Exhibit No. ___ (JAR-12)
Docket No. UE-050684
Witness: James A. Rothschild

BEFORE THE WASHINGTON UTILITIES AND TRANSPORTATION COMMISSION

WASHINGTON UTILITIES AND TRANSPORTATION COMMISSION, Complainant, v. PACIFICORP, d/b/a Pacific Power & Light Company, Respondent.	DOCKET NO. UE-050684

EXHIBIT TO
TESTIMONY OF

JAMES A. ROTHSCHILD

For
STAFF OF
WASHINGTON UTILITIES AND
TRANSPORTATION COMMISSION

Geometric Average vs. Arithmetic Average

November 3, 2005

Q. FOR THE PURPOSE OF DETERMINING ACTUAL RISK PREMIUMS, HOW SHOULD HISTORIC ACTUAL RETURNS BE COMPUTED?

A. Historic returns should be computed by determining the compound annual returns actually achieved. Compound annual returns are often referred to as geometric average returns. Geometric average returns are computed by determining the return rate that would have to be earned on average between the beginning year and the ending year of an investment. For example, if an investor invested $1,000 ten years ago and has $2,000 today, the compound annual return would be 7.177% because $1,000 invested for ten years at a compound annual return of 7.177% would be exactly $2,000 in ten years.

Q. IS THERE AN ALTERNATIVE APPROACH THAT IS USED?

A. There is no alternative approach that is accurate. However, a common mistake that is made is to simply compute the average of the annual returns each year and incorrectly conclude that this arithmetic average of actual returns is somehow the return rate investors achieved. For example, if the same $1,000 investment hypothesized above earned 100% in the first year and therefore was worth $2,000 after one year, subsequently lost 50% in the second year and was again worth $1,000 and continued to alternate in such a wild fashion year after year such that after the 9th year the investment was worth $2,000 and stayed at the same $2,000 in the 10th year, the arithmetic averaging method would average the +100%,-50% etc. to arrive at the incorrect conclusion that this investor who started with $1,000

in the first year and had $2,000 ten years later would have earned 30% per year (average of +100% -50%+100%-50%+100%-50% +100% -50% +100% +0%) even though the real return received on the original investment was the equivalent of a compound annual 7.177%.

Q. IS THE PROBLEM WITH USING THE ARITHMETIC AVERAGING METHOD WIDELY RECOGNIZED?

A. Yes. As will be explained in detail, textbooks, the U.S. Securities and Exchange Commission ("SEC"), and Value Line have all recognized that the only proper way to measure long-term historic actual earned returns is to use the geometric mean, not the arithmetic mean. The arithmetic mean is specifically identified by several sources as a method that will specifically result in an answer that is upwardly biased.

Q. IS THERE A MATHEMATICAL RELATIONSHIP BETWEEN THE GEOMETRIC AVERAGE AND THE ARITHMETIC AVERAGE?

A. Yes. Page 24 of the third edition of *Stocks for the Long Run* by Professor Jeremy J. Siegel © 2002 contains the following:

> The geometric return is approximately equal to the arithmetic return minus one-half of the variance σ^2 of yearly returns $r_G=r_A-1/2\ \sigma^2$.
>
> Investors can be expected to realize geometric returns only over long periods of time. The average geometric return is always less than the average arithmetic return except when all yearly returns are exactly equal. This difference is related to the volatility of yearly returns.

As correctly explained above, the only reason the arithmetic average is higher than the geometric average is because of the volatility of yearly returns. Therefore, from the perspective of the cost of equity to allow a regulated utility, the correct return is the geometric return. The geometric return, if allowed, will be the return the utility company is given a reasonable opportunity to earn. If there is a difference between the geometric return and the arithmetic return, for a regulated utility this difference will occur simply because a utility company's stock price will fluctuate up and down even though the allowed return on equity remains fixed at least until the next rate case.

Q. HAVE YOU SEEN COMPANY WITNESSES WHO USE THE ARITHMETIC AVERAGE CLAIM THAT THE GEOMETRIC AVERAGE IS THE CORRECT AVERAGE TO USE WHEN MEASURING HISTORIC RETURNS, BUT THE ARITHMETIC AVERAGE IS SOMEHOW CORRECT FOR FORECASTING FUTURE RETURNS?

