortality data are used to predict how long similar groups of people who are alive today will live. Insurance companies rely on actuarial analysis in determining premiums for life insurance policies.

The study of human mortality is analogous to estimating service lives of industrial property groups. While some humans die solely from chance, most deaths are related to age; that is, death rates generally increase as age increases. Similarly, physical plant is also subject to forces of retirement. These forces include physical, functional, and contingent factors, as shown in the table below.<sup>1</sup>

**Figure 1:  
Forces of Retirement**

<u>Physical Factors</u>	<u>Functional Factors</u>	<u>Contingent Factors</u>
Wear and tear Decay or deterioration Action of the elements	Inadequacy Obsolescence Changes in technology Regulations Managerial discretion	Casualties or disasters Extraordinary obsolescence

While actuaries study historical mortality data in order to predict how long a group of people will live, depreciation analysts must look at a utility’s historical data in order to estimate the average lives of property groups. A utility’s historical data is often contained in the Continuing Property Records (“CPR”). Generally, a CPR should contain 1) an inventory of property record

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<sup>1</sup> National Association of Regulatory Utility Commissioners, Public Utility Depreciation Practices 14-15 (NARUC 1996).

units; 2) the association of costs with such units; and 3) the dates of installation and removal of plant. Since actuarial analysis includes the examination of historical data to forecast future retirements, the historical data used in the analysis should not contain events that are anomalous or unlikely to recur.<sup>2</sup> Historical data is used in the retirement rate actuarial method, which is discussed further below.

### The Retirement Rate Method

There are several systematic actuarial methods that use historical data to calculate observed survivor curves for property groups. Of these methods, the retirement rate method is superior, and is widely employed by depreciation analysts.<sup>3</sup> The retirement rate method is ultimately used to develop an observed survivor curve, which can be fitted with an Iowa curve discussed in Appendix B to forecast average life. The observed survivor curve is calculated by using an observed life table (“OLT”). The figures below illustrate how the OLT is developed. First, historical property data are organized in a matrix format, with placement years on the left forming rows, and experience years on the top forming columns. The placement year (a.k.a. “vintage year” or “installation year”) is the year of placement into service of a group of property. The experience year (a.k.a. “activity year”) refers to the accounting data for a particular calendar year. The two matrices below use aged data—that is, data for which the dates of placements, retirements, transfers, and other transactions are known. Without aged data, the retirement rate actuarial method may not be employed. The first matrix is the exposure matrix, which shows the exposures

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<sup>2</sup> *Id.* at 112–13.

<sup>3</sup> Anson Marston, Robley Winfrey & Jean C. Hempstead, *Engineering Valuation and Depreciation* 154 (2nd ed., McGraw-Hill Book Company, Inc. 1953).

at the beginning of each year.<sup>4</sup> An exposure is simply the depreciable property subject to retirement during a period. The second matrix is the retirement matrix, which shows the annual retirements during each year. Each matrix covers placement years 2003–2015, and experience years 2008–2015. In the exposure matrix, the number in the 2012 experience column and the 2003 placement row is \$192,000. This means at the beginning of 2012, there was \$192,000 still exposed to retirement from the vintage group placed in 2003. Likewise, in the retirement matrix, \$19,000 of the dollars invested in 2003 were retired during 2012.

**Figure 2:  
 Exposure Matrix**

Placement Years	Experience Years								Total at Start of Age Interval	Age Interval
	Exposures at January 1 of Each Year (Dollars in 000's)									
	2008	2009	2010	2011	2012	2013	2014	2015		
2003	261	245	228	211	<b>192</b>	173	152	131	131	11.5 - 12.5
2004	267	252	236	220	202	<b>184</b>	165	145	297	10.5 - 11.5
2005	304	291	277	263	248	232	<b>216</b>	198	536	9.5 - 10.5
2006	345	334	322	310	298	284	270	<b>255</b>	<b>847</b>	8.5 - 9.5
2007	367	357	347	335	324	312	299	286	1,201	7.5 - 8.5
2008	375	366	357	347	336	325	314	302	1,581	6.5 - 7.5
2009		377	366	356	346	336	327	319	1,986	5.5 - 6.5
2010			381	369	358	347	336	327	2,404	4.5 - 5.5
2011				386	372	359	346	334	2,559	3.5 - 4.5
2012					395	380	366	352	2,722	2.5 - 3.5
2013						401	385	370	2,866	1.5 - 2.5
2014							410	393	2,998	0.5 - 1.5
2015								416	3,141	0.0 - 0.5
<b>Total</b>	<b>1919</b>	<b>2222</b>	<b>2514</b>	<b>2796</b>	<b>3070</b>	<b>3333</b>	<b>3586</b>	<b>3827</b>	<b>23,268</b>	

