

**BEFORE THE WASHINGTON
UTILITIES & TRANSPORTATION COMMISSION**

WASHINGTON UTILITIES AND TRANSPORTATION COMMISSION,

Complainant,

v.

PUGET SOUND ENERGY, INC.

Respondent.

DOCKETS UE-240004 & UG-240005 (*Consolidated*)

**DAVID J. GARRETT
ON BEHALF OF THE
WASHINGTON STATE OFFICE OF THE ATTORNEY GENERAL
PUBLIC COUNSEL UNIT**

Appendix B

Iowa Curves

August 6, 2024

APPENDIX B:

IOWA CURVES

Early work in the analysis of the service life of industrial property was based on models that described the life characteristics of human populations.¹ This history explains why the word “mortality” is often used in the context of depreciation analysis. In fact, a group of property installed during the same accounting period is analogous to a group of humans born during the same calendar year. Each period the group will incur a certain fraction of deaths / retirements until there are no survivors. Describing this pattern of mortality is part of actuarial analysis and is regularly used by insurance companies to determine life insurance premiums. The pattern of mortality may be described by several mathematical functions, particularly the survivor curve and frequency curve. Each curve may be derived from the other so that if one curve is known, the other may be obtained. A survivor curve is a graph of the percent of units remaining in service expressed as a function of age.² A frequency curve is a graph of the frequency of retirements as a function of age. Several types of survivor and frequency curves are illustrated in the figures below.

1. Development

The survivor curves used by analysts today were developed over several decades from extensive analysis of utility and industrial property. In 1931, Edwin Kurtz and Robley Winfrey used extensive data from a range of 65 industrial property groups to create survivor curves representing the life characteristics of each group of property.³ They generalized the 65 curves into 13 survivor curve types and published their results in *Bulletin 103: Life Characteristics of Physical Property*. The 13 type curves were designed to be used as valuable aids in forecasting

¹ Wolf & W. Chester Fitch, *Depreciation Systems* 276 (Iowa State University Press 1994).

² *Id.* at 23.

³ *Id.* at 34.

probable future service lives of industrial property. Over the next few years, Winfrey continued gathering additional data, particularly from public utility property and expanded the examined property groups from 65 to 176.⁴ This research resulted in 5 additional survivor curve types for a total of 18 curves. In 1935, Winfrey published *Bulletin 125: Statistical Analysis of Industrial Property Retirements*. According to Winfrey, “[t]he 18 type curves are expected to represent quite well all survivor curves commonly encountered in utility and industrial practices.”⁵ These curves are known as the “Iowa curves” and are used extensively in depreciation analysis in order to obtain the average service lives of property groups. (Use of Iowa curves in actuarial analysis is further discussed in Appendix C.)

In 1942, Winfrey published *Bulletin 155: Depreciation of Group Properties*. In Bulletin 155, Winfrey made some slight revisions to a few of the 18 curve types, and published the equations, tables of the percent surviving, and probable life of each curve at five-percent intervals.⁶ Rather than using the original formulas, analysts typically rely on the published tables containing the percentages surviving. This reliance is necessary because, absent knowledge of the integration technique applied to each age interval, it is not possible to recreate the exact original published table values. In the 1970s, John Russo collected data from over 2,000 property accounts reflecting observations during the period 1965 – 1975 as part of his Ph.D. dissertation at Iowa State. Russo essentially repeated Winfrey’s data collection, testing, and analysis methods used to develop the original Iowa curves, except that Russo studied industrial property in service several decades after

⁴ *Id.*

⁵ Robley Winfrey, *Bulletin 125: Statistical Analyses of Industrial Property Retirements* 85, Vol. XXXIV, No. 23 (Iowa State College of Agriculture and Mechanic Arts 1935).

⁶ Robley Winfrey, *Bulletin 155: Depreciation of Group Properties* 121-28, Vol XLI, No. 1 (The Iowa State College Bulletin 1942); see also Wolf *supra* n.7, at 305–38 (publishing the percent surviving for each Iowa curve, including “O” type curve, at one percent intervals).

Winfrey published the original Iowa curves. Russo drew three major conclusions from his research:⁷

1. No evidence was found to conclude that the Iowa curve set, as it stands, is not a valid system of standard curves;
2. No evidence was found to conclude that new curve shapes could be produced at this time that would add to the validity of the Iowa curve set; and
3. No evidence was found to suggest that the number of curves within the Iowa curve set should be reduced.

Prior to Russo's study, some had criticized the Iowa curves as being potentially obsolete because their development was rooted in the study of industrial property in existence during the early 1900s. Russo's research, however, negated this criticism by confirming that the Iowa curves represent a sufficiently wide range of life patterns and that, though technology will change over time, the underlying patterns of retirements remain constant and can be adequately described by the Iowa curves.⁸

Over the years, several more curve types have been added to Winfrey's 18 Iowa curves. In 1967, Harold Cowles added four origin-modal curves. In addition, a square curve is sometimes used to depict retirements which are all planned to occur at a given age. Finally, analysts commonly rely on several "half curves" derived from the original Iowa curves. Thus, the term "Iowa curves" could be said to describe up to 31 standardized survivor curves.