A. Yes, I have seen this argument. But, given that the difference between the geometric return and the arithmetic return is due to volatility and not the true return actually being achieved, such an argument that claims a different measurement technique applies to historic data than to forecast data is incorrect. Consider the following example. Assume that the U.S. Government issued a 30-year bond 15 years ago that pays an annual interest rate of 5.0% on the face amount of the bond. Further assume that although interest rates fluctuated over the last 15 years, the current interest rate demanded by investors happens to be

5% today. Under these assumptions, over the last 15 years, the price of the bond has gone up in some years and gone down in other years. But, if the current interest rate demanded by investors on this bond is still the same 5% as was demanded by investors at the time of the original issuance, the bond will be selling for the same price as it did when originally issued 15 years ago. Because of this fluctuation, if the total return (price appreciation or price depreciation plus the 5% interest income) is measured using the arithmetic average, then the measured return will include the 5% real return actually obtained by investors plus an additional illusory return cause by volatility rather than an actual return received by the investor. From the perspective of the investor who is forecasting the return on this 5% government bond with 15 years remaining, we know with certainty that the accurate forecasted future return will be 5% per year. We also can be confident that interest rates will fluctuate over the next 15 years. Therefore, this fluctuation will cause the arithmetic return measurement to be higher than the 5% annual return even though the 5% return is the only possible return an investor who holds this bond to maturity could get.

Q. IS IT THE 5% RETURN ON THE TREASURY BOND OR IS IT THE ARITHMETIC AVERAGE RETURN THAT IS ANALAGOUS TO THE ALLOWED RETURN ON EQUITY TO A REGULATED UTILITY COMPANY?

A. The 5% coupon return is the return that is analogous to the allowed return. Therefore, even if we were to attempt to satisfy the investor who was incorrectly

led to believe that he or she would achieve the arithmetic average and not the geometric average, the return based upon the geometric average should form the return allowed. Then, an investor who wishes to be fooled into achieving a higher return than is achieved by the geometric average will continue to be under the misconception that he or he is earning more than the geometric average. This can happen because the stock price fluctuation will still produce annual returns that, under the arithmetic average method, will appear to be higher than the allowed geometric return.

Consider the problem that would develop if allowed returns were errantly set based upon the arithmetic average rather than the geometric average. If a utility company is allowed to earn a return on rate base equal to the arithmetic average, then the normal stock price fluctuations would cause the new arithmetic average measured result to continue to exceed the old allowed arithmetic average. A repetition of the error caused by using the arithmetic average, if repeated in the next rate case, would cause yet a further ratcheting up of the allowed return in each future rate case where this mistake to use the arithmetic average is repeated.

Q. DOES THE FINANCIAL COMMUNITY COMPUTE HISTORIC ACTUAL ACHIEVED RETURNS BASED UPON ARITHMETIC MEANS OR GEOMETRIC MEANS?

A. As shown earlier in this testimony, the financial community (as represented by articles from *The Wall Street Journal* and from *Business Week*) refers to

geometric averages when evaluating historic returns. Additionally, an article on page 92 of the August 16, 1999 issue of *Fortune* magazine refers to the return that is equal to the geometric mean from Ibbotson Associates as "...the oft-quoted calculation..." of historic actual returns on common stocks. The article does not even mention the number that is equal to the historic arithmetic return.

Q. DO FINANCIAL TEXTBOOKS SUPPORT THE USE OF THE GEOMETRIC AVERAGE FOR COMPUTING HISTORIC ACTUAL RETURNS?

A. Yes. For example, the textbook *Valuation. Measuring and Managing the Value of Companies*, by Copeland, Koller, and Murrin of McKinsey & Co. , John Wiley & Sons, 1994, in a description of how to use the Ibbotson Associates data states the following on pages 261-262:

> We use a geometric average of rates of return because arithmetic averages are biased by the measurement period. An arithmetic average estimates the rates of return by taking a simple average of the single period rates of return. Suppose you buy a share of a nondividend-paying stock for $50. After one year the stock is worth $100. After two years the stock falls to $50 once again. The first period return is 100 percent; the second period return is -50 percent. The arithmetic average return is 25 percent [(100 percent - 50 percent)/2]. The geometric average is zero. (The geometric average is the compound rate of return that equates the beginning and ending value.) **We believe that the geometric average represents a better estimate of investors' expected returns over long periods of time.** [Emphasis added]

Similarly, in another textbook discussion that specifically addresses the use of the Ibbotson data, *Financial Market Rates & Flows*, by James C. Van Horne, Prentice Hall, 1990, states the following on page 80:

> The geometric mean is a geometric average of annual returns, whereas the arithmetic mean is an arithmetic average. For cumulative

wealth changes over long sweeps of time, the geometric mean is the appropriate measure.