<sup>4</sup> Technically, the last numbers in each column are “gross additions” rather than exposures. Gross additions do not include adjustments and transfers applicable to plant placed in a previous year. Once retirements, adjustments, and transfers are factored in, the balance at the beginning of the next accounting period is called an “exposure” rather than an addition.

**Figure 3:  
 Retirement Matrix**

Placement Years	Experience Years								Total During Age Interval	Age Interval
	Retirements During the Year (Dollars in 000's)									
	2008	2009	2010	2011	2012	2013	2014	2015		
2003	16	17	18	19	<b>19</b>	20	21	23	23	11.5 - 12.5
2004	15	16	17	17	18	<b>19</b>	20	21	43	10.5 - 11.5
2005	13	14	14	15	16	17	<b>17</b>	18	59	9.5 - 10.5
2006	11	12	12	13	13	14	15	<b>15</b>	71	8.5 - 9.5
2007	10	11	11	12	12	13	13	14	82	7.5 - 8.5
2008	9	9	10	10	11	11	12	13	91	6.5 - 7.5
2009		11	10	10	9	9	9	8	95	5.5 - 6.5
2010			12	11	11	10	10	9	100	4.5 - 5.5
2011				14	13	13	12	11	93	3.5 - 4.5
2012					15	14	14	13	91	2.5 - 3.5
2013						16	15	14	93	1.5 - 2.5
2014							17	16	100	0.5 - 1.5
2015								18	112	0.0 - 0.5
Total	74	89	104	121	139	157	175	194	1,052	

These matrices help visualize how exposure and retirement data are calculated for each age interval. An age interval is typically one year. A common convention is to assume that any unit installed during the year is installed in the middle of the calendar year (i.e., July 1st). This convention is called the “half-year convention” and effectively assumes that all units are installed uniformly during the year.<sup>5</sup> Adoption of the half-year convention leads to age intervals of 0–0.5 years, 0.5–1.5 years, etc., as shown in the matrices.

The purpose of the matrices is to calculate the totals for each age interval, which are shown in the second column from the right in each matrix. This column is calculated by adding each number from the corresponding age interval in the matrix. For example, in the exposure matrix, the total amount of exposures at the beginning of the 8.5–9.5 age interval is \$847,000. This number was calculated by adding the numbers shown on the “stairs” to the left (192+184+216+255=847). The same calculation is applied to each number in the column. The amounts retired during the

<sup>5</sup> Frank K. Wolf & W. Chester Fitch, *Depreciation Systems* 22 (Iowa State University Press 1994).

year in the retirements matrix affect the exposures at the beginning of each year in the exposures matrix. For example, the amount exposed to retirement in 2008 from the 2003 vintage is \$261,000. The amount retired during 2008 from the 2003 vintage is \$16,000. Thus, the amount exposed to retirement at the beginning of 2009 from the 2003 vintage is \$245,000 (\$261,000 - \$16,000). The company's property records may contain other transactions which affect the property, including sales, transfers, and adjusting entries. Although these transactions are not shown in the matrices above, they would nonetheless affect the amount exposed to retirement at the beginning of each year.