2. Classification

The Iowa curves are classified by three variables: modal location, average life, and variation of life. First, the mode is the percent life that results in the highest point of the frequency

⁷ See Wolf *supra* n. 1, at 37.

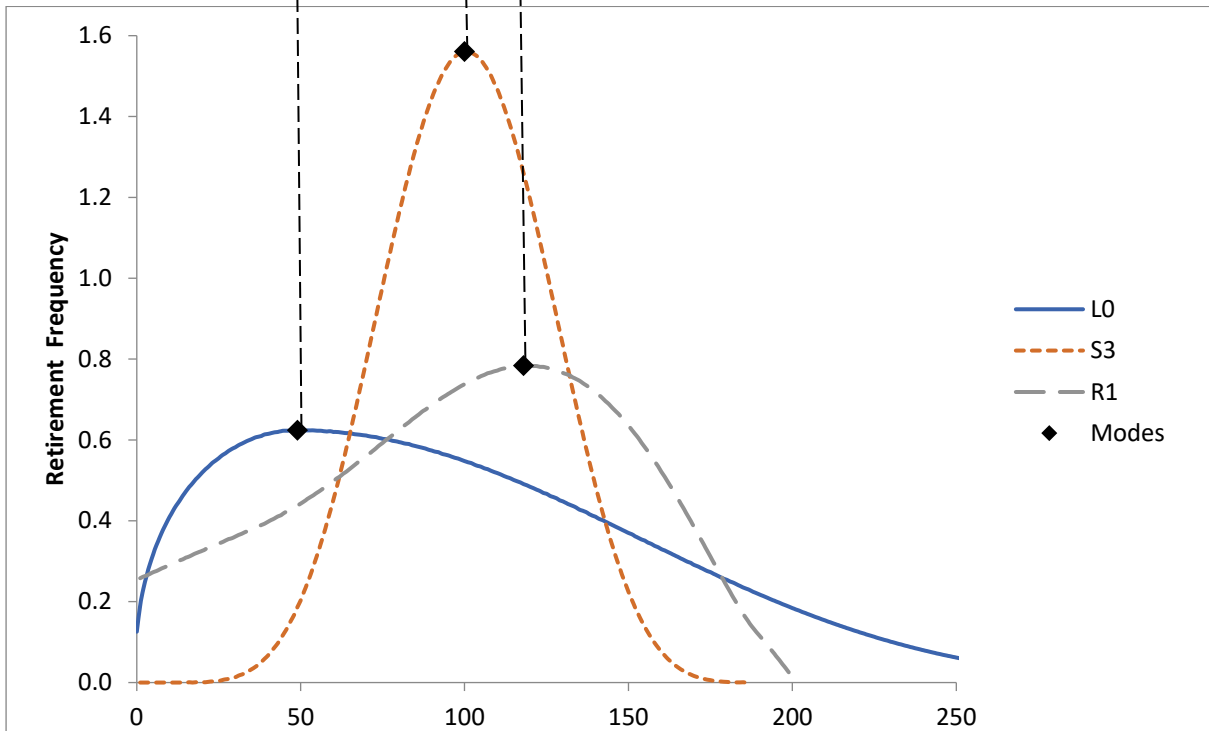
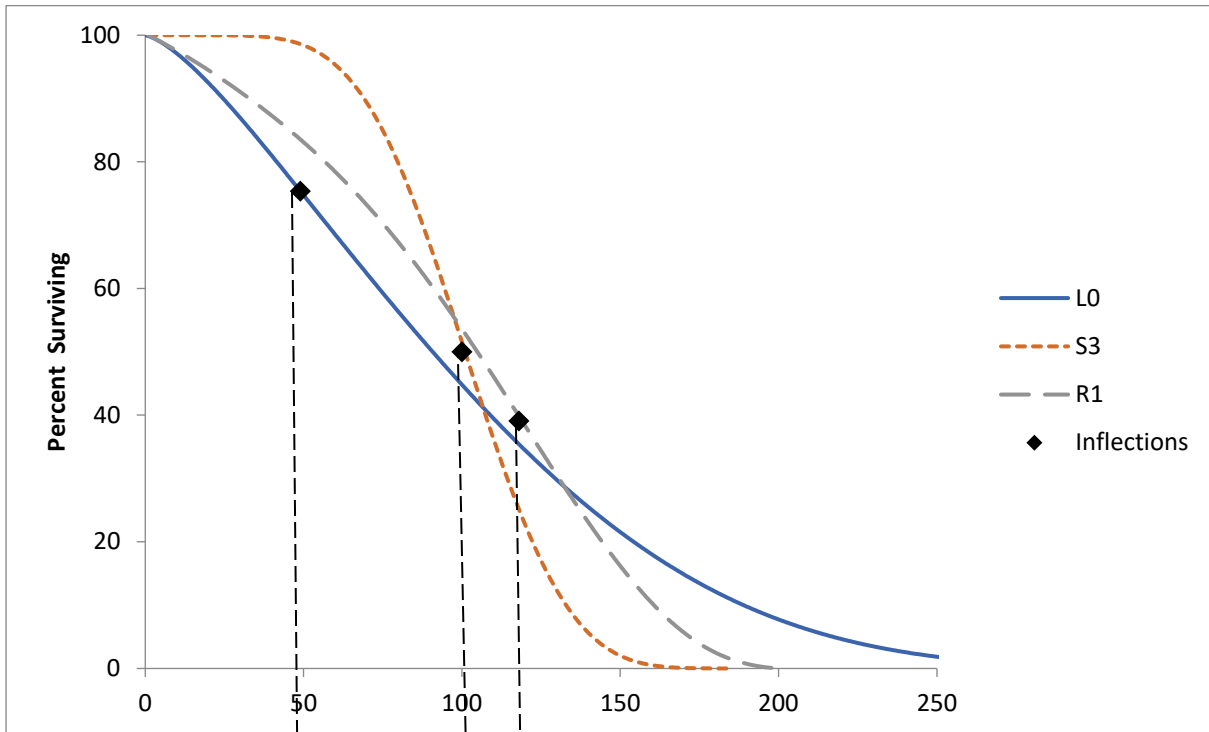
⁸ *Id.*

