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On the CAPM Approach to the Estimation of A Public Utility's Cost of Equity Capital

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I. Introduction

IN RECENT YEARS the Capital Asset Pricing Model (CAPM) has been used in several public utility rate cases to measure the cost of equity capital. In actual application, the cost of equity capital is frequently estimated as the annualized 90 day Treasury Bill rate plus a risk premium. The risk premium is obtained as the product of the average annual excess rate of return on a value weighted index of NYSE stocks (where the average is taken over a long period of time) and an estimate of the utility's NYSE beta.

Underlying this procedure is the assumption that risk premiums are *strictly* proportional to NYSE betas. However, this assumption is inconsistent with the academic empirical literature on CAPM. This literature supports a (non-proportional) linear relationship between risk premiums and NYSE betas with a positive intercept. Other empirical studies suggest that, in addition to betas, risk premiums are influenced by dividend yields and systematic skewness. Evidence presented in this literature is consistent with the predictions of CAPM models that account for margin restrictions on the borrowing of investors, divergent borrowing and lending rates, the existence of risky assets (such as bonds, residential real estate, unincorporated businesses, and human capital) that are not included in the value weighted NYSE stock index, taxes and skewness preference.

The version of the CAPM that should be employed in estimating a public utility's cost of equity capital cannot be conclusively demonstrated by theoretical arguments. A positive theory of the valuation of risking assets should not be judged upon the realism of its assumptions but rather on the accuracy of its predictions. The relationship between risk premiums and betas that is used to estimate the cost of equity capital should therefore be estimated econometrically rather than specified *a priori*.

Section 2 compares the predictions of alternative versions of the CAPM. The assertion that risk premiums are proportional to NYSE betas is shown to result in a downward (upward) biased prediction of the cost of equity capital for a public utility having a NYSE beta that is less (greater) than unity, a dividend yield higher (lower) than the yield on the value weighted NYSE stock index, and/or a systematic skewness that exceeds (is less than) its beta.

Section 3 discusses problems that arise in implementing CAPM approaches and presents possible solutions. Section 4 describes econometric procedures for

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estimating the relationship between risk premiums and NYSE betas. Section 5 presents estimates of CAPM parameters, and, Section 6, using two utilities as examples, illustrates how these estimates can be used to measure the cost of equity capital.

II. Alternative versions of the CAPM: Theory and Evidence

The versions of the CAPM discussed below all assume that investors are risk averse and have homogeneous beliefs. They also assume that a riskless asset exists, that all assets are marketable, and that there are no transactions costs or indivisibilities. The mean-variance versions assume that expected utility is completely defined over the first two moments of the rate of return on investors portfolios. The three moment CAPM assumes that investors have utility functions displaying non-increasing absolute risk aversion and that expected utility is defined over the first three moments of the rate of return on investors portfolios. The before-tax versions ignore taxes while the after-tax versions account for the differential taxation of dividends and capital gains. The constrained borrowing versions allow unlimited short selling of risky securities while the unconstrained borrowing versions allow unlimited short selling of the riskless security (i.e., unlimited borrowing).

The Traditional Version of the CAPM

The traditional version of the CAPM developed by Sharpe [1964] and Lintner [1965] predicts the following relationship between risk premiums and betas,

$$E(\tilde{r}_i) = E(\tilde{r}_m)\beta_i, \tag{1}$$

where:

$E(\tilde{r}_i)$ = the risk premium, or expected excess rate of return above the riskless rate of interest, on the i-th security,
$E(\tilde{r}_m)$ = the risk premium on the market portfolio of all assets, and
$\beta_i = \text{Cov}(\tilde{r}_i, \tilde{r}_m)/\text{Var}(\tilde{r}_m)$, the beta of the i-th security measured against the true market portfolio of all assets.

Before-Tax Constrained Borrowing Versions of the CAPM

Constrained borrowing versions of the CAPM have been developed by Lintner [1969], Vasicek [1971], Black [1972], Brennan [1972], and Fama [1976]. They predict the following relationship between risk premiums and betas,

$$E(\tilde{r}_i) = E(\tilde{r}_m)\beta_i + E(\tilde{r}_z)(1 - \beta_i), \tag{2}$$

$$\text{or} \quad E(\tilde{r}_i) = E(\tilde{r}_z) + \beta_i(E(\tilde{r}_m) - E(\tilde{r}_z)) \tag{2A}$$

where:

$E(\tilde{r}_z)$ = the risk premium on the minimum variance zero beta portfolio.