The totaled amounts for each age interval in both matrices are used to form the exposure and retirement columns in the OLT, as shown in the chart below. This chart also shows the retirement ratio and the survivor ratio for each age interval. The retirement ratio for an age interval is the ratio of retirements during the interval to the property exposed to retirement at the beginning of the interval. The retirement ratio represents the probability that the property surviving at the beginning of an age interval will be retired during the interval. The survivor ratio is simply the complement to the retirement ratio ( $1 - \text{retirement ratio}$ ). The survivor ratio represents the probability that the property surviving at the beginning of an age interval will survive to the next age interval.

**Figure 4:  
 Observed Life Table**

Age at Start of Interval	Exposures at Start of Age Interval	Retirements During Age Interval	Retirement Ratio	Survivor Ratio	Percent Surviving at Start of Age Interval
A	B	C	D = C / B	E = 1 - D	F
0.0	3,141	112	0.036	0.964	<b>100.00</b>
0.5	2,998	100	0.033	0.967	<b>96.43</b>
1.5	2,866	93	0.032	0.968	<b>93.21</b>
2.5	2,722	91	0.033	0.967	<b>90.19</b>
3.5	2,559	93	0.037	0.963	<b>87.19</b>
4.5	2,404	100	0.042	0.958	<b>84.01</b>
5.5	1,986	95	0.048	0.952	<b>80.50</b>
6.5	1,581	91	0.058	0.942	<b>76.67</b>
7.5	1,201	82	0.068	0.932	<b>72.26</b>
8.5	847	71	0.084	0.916	<b>67.31</b>
9.5	536	59	0.110	0.890	<b>61.63</b>
10.5	297	43	0.143	0.857	<b>54.87</b>
11.5	131	23	0.172	0.828	<b>47.01</b>
<b>Total</b>	<b>23,268</b>	<b>1,052</b>			<b>38.91</b>

Column F on the right shows the percentages surviving at the beginning of each age interval. This column starts at 100 percent surviving. Each consecutive number below is calculated by multiplying the percent surviving from the previous age interval by the corresponding survivor ratio for that age interval. For example, the percent surviving at the start of age interval 1.5 is 93.21 percent, which was calculated by multiplying the percent surviving for age interval 0.5 (96.43 percent) by the survivor ratio for age interval 0.5 (0.967).<sup>6</sup>

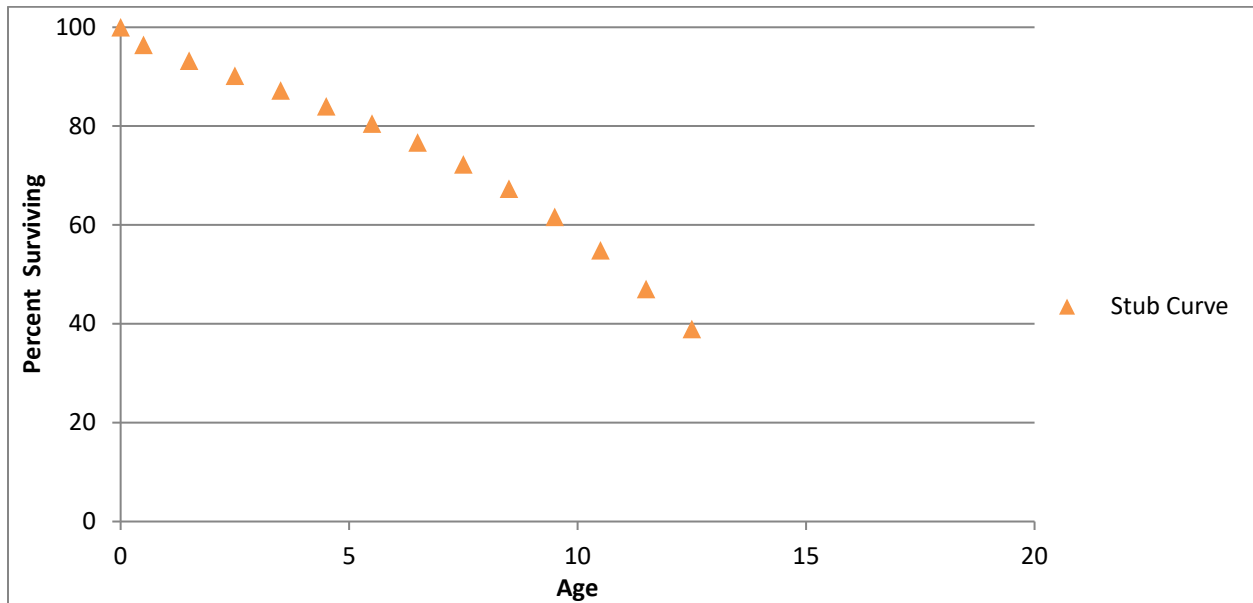
The percentages surviving in Column F are the numbers that are used to form the original survivor curve. This particular curve starts at 100 percent surviving and ends at 38.91 percent surviving. An observed survivor curve such as this that does not reach zero percent surviving is