curve and the “inflection point” on the survivor curve. The modal age is the age at which the greatest rate of retirement occurs. As illustrated in the figure below, the modes appear at the steepest point of each survivor curve in the top graph, as well as the highest point of each corresponding frequency curve in the bottom graph.

The classification of the survivor curves was made according to whether the mode of the retirement frequency curves was to the left, to the right, or coincident with average service life. There are three modal “families” of curves: six left modal curves (L0, L1, L2, L3, L4, L5); five right modal curves (R1, R2, R3, R4, R5); and seven symmetrical curves (S0, S1, S2, S3, S4, S5, S6).⁹ In the figure below, one curve from each family is shown: L0, S3 and R1, with average life at 100 on the x-axis. It is clear from the graphs that the modes for the L0 and R1 curves appear to the left and right of average life respectively, while the S3 mode is coincident with average life.

⁹ In 1967, Harold A. Cowles added four origin-modal curves known as “O type” curves. There are also several “half” curves and a square curve, so the total amount of survivor curves commonly called “Iowa” curves is about 31.

**Figure 1:
Modal Age Illustration**



The second Iowa curve classification variable is average life. The Iowa curves were designed using a single parameter of age expressed as a percent of average life instead of actual age. This design was necessary for the curves to be of practical value. As Winfrey notes:

Since the location of a particular survivor on a graph is affected by both its span in years and the shape of the curve, it is difficult to classify a group of curves unless one of these variables can be controlled. This is easily done by expressing the age in percent of average life.”¹⁰

Because age is expressed in terms of percent of average life, any particular Iowa curve type can be modified to forecast property groups with various average lives.

The third variable, variation of life, is represented by the numbers next to each letter. A lower number (e.g., L1) indicates a relatively low mode, large variation, and large maximum life; a higher number (e.g., L5) indicates a relatively high mode, small variation, and small maximum life. All three classification variables – modal location, average life, and variation of life – are used to describe each Iowa curve. For example, a 13-L1 Iowa curve describes a group of property with a 13-year average life, with the greatest number of retirements occurring before (or to the left of) the average life, and a relatively low mode. The graphs below show these 18 survivor curves, organized by modal family.

¹⁰ Winfrey *supra* n. 6, at 60.