With diverse investor preferences and no borrowing (Vasicek [1972] and Black

[1972]), divergent borrowing and lending rates (Brennan [1972]), or margin restrictions (Fama [1976]), the risk premium on the zero beta portfolio is positive (i.e., $E(\tilde{r}_z) > 0$). The first term on the RHS of relation (2) is the risk premium on security i that is predicted by the traditional CAPM. The second term is the bias inherent in that prediction when investor borrowing is constrained. Because $E(\tilde{r}_z) > 0$, the traditional CAPM's prediction of the risk premium would be biased downward (upward) for a public utility having a beta less (greater) than unity.

After-Tax Versions of the CAPM

After-tax versions of the CAPM have been developed by Brennan [1973] under the assumption of unlimited borrowing and lending and by Litzenberger and Ramaswamy [1979] under constrained borrowing. They predict the following relationship between risk premiums, betas and dividend yields,

$$E(\tilde{r}_i) = E(\tilde{r}_m)\beta_i + E(\tilde{r}'_z)(1 - \beta_i) + E(\tilde{r}_h)(d_i - \beta_i d_m), \tag{3}$$

where:

$E(\tilde{r}'_z)$ = the risk premium on a portfolio having a zero beta and zero dividend yield,
$E(\tilde{r}_h)$ = the expected rate of return on a hedge portfolio having a zero beta and a dividend yield of unity,
d_i = the dividend yield on stock i, and
d_m = the dividend yield on the market portfolio.

The first term on the RHS of relation (3) is once again the prediction of the traditional CAPM. The sum of the second and third terms indicates the bias inherent in this prediction. With constrained borrowing, the sign of $E(\tilde{r}'_z)$ cannot be determined theoretically; however, econometric estimates indicate that $E(\tilde{r}'_z) > 0$. This result implies that the second term on the RHS of relation (3) is positive (negative) for public utilities having betas less (greater) than unity. With the taxation of corporate dividends and the preferential taxation of capital gains, $E(\tilde{r}_h) > 0$. Therefore, the third term on the RHS of relation (3) would be positive (negative) for a public utility having a beta less (greater) than unity and a dividend yield that is higher (lower) than the dividend yield on the market portfolio. Thus, the sum of the second and third terms is positive (negative) for public utilities having betas less (greater) than unity and higher (lower) than average dividend yields, indicating that the prediction of the traditional version of the CAPM would be downward (upward) biased.

The Three Moment Version of the CAPM

The three moment CAPM, developed by Rubinstein [1973] and Kraus and Litzenberger [1976], predicts the following relationship between risk premiums, betas, and gammas (systematic skewness),

$$E(\tilde{r}_i) = E(\tilde{r}_m)\beta_i + E(\tilde{r}_w)(\gamma_i - \beta_i), \tag{4}$$

where:

$$\gamma_i = \frac{E[(\tilde{r}_i - E(r_i))(\tilde{r}_m - E(r_m))^2]}{E[(\tilde{r}_m - E(\tilde{r}_m))^3]}, \text{ the systematic skewness of security } i$$

$E(\tilde{r}_w)$ = the expected risk premium on a security having a zero beta and a gamma of unity.

With non-increasing absolute risk aversion, $E(\tilde{r}_w) > 0$. The second term on the RHS of relation (4) is the bias inherent in the traditional version of the CAPM. For a public utility whose future profitability is constrained by the regulatory process, gamma may be less than beta and, the risk premium predicted by the traditional version of the CAPM may be downward biased.