The textbook *Investments* by Nancy L. Jacob and R. Richardson Pettit, Irwin, 1988, puts it well when it says:

The existence of uncertainty as reflected in a distribution of possible values makes the **expected value**, or arithmetic average rate of return, a misleading and biased representation of the wealth increments which will be generated from multiperiod investment opportunities.

The average *annual* rate of wealth accumulation over the investment period, termed the **average annual geometric rate of return**, correctly measures the average annual accumulation to wealth when multiple periods are involved.

[Emphasis is contained in the original]

Q. HAS VALUE LINE SAID ANYTHING REGARDING THE USE OF AN ARITHMETIC AVERAGE OR A GEOMETRIC AVERAGE?

A. Yes. On May 9, 1997, Value Line issued a report entitled "The Differences in Averaging". This report was contained on pages 6844-6845 of the "Value Line Selection & Opinion" portion of its weekly mailings to subscribers. This report says that:

(t)he arithmetic average has an upward bias, though it is the simplest to calculate. The geometric average does not have any bias, and thus is the best to use when compounding (over a number of years) is involved.

The Value Line report then goes on to provide examples that show why the arithmetic average overstates the achieved returns while the geometric average produces the correct result.

Ibbotson Associates has also said that it is the geometric average that is "... the correct average to compare with a bond yield..."[1]

On October 8, 2003, The Wall Street Journal published an article entitled "Financial Advisers and Fuzzy Math". This article starts out by saying:

> Next time your financial adviser makes a prediction for an average rate of return during an investment pitch, you might want to doublecheck the math.
>
> Some financial advisers rely too heavily on a formula known as an arithmetic average, which can be misleading when investing for the long term. Financial advisers who use this formula may be overstating your potential profit and leading you to take risks you might otherwise avoid.

Q. HAVE YOU COMPARED GRAPHICALLY THE CAPITAL APPRECIATION GROWTH RATE USING THE ARITHMETIC AVERAGE METHOD WITH THE CAPITAL APPRECIATION GROWTH RATE THAT IS OBTAINED USING THE SEC METHOD?

A. Yes. In the following graph I show the actual movement of the S&P Utility index from 1928 through 2003. I also show how the index would have behaved on a year-by-year basis using the average growth obtained from the SEC method and using the arithmetic average historic growth rate methodology. The graph illustrates that the arithmetic average calculation of historic actual returns deviates at an ever-increasing rate over time from the actual S&P Utility Index, overstating the total return from 1928-2003 by about 500%. By contrast, the historic actual returns computed using the SEC method is a dramatically more reasonable track