**Figure 2:
Type L Survivor and Frequency Curves**

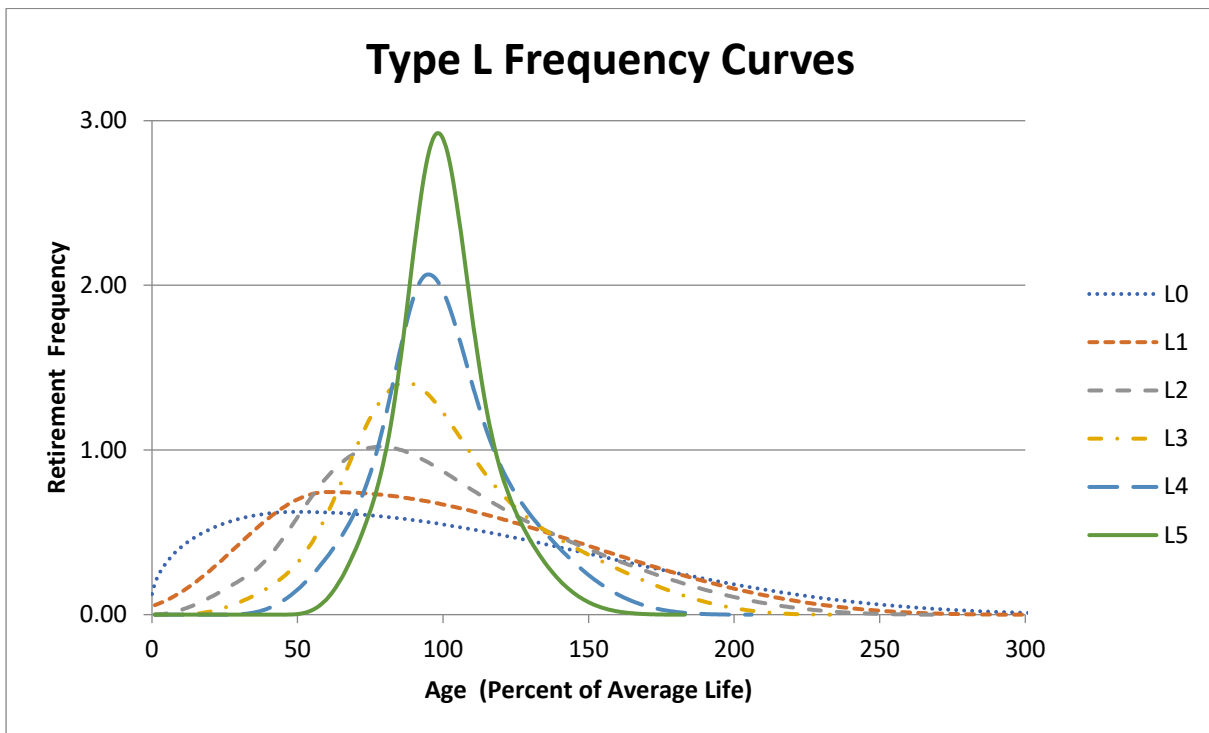
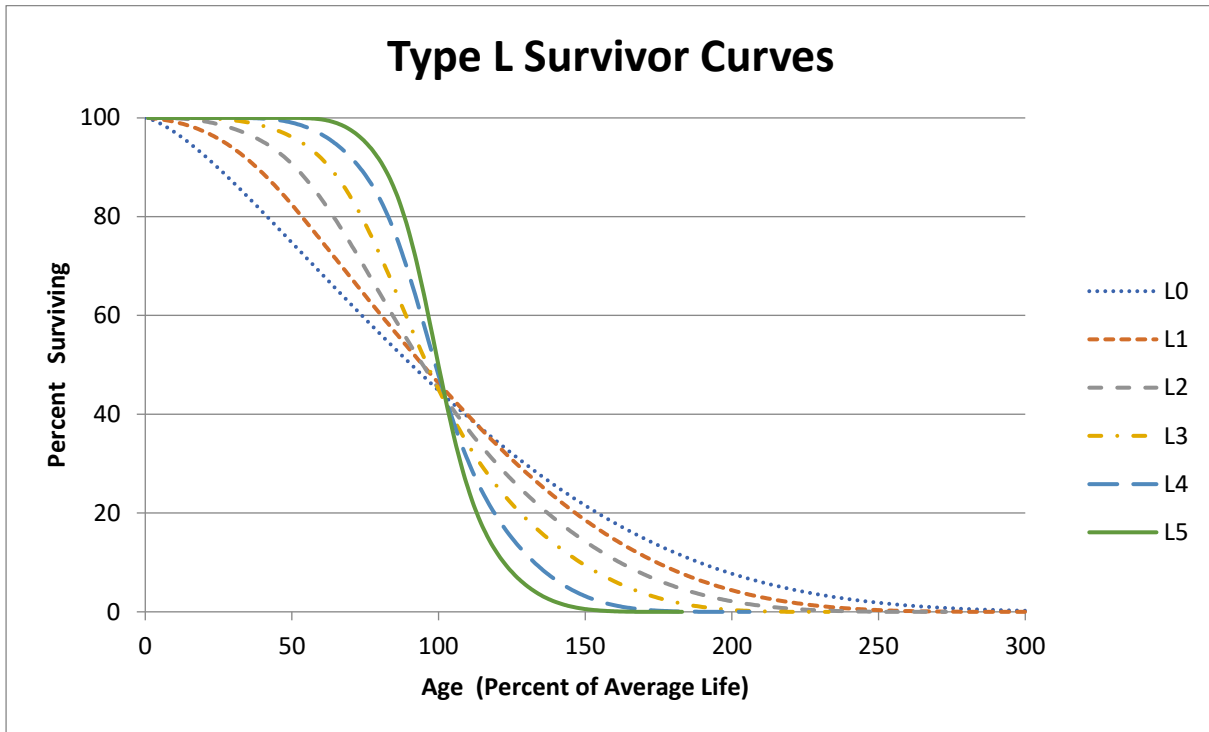