Missing Asset Version of the CAPM

Many classes of assets such as human capital, residential real estate, unincorporated business, and bonds are not included in the value weighted index of NYSE stocks. This "missing assets" problem has been analyzed by Mayers [1972], Sharpe [1977] and Roll [1977]. If the traditional version of the CAPM were valid (i.e., if risk premiums were proportional to true betas) it can be shown that,[1]

$$E(\tilde{r}_i) = E(\tilde{r}_s)\beta_{i,s} + E(\tilde{r}_{zs})(1 - \beta_{i,s}) + u_i \tag{5}$$

where:

$$u_i = E(\tilde{r}_m)\beta_{e_i,zs} - E(\tilde{r}_{zs})\{\beta_{i,zs} - (1 - \beta_{i,s})\}$$

and:

$\beta_{i,s}$ = the beta of security i w.r.t. the NYSE index,
$E(\tilde{r}_{zs})$ = the risk premium on the minimum variance zero NYSE beta portfolio,

[1] To obtain relation (5) note that without loss of generality the return on any security i may be expressed as,

$$\tilde{r}_i - E(\tilde{r}_i) = \beta_{i,s}[\tilde{r}_s - E(r_s)] + \beta_{i,zs}[\tilde{r}_{zs} - E(\tilde{r}_{zs})] + \tilde{e}_i$$

where:

$$E(e_i) = \text{Cov}(e_i, r_s) = \text{Cov}(e_i, r_{zs}) = 0$$

Multiplying both sides by $\tilde{r}_m$, taking expectations and dividing by the variance of $\tilde{r}_m$ yields.

$$\beta_i = \beta_{i,s}\beta_s + \beta_{i,zs}\beta_{zs} + \beta_{e_i},$$

where z is used here to refer to the zero beta portfolio related to NYSE index.

Substituting the RHS of the above relation for β_i in relation (1) yields

$$E(\tilde{r}_i) = [E(\tilde{r}_m)\beta_s]\beta_{i,s} + [E(r_m)\beta_{zs}]\beta_{i,zs} + E(r_m)\beta_{e_i}$$

Using the traditional CAPM to evaluate the terms in $[\cdot]$'s yields

$$E(\tilde{r}_i) = E(\tilde{r}_s)\beta_{i,s} + E(r_s)\beta_{i,zs} + E(\tilde{r}_m)\beta_{e_i}$$

which, when rearranged, is relation (5) in text.

$\beta_{e_i,zs}$ = the beta of the residual of security i measured using a two factor model where the factors are the value weighted NYSE index and the minimum variance zero NYSE beta portfolio.

The first term on the RHS of relation (5) is the predicted return on security i obtained by naively assuming that the NYSE portfolio is the true market portfolio. If the NYSE portfolio were on the efficient frontier then the third term, u_i, would be zero for all i and the second term would be the bias inherent in this naive application of the traditional model. Thus, even if the NYSE portfolio were efficient and risk premiums were proportional to true market betas, risk premiums would not in general be proportional to NYSE betas. For example, if the NYSE portfolio was efficient, but riskier than the true market portfolio, there would be an *ex-ante* linear relationship between risk premiums and NYSE betas with a positive intercept (i.e., $E(\tilde{r}_i) = E(\tilde{r}_{zs}) + \beta_{i,s}(E(\tilde{r}_s) - E(\tilde{r}_{zs}))$).

However, there is no reason to believe that the NYSE portfolio is on the efficient frontier. Here the error term on the RHS of relation (5) would no longer be identically zero for all securities. However, the value weighted average of the error term on the RHS of relation (5) is zero.[2] Thus, for a randomly selected NYSE stock (i) where its probability of selection is proportional to its weight in the NYSE index, the expectation of u_i would be zero. Thus, when the NYSE portfolio is not efficient, *ex-ante* risk premiums would be linear functions of NYSE betas plus an error term. If the minimum variance zero-NYSE beta portfolio had a positive beta with respect to the true market, then its risk premium would be positive (i.e., $E(\tilde{r}_{zs} > 0)$). This would imply the existence of a (non-proportional) linear relationship between risk premiums and NYSE betas (with a positive intercept) plus an error term.

Other Versions of the CAPM

Other versions of the CAPM have been developed. Merton [1971], Cox, Ingersoll and Ross [1978], Breeden and Litzenberger [1978] and Breeden [1980] have derived intertemporal CAPM's that account for shifts in the investment opportunity set. The Merton and the Cox, Ingersoll and Ross studies present multi-beta equilibrium models. The Breeden and Litzenberger, and the Breeden studies, respectively, indicate that the relevant measure of risk is covariance with the marginal utility of consumption and a beta measured relative to aggregate consumption.

While the CAPM theories previously discussed were developed in terms of a single good model, they have been implemented using nominal rates of return. Gonzalez-Gaverra [1973] developed a model that accounts for unanticipated inflation. It suggests that *nominal* risk premiums are linearly related to *real* betas rather than nominal betas.