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<sup>6</sup> Multiplying 96.43 by 0.967 does not equal 93.21 exactly due to rounding.

called a “stub” curve. The figure below illustrates the stub survivor curve derived from the OLT above.

**Figure 5:  
Original “Stub” Survivor Curve**



The matrices used to develop the basic OLT and stub survivor curve provide a basic illustration of the retirement rate method in that only a few placement and experience years were used. In reality, analysts may have several decades of aged property data to analyze. In that case, it may be useful to use a technique called “banding” in order to identify trends in the data.

### Banding

The forces of retirement and characteristics of industrial property are constantly changing. A depreciation analyst may examine the magnitude of these changes. Analysts often use a technique called “banding” to assist with this process. Banding refers to the merging of several years of data into a single data set for further analysis, and it is a common technique associated

with the retirement rate method.<sup>7</sup> There are three primary benefits of using bands in depreciation analysis:

1. Increasing the sample size. In statistical analyses, the larger the sample size in relation to the body of total data, the greater the reliability of the result;
2. Smooth the observed data. Generally, the data obtained from a single activity or vintage year will not produce an observed life table that can be easily fit; and
3. Identify trends. By looking at successive bands, the analyst may identify broad trends in the data that may be useful in projecting the future life characteristics of the property.<sup>8</sup>

Two common types of banding methods are the “placement band” method and the “experience band” method.” A placement band, as the name implies, isolates selected placement years for analysis. The figure below illustrates the same exposure matrix shown above, except that only the placement years 2005–2008 are considered in calculating the total exposures at the beginning of each age interval.

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<sup>7</sup> NARUC *supra* n. 1, at 113.

<sup>8</sup> *Id.*



**Figure 6:  
 Placement Bands**

Placement Years	Experience Years								Total at Start of Age Interval	Age Interval
	Exposures at January 1 of Each Year (Dollars in 000's)									
	2008	2009	2010	2011	2012	2013	2014	2015		
2003	261	245	228	211	192	173	152	131		11.5 - 12.5
2004	267	252	236	220	202	184	165	145		10.5 - 11.5
2005	304	291	277	263	248	232	216	198	198	9.5 - 10.5
2006	345	334	322	310	298	284	270	255	471	8.5 - 9.5
2007	367	357	347	335	324	312	299	286	788	7.5 - 8.5
2008	375	366	357	347	336	325	314	302	1,133	6.5 - 7.5
2009		377	366	356	346	336	327	319	1,186	5.5 - 6.5
2010			381	369	358	347	336	327	1,237	4.5 - 5.5
2011				386	372	359	346	334	1,285	3.5 - 4.5
2012					395	380	366	352	1,331	2.5 - 3.5
2013						401	385	370	1,059	1.5 - 2.5
2014							410	393	733	0.5 - 1.5
2015								416	375	0.0 - 0.5
Total	1919	2222	2514	2796	3070	3333	3586	3827	9,796	

The shaded cells within the placement band equal the total exposures at the beginning of age interval 4.5–5.5 (\$1,237). The same placement band would be used for the retirement matrix covering the same placement years of 2005–2008. This use of course would result in a different OLT and original stub survivor curve than those that were calculated above without the restriction of a placement band.