Figure 3:
Type S Survivor and Frequency Curves

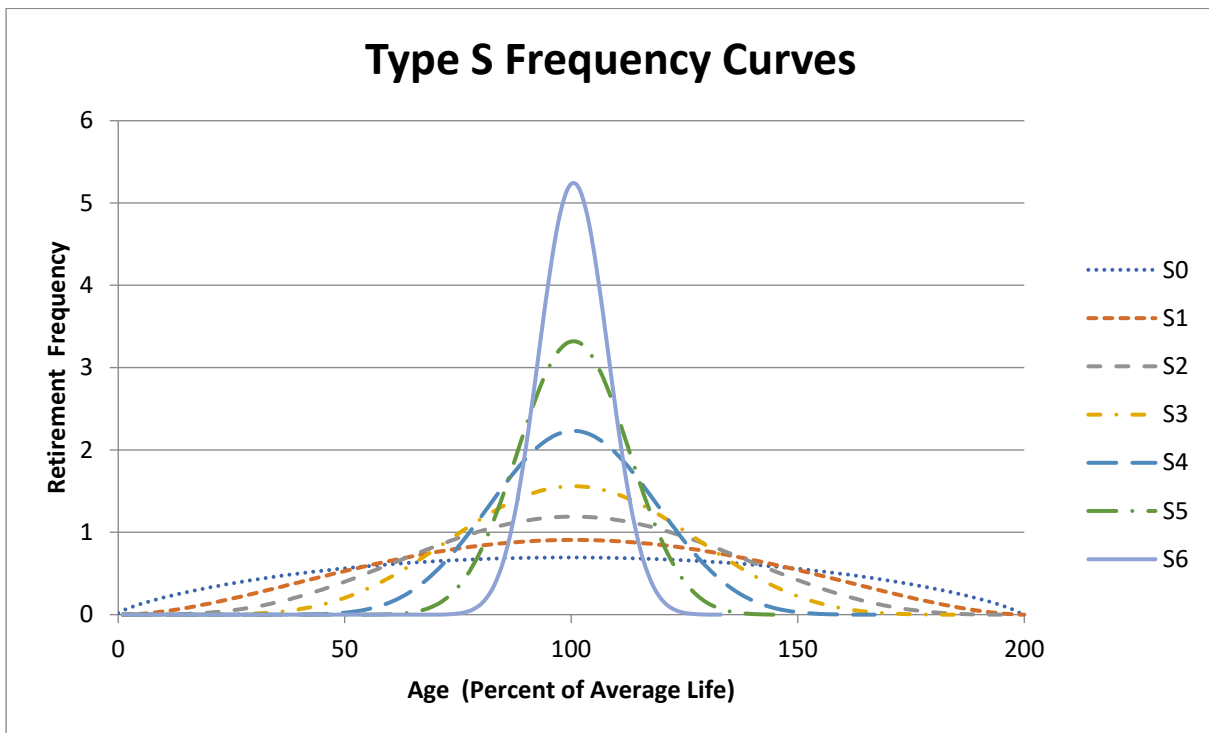