[2] This follows because for the value weighted index of NYSE stocks $\beta_{es,zs} = \beta_{sz} = (1 - \beta_{ss}) = 0$ by construction.

Implications of Empirical Evidence

Empirical studies by Black, Jensen and Scholes [1972], Fama and MacBeth [1973] and Friend and Blume [1973] find that the relationship between average excess rates of return and NYSE betas is linear, with a positive intercept, rather than proportional. There are at least three possible explanations for these results:

1. Constraints on investor borrowing;
2. Misspecification caused by the exclusion of classes of assets such as bonds, residential real estate, unincorporated business, and human capital from the index; and/or,
3. Misspecification caused by exclusion of other independent variables such as systematic skewness and/or dividend yield from the model.

Each of these explanations yields predictions that are inconsistent with the proportional relationship between risk premiums and NYSE betas that has been asserted in several recent rate cases that use CAPM. To the extent that the NYSE index is a good surrogate for the true market index, the first explanation suggests that a linear relationship between NYSE betas and risk premiums should be estimated and used to calculate the cost of equity capital. The second explanation suggests that a broadly based index should be used to calculate betas. Unfortunately, rate of return data do not exist for some classes of assets and are difficult to obtain for other classes of assets. This suggests that an exact linear relationship between risk premiums and NYSE betas does not exist. However, the NYSE betas of common stocks may be highly correlated with the true unknown betas (measured relative to the true market index). This suggests that the empirical relationship between risk premiums and NYSE betas should be estimated empirically rather than asserted *a priori*.

The third explanation suggests that the effect of other independent variables on risk premiums should be estimated and used in calculating the cost of equity capital. Empirical studies by Rosenberg and Marathé [1979], Litzenberger and Ramaswamy, and Blume [1979] find that, in addition to beta, dividend yield has a significant positive association with average excess rates of return. This result is consistent with the after-tax version of the CAPM and suggests that the relationship between risk premiums, NYSE betas, and dividend yields should be estimated and used to calculate the cost of equity capital. However, Litzenberger and Ramaswamy also present preliminary evidence indicating that the relationship between risk premiums, NYSE betas and yields is non-linear. This result is inconsistent with the Brennan, and Litzenberger and Ramaswamy versions of after-tax CAPM and therefore the use of a linear relationship between risk premiums, betas and dividend yield to calculate the cost of equity capital should be viewed as an approximation to a more complex non-linear relationship.

An empirical study by Kraus and Litzenberger [1976] found that, in addition to beta, systematic skewness (gamma) has a significant negative association with average excess rates of return. However, estimates of gamma are not stable over time and therefore it is not possible to obtain accurate *ex-ante* estimates of the systematic skewness of individual securities. Betas and gammas have a strong

positive association, and, therefore, the use of a linear relationship between risk premiums and betas may again be viewed as approximation to a more complex relationship.

III. Implementing the CAPM Approach

This section discusses econometric problems that are associated with implementing the CAPM approach and presents possible solutions.

Measuring Expectations

The alternative versions of the CAPM discussed above are positive theories of the relationship between *ex-ante* risk premiums and betas.

Ex-ante risk premiums are not, however, directly observable. To handle this problem it is assumed that investors have rational expectations, that the excess rate of return (realized rate of return less the riskless rate of interest) on any portfolio or security in a given month is an unbiased estimate of its risk premium, and that the excess rates of return on each portfolio are independently and identically distributed over time.

Computing Beta

Estimates of the unadjusted betas for each security are obtained from an OLS regression of its excess rate of return on the value weighted NYSE index over a 60 month period. An advantage of using monthly data is that it mitigates the effect of the nonsimultaneity of closing prices. Recently Scholes and Williams [1978] have suggested the use of lagged rates of return as an instrumental variable for the errors in variables problem. Unfortunately, the CRSP daily data file is not available over a sufficiently long time period to be useful in estimating the parameters of the relationship between risk premiums and NYSE betas. Beaver, Kettler and Scholes [1970] and Rosenberg and McKibben [1973] have shown that accounting measures of risk are useful in predicting future betas. However, the Compustat data file, which would be necessary to estimate betas using either of their procedures, does not cover the 1926 to 1947 period.