Analysts often use placement bands for comparing the survivor characteristics of properties with different physical characteristics.<sup>9</sup> Placement bands allow analysts to isolate the effects of changes in technology and materials that occur in successive generations of plant. For example, if in 2005 an electric utility began placing transmission poles into service with a special chemical treatment that extended the service lives of those poles, an analyst could use placement bands to isolate and analyze the effect of that change in the property group’s physical characteristics. While placement bands are very useful in depreciation analysis, they also possess an intrinsic dilemma.

<sup>9</sup> Wolf *supra* n. 5, at 182.

A fundamental characteristic of placement bands is that they yield fairly complete survivor curves for older vintages. However, with newer vintages, which are arguably more valuable for forecasting, placement bands yield shorter survivor curves. Longer “stub” curves are considered more valuable for forecasting average life. Thus, an analyst must select a band width broad enough to provide confidence in the reliability of the resulting curve fit yet narrow enough so that an emerging trend may be observed.<sup>10</sup>

Analysts also use “experience bands.” Experience bands show the composite retirement history for all vintages during a select set of activity years. The figure below shows the same data presented in the previous exposure matrices, except that the experience band from 2011–2013 is isolated, resulting in different interval totals.

**Figure 7:  
Experience Bands**

Placement Years	Experience Years								Total at Start of Age Interval	Age Interval
	Exposures at January 1 of Each Year (Dollars in 000's)									
	2008	2009	2010	2011	2012	2013	2014	2015		
2003	261	245	228	211	192	173	152	131		11.5 - 12.5
2004	267	252	236	220	202	184	165	145		10.5 - 11.5
2005	304	291	277	263	248	232	216	198	173	9.5 - 10.5
2006	345	334	322	310	298	284	270	255	376	8.5 - 9.5
2007	367	357	347	335	324	312	299	286	645	7.5 - 8.5
2008	375	366	357	347	336	325	314	302	752	6.5 - 7.5
2009		377	366	356	346	336	327	319	872	5.5 - 6.5
2010			381	369	358	347	336	327	959	4.5 - 5.5
2011				386	372	359	346	334	1,008	3.5 - 4.5
2012					395	380	366	352	1,039	2.5 - 3.5
2013						401	385	370	1,072	1.5 - 2.5
2014							410	393	1,121	0.5 - 1.5
2015								416	1,182	0.0 - 0.5
Total	1919	2222	2514	2796	3070	3333	3586	3827	9,199	

The shaded cells within the experience band equal the total exposures at the beginning of age interval 4.5–5.5 (\$1,237). The same experience band would be used for the retirement matrix

<sup>10</sup> NARUC *supra* n. 1, at 114.

covering the same experience years of 2011–2013. This use of course would result in a different OLT and original stub survivor than if the band had not been used. Analysts often use experience bands to isolate and analyze the effects of an operating environment over time.<sup>11</sup> Likewise, the use of experience bands allows analysis of the effects of an unusual environmental event. For example, if an unusually severe ice storm occurred in 2013, destruction from that storm would affect an electric utility's line transformers of all ages. That is, each of the line transformers from each placement year would be affected, including those recently installed in 2012, as well as those installed in 2003. Using experience bands, an analyst could isolate or even eliminate the 2013 experience year from the analysis. In contrast, a placement band would not effectively isolate the ice storm's effect on life characteristics. Rather, the placement band would show an unusually large rate of retirement during 2013, making it more difficult to accurately fit the data with a smooth Iowa curve. Experience bands tend to yield the most complete stub curves for recent bands because they have the greatest number of vintages included. Longer stub curves are better for forecasting. The experience bands, however, may also result in more erratic retirement dispersion making the curve-fitting process more difficult.

Depreciation analysts must use professional judgment in determining the types of bands to use and the band widths. In practice, analysts may use various combinations of placement and experience bands in order to increase the data sample size, identify trends and changes in life characteristics, and isolate unusual events. Regardless of which bands are used, observed survivor curves in depreciation analysis rarely reach zero percent. They rarely reach zero percent because, as seen in the OLT above, relatively newer vintage groups have not yet been fully retired at the

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<sup>11</sup> *Id.*

time the property is studied. An analyst could confine the analysis to older, fully retired vintage groups to get complete survivor curves, but such analysis would ignore some of the property currently in service and would arguably not provide an accurate description of life characteristics for current plant in service. Because a complete curve is necessary to calculate the average life of the property group, however, curve-fitting techniques using Iowa curves or other standardized curves may be employed in order to complete the stub curve.