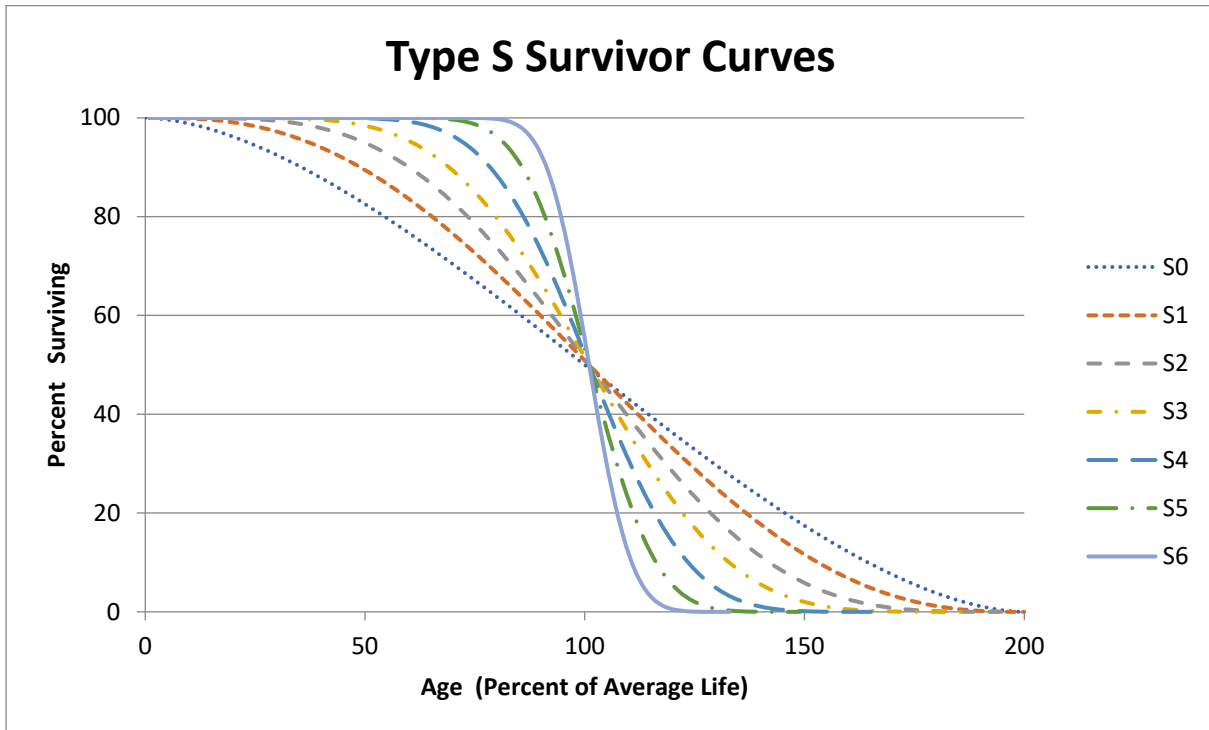
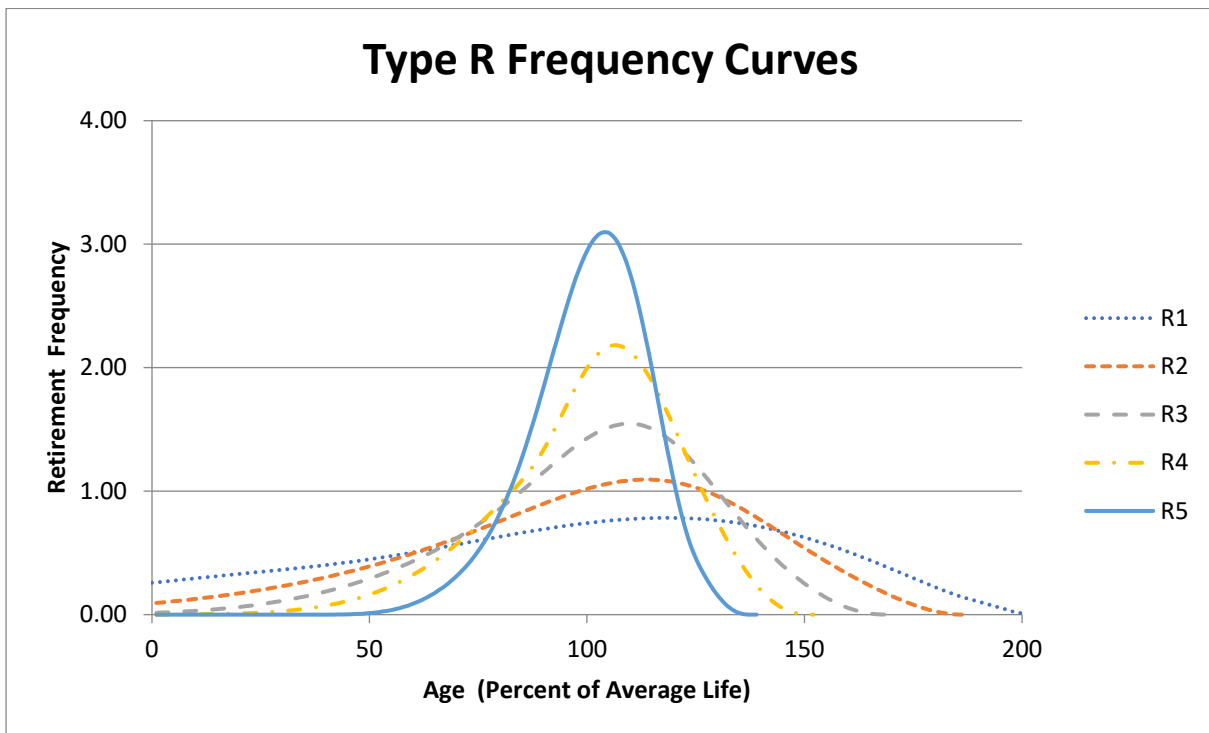
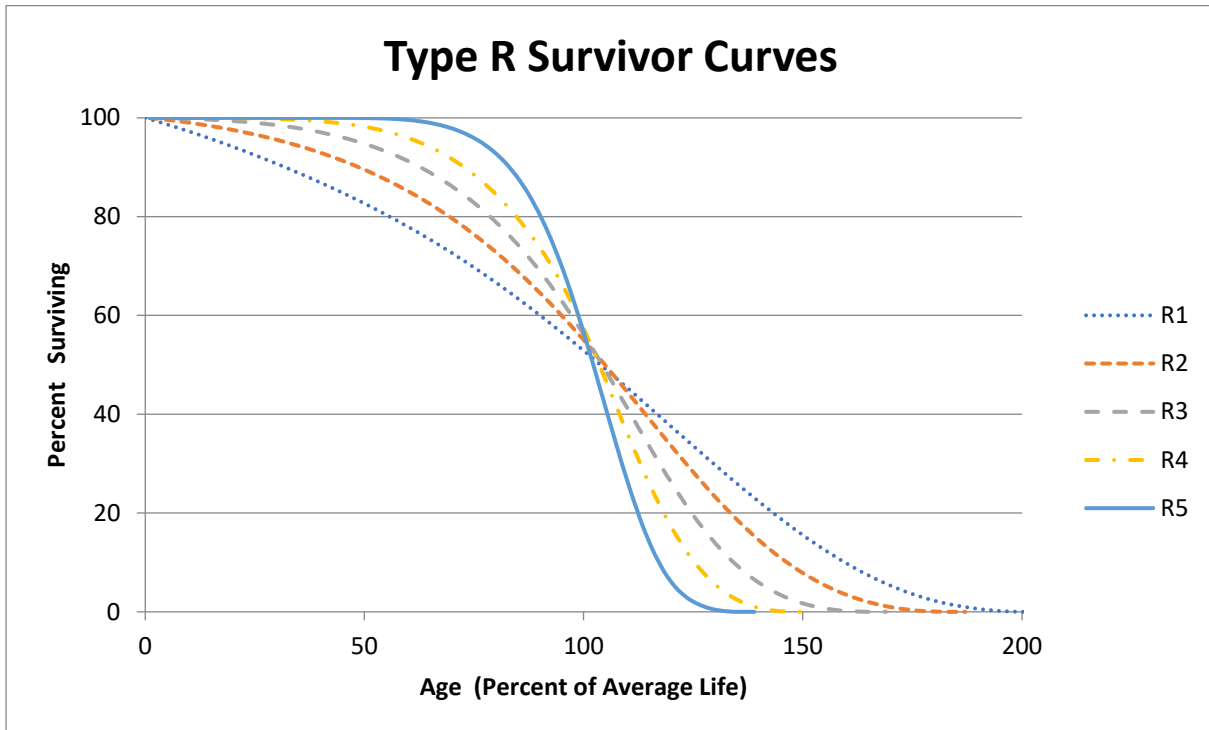