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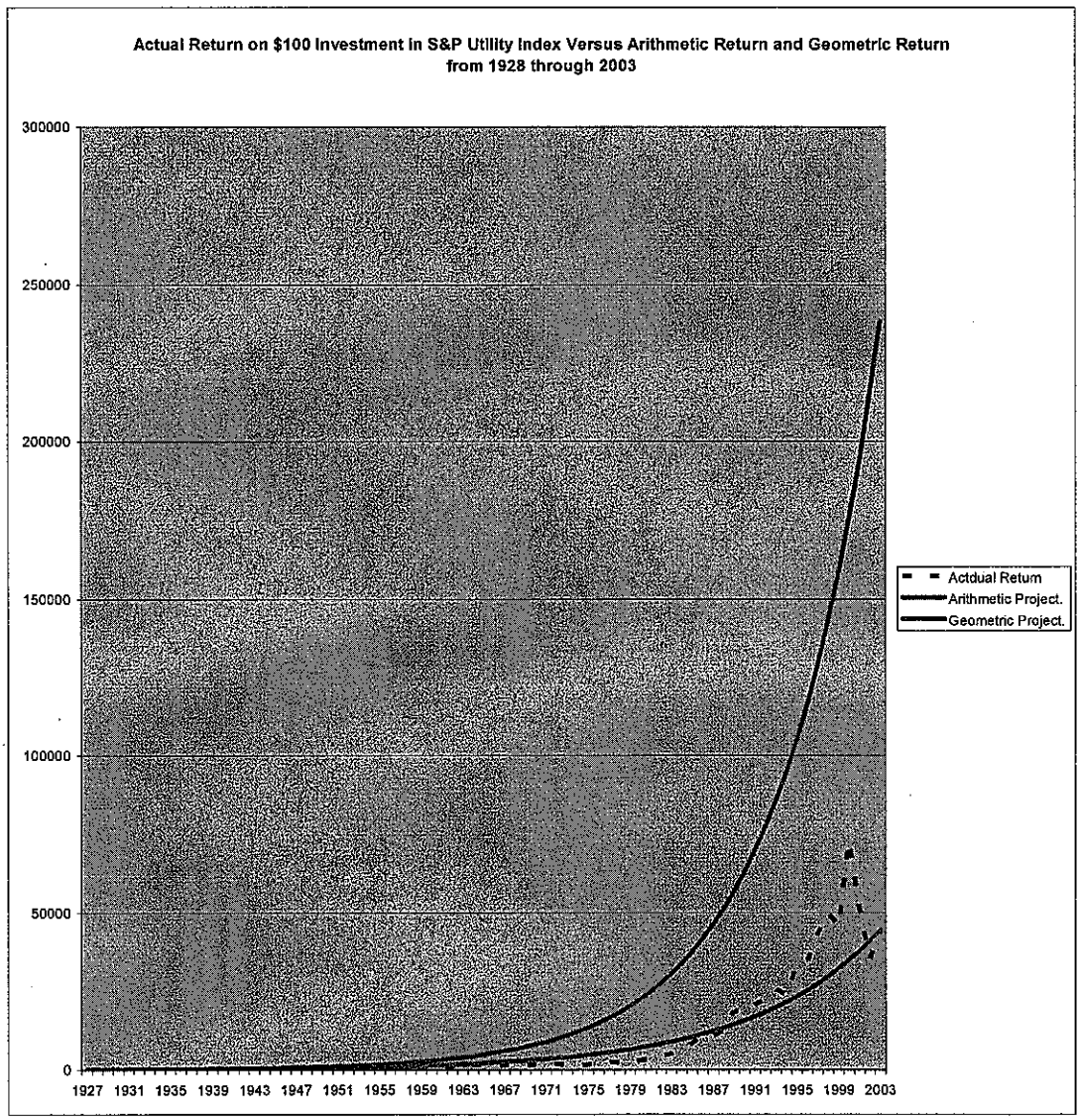
[1] Page 75 of Stocks, Bonds, Bills, and Inflation 1986 Yearbook.

of the growth of the S&P utility over time and thus is the proper measure of historic actual return rates realized by investors.



In the above chart, the top line shows that if $100 had been invested in public utility common stocks from the beginning of 1928 through 2003 and had earned the arithmetic return, the $100 would have grown to about $238,000. The dotted line in the graph shows what actually would have happened to a real $100 investment if it had been invested in public utility common stocks. As shown on the graph, the $100 investment would have actually grown to about $44,000. While the increase from

$100 to $44,000 is a very sizeable return, it is far less than the $238,000 return that would have been achieved if the arithmetic return methodology had been achieved. The smooth line that ends at the same place as the dotted actual return line is the ongoing value of $100 invested in 1928 that grew at the geometric return rate. Note that the $100 invested at the geometric return rate is, by 2003, exactly equal to the actual return. Therefore, the geometric return accurately measures the actual return that was achieved from 1928 through 2003, but the arithmetic average return exaggerates the actual return by over five times.

Q. HOW MUCH HIGHER IS THE RISK PREMIUM DIFFERENCE BASED UPON AN ARITHMETIC AVERAGE THAN IT IS BASED UPON A GEOMETRIC AVERAGE?

A. From 1928 to 2003, the arithmetic average method produced an indicated risk premium that was 2.13% higher for public utility stocks versus public utility bonds than the risk premium indicated by using the SEC, or geometric average method. The arithmetic median method is essentially identical to the arithmetic mean method and therefore produces an error that is similar to the error produced by the arithmetic average method.