### Curve Fitting

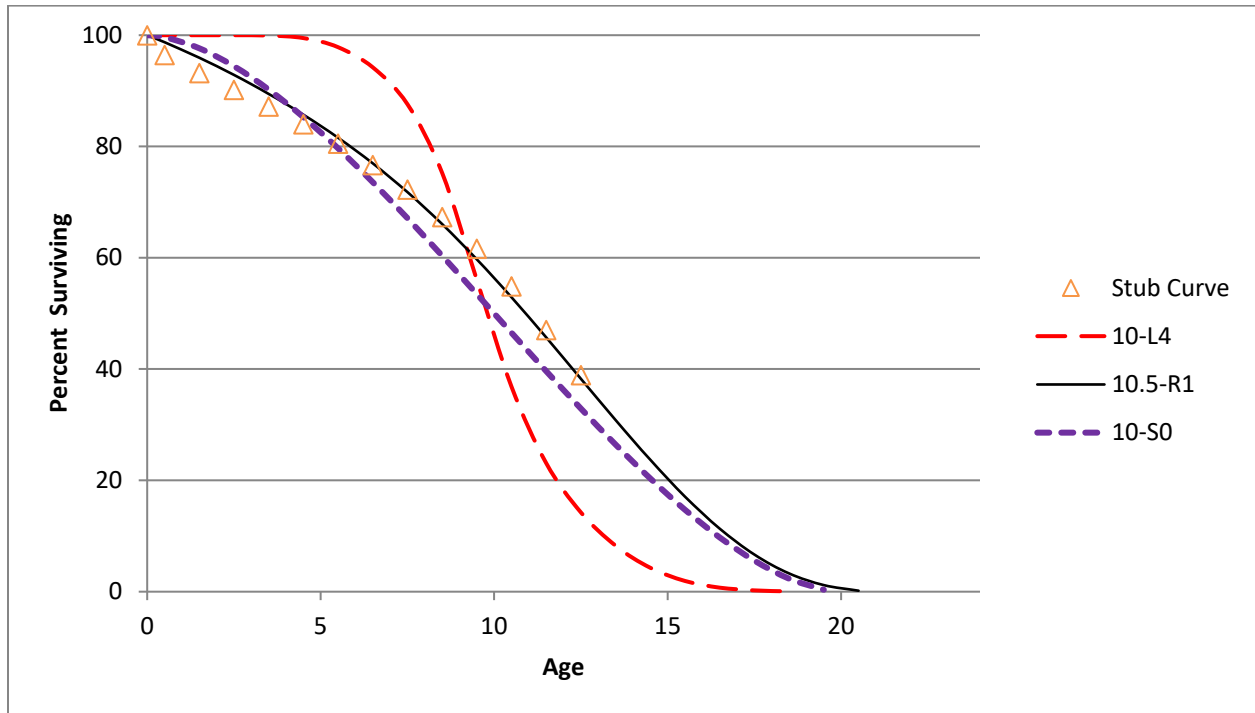
Depreciation analysts typically use the survivor curve rather than the frequency curve to fit the observed stub curves. The most commonly used generalized survivor curves in the curve-fitting process are the Iowa curves discussed above. As Wolf notes, if “the Iowa curves are adopted as a model, an underlying assumption is that the process describing the retirement pattern is one of the 22 [or more] processes described by the Iowa curves.”<sup>12</sup>

Curve fitting may be done through visual matching or mathematical matching. In visual curve fitting, the analyst visually examines the plotted data to make an initial judgment about the Iowa curves that may be a good fit. The figure below illustrates the stub survivor curve shown above. It also shows three different Iowa curves: the 10-L4, the 10.5-R1, and the 10-S0. Visually, the 10.5-R1 curve is clearly a better fit than the other two curves.