Figure 4:
Type R Survivor and Frequency Curves



As shown in the graphs above, the modes for the L family frequency curves occur to the left of average life (100% on the x-axis), while the S family modes occur at the average, and the R family modes occur after the average.

3. Types of Lives

Several other important statistical analyses and types of lives may be derived from an Iowa curve. These include: 1) average life; 2) realized life; 3) remaining life; and 4) probable life. The figure below illustrates these concepts. It shows the frequency curve, survivor curve, and probable life curve. Age M_x on the x-axis represents the modal age, while age AL_x represents the average age. Thus, this figure illustrates an “L type” Iowa curve since the mode occurs before the average.¹¹

First, average life is the area under the survivor curve from age zero to maximum life. Because the survivor curve is measured in percent, the area under the curve must be divided by 100% to convert it from percent-years to years. The formula for average life is as follows:¹²

**Equation 1:
Average Life**

$$\text{Average Life} = \frac{\text{Area Under Survivor Curve from Age 0 to Max Life}}{100\%}$$

Thus, average life may not be determined without a complete survivor curve. Many property groups being analyzed will not have experienced full retirement. This dynamic results in a “stub” survivor curve. Iowa curves are used to extend stub curves to maximum life in order to make the average life calculation (see Appendix C).

¹¹ From age zero to age M_x on the survivor curve, it could be said that the percent surviving from this property group is decreasing at an increasing rate. Conversely, from point M_x to maximum on the survivor curve, the percent surviving is decreasing at a decreasing rate.

¹² National Association of Regulatory Utility Commissioners, Public Utility Depreciation Practices 71 (NARUC 1996).

Realized life is similar to average life, except that realized life is the average years of service experienced to date from the vintage's original installations.¹³ As shown in the figure below, realized life is the area under the survivor curve from zero to age RL_x . Likewise, unrealized life is the area under the survivor curve from age RL_x to maximum life. Thus, it could be said that average life equals realized life plus unrealized life.

Average remaining life represents the future years of service expected from the surviving property.¹⁴ Remaining life is sometimes referred to as "average remaining life" and "life expectancy." To calculate average remaining life at age x , the area under the estimated future portion of the survivor curve is divided by the percent surviving at age x (denoted S_x). Thus, the average remaining life formula is:

**Equation 2:
Average Remaining Life**

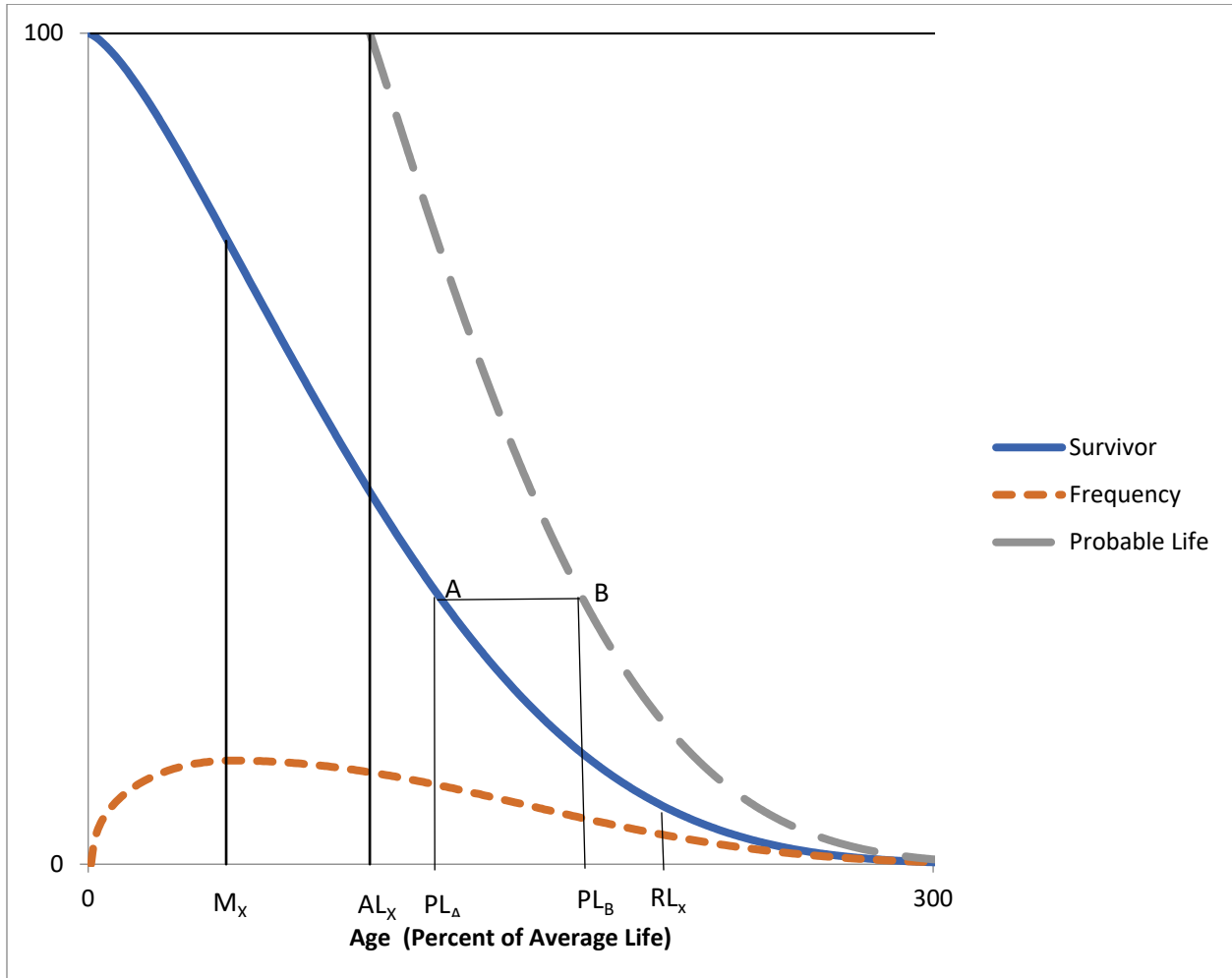
$$\text{Average Remaining Life} = \frac{\text{Area Under Survivor Curve from Age } x \text{ to Max Life}}{S_x}$$

It is necessary to determine average remaining life to calculate the annual accrual under the remaining life technique.

¹³ *Id.* at 73.

¹⁴ *Id.* at 74.

**Figure 5:
 Iowa Curve Derivations**



Finally, the probable life may also be determined from the Iowa curve. The probable life of a property group is the total life expectancy of the property surviving at any age and is equal to the remaining life plus the current age.¹⁵ The probable life is also illustrated in this figure. The probable life at age PL_A is the age at point PL_B . Thus, to read the probable life at age PL_A , see the corresponding point on the survivor curve above at point “A,” then horizontally to point “B” on

¹⁵ Wolf *supra* n. 1, at 28.

the probable life curve, and back down to the age corresponding to point “B.” It is no coincidence that the vertical line from AL_x connects at the top of the probable life curve. This connection occurs because at age zero, probable life equals average life.