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<sup>12</sup> Wolf *supra* n. 5, at 46 (22 curves includes Winfrey’s 18 original curves plus Cowles’s four “O” type curves).

**Figure 8:  
Visual Curve Fitting**



In mathematical fitting, the least squares method is used to calculate the best fit. This mathematical method would be excessively time consuming if done by hand. With the use of modern computer software however, mathematical fitting is an efficient and useful process. The typical logic for a computer program, as well as the software employed for the analysis in this testimony is as follows:

First (an Iowa curve) curve is arbitrarily selected. . . . If the observed curve is a stub curve, . . . calculate the area under the curve and up to the age at final data point. Call this area the realized life. Then systematically vary the average life of the theoretical survivor curve and calculate its realized life at the age corresponding to the study date. This trial and error procedure ends when you find an average life such that the realized life of the theoretical curve equals the realized life of the observed curve. Call this the average life.

Once the average life is found, calculate the difference between each percent surviving point on the observed survivor curve and the corresponding point on the Iowa curve. Square each difference and sum them. The sum of squares is used as a measure of goodness of fit for that particular Iowa type curve. This procedure is

repeated for the remaining 21 Iowa type curves. The “best fit” is declared to be the type of curve that minimizes the sum of differences squared.<sup>13</sup>

Mathematical fitting requires less judgment from the analyst and is thus less subjective. Blind reliance on mathematical fitting, however, may lead to poor estimates. Thus, analysts should employ both mathematical and visual curve fitting in reaching their final estimates. This way, analysts may utilize the objective nature of mathematical fitting while still employing professional judgment. As Wolf notes: “The results of mathematical curve fitting serve as a guide for the analyst and speed the visual fitting process. But the results of the mathematical fitting should be checked visually, and the final determination of the best fit be made by the analyst.”<sup>14</sup>

In the graph above, visual fitting was sufficient to determine that the 10.5-R1 Iowa curve was a better fit than the 10-L4 and the 10-S0 curves. Using the sum of least squares method, mathematical fitting confirms the same result. In the chart below, the percentages surviving from the OLT that formed the original stub curve are shown in the left column, while the corresponding percentages surviving for each age interval are shown for the three Iowa curves. The right portion of the chart shows the differences between the points on each Iowa curve and the stub curve. These differences are summed at the bottom. Curve 10.5-R1 is the best fit because the sum of the squared differences for this curve is less than the same sum for the other two curves. Curve 10-L4 is the worst fit, which was also confirmed visually.

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<sup>13</sup> Wolf *supra* n. 5, at 47.

<sup>14</sup> *Id.* at 48.

**Figure 9:  
 Mathematical Fitting**

Age Interval	Stub Curve	Iowa Curves			Squared Differences		
		10-L4	10-S0	10.5-R1	10-L4	10-S0	10.5-R1
0.0	100.0	100.0	100.0	100.0	0.0	0.0	0.0
0.5	96.4	100.0	99.7	98.7	12.7	10.3	5.3
1.5	93.2	100.0	97.7	96.0	46.1	19.8	7.6
2.5	90.2	100.0	94.4	92.9	96.2	18.0	7.2
3.5	87.2	100.0	90.2	89.5	162.9	9.3	5.2
4.5	84.0	99.5	85.3	85.7	239.9	1.6	2.9
5.5	80.5	97.9	79.7	81.6	301.1	0.7	1.2
6.5	76.7	94.2	73.6	77.0	308.5	9.5	0.1
7.5	72.3	87.6	67.1	71.8	235.2	26.5	0.2
8.5	67.3	75.2	60.4	66.1	62.7	48.2	1.6
9.5	61.6	56.0	53.5	59.7	31.4	66.6	3.6
10.5	54.9	36.8	46.5	52.9	325.4	69.6	3.9
11.5	47.0	23.1	39.6	45.7	572.6	54.4	1.8
12.5	38.9	14.2	32.9	38.2	609.6	36.2	0.4
<b>SUM</b>					<b>3004.2</b>	<b>371.0</b>	<b>41.0</b>