It has been observed by Blume [1971] that historical betas which are adjusted towards unity are better predictors of future betas (in a mean square forecast error sense) than are unadjusted betas. One explanation of this phenomenon is that the true underlying betas follow a mean reverting process where the mean is unity. Another is that the true underlying beta is constant, the historical beta is a sample estimate of the true underlying beta, and the prior of the beta is unity. These explanations are not mutually exclusive and Blume [1975] has presented preliminary empirical evidence that the true underlying betas display reversion towards the population mean of unity.

Regardless of the cause of the phenomenon, the existence of reversion towards unity suggests that "adjusted" betas, computed as convex combinations of the historical beta and unity, are better predictors than are unadjusted betas. A possible approach is to assume that the same weight ω, $(0 < \omega < 1)$ is applicable

to all securities such that,

$$\beta_i(\text{predicted}) = \omega\beta_{i(\text{historical})} + (1 - \omega)1.$$

This is the procedure used by Blume [1971] and by Merrill Lynch and is called a global adjustment approach. This approach implies a linear relationship between future betas and historical betas and suggests that unadjusted betas may be used to predict risk premiums. For example, consider the following relationship between excess rates of returns and globally adjusted betas,

$$\tilde{r}_i = a + b[\omega\beta_{i(\text{historical})} + (1 - \omega)1] + \tilde{e}_i.$$

This relationship reduces to the following relationship between excess rates of return and historical betas,

$$\tilde{r}_i = a' + b'\beta_{i(\text{historical})} + \tilde{e}_i$$

where

$$a' = a + b(1 - \omega), \quad \text{and}$$

$$b' = b\omega.$$

Note that for predictive purposes, a' and b' may be estimated directly; knowledge of ω is not required. If the ω used were constant over time, then the cost of equity capital estimates obtained using CAPM parameters measured using this global procedure would be identical to those obtained using unadjusted betas. This global adjustment procedure has the advantage of not depending on the exact cause or combination of causes for the empirical tendency of beta estimates to revert towards unity.

Another approach to adjusting betas is to use an individual Bayesian-adjustment procedure. This approach recognizes that the variances of sample betas (obtained from an OLS time series regression of stock returns on the NYSE index) are not identical. This approach is, however, based on the assumption that the true underlying beta is stationary which is inconsistent with Blume's preliminary empirical evidence. Under this approach, the probability of selecting a given stock is assumed to be proportional to its weight in the value weighted portfolio. Therefore, the diffuse prior estimate of its beta is unity. The variance of this prior is computed as

$$\text{Var}(\beta_{i,\text{prior}}) = \sum_{i=1}^{N_t}[[V_i/\sum_{i=1}^{N_t} V_i](\beta_{i,\text{sample}} - 1.0)^2] \tag{6}$$

where V_i is the value of firm i. Thus, the variance of the prior is the cross-sectional variation in sample betas around the value weighted mean of unity. It differs from the Vasicek [1971] adjustment, which computes the prior variance as,

$$\text{Var}(\beta_{i,\text{prior}}) = \sum_{i=1}^{N}(\beta_{i,\text{sample}} - 1.0)^2/N$$

thus giving equal weight to each security. With either the global adjustment or the individual adjustment, the posterior estimate of beta has variance given by

$$\text{Var}(\beta_{i,\text{prior}}) = \omega_i \text{ Var}(\beta_{i,\text{sample}}) + (1 - \omega_i)^2 \text{ Var}(\beta_{i,\text{prior}}) \quad (7)$$

This information is useful in estimating the model coefficients.

Knowing the variance of the measurement error allows implementation of the classical approach to errors in variables and therefore yields a consistent estimator of $\hat{a}_2 = [E(\tilde{R}_{zs}) - R_f]$ (see the next section).

Computing the Risk-Free Rate

In choosing the appropriate proxy for the riskless rate of interest, explicit cognizance should be taken of the fact that the fair rate of return determined in a rate case is applicable throughout a future period. Therefore, the risk-free rate that is chosen should correspond to a risk free return that would be expected to prevail during the period that the pending rate order is expected to be in force.

One simple procedure is to compute the risk free rate as a simple average of monthly forward Treasury Bill rates for the period the pending rate order is expected to be in effect. The Treasury-Bill futures market or McCulloch's [1971] procedure of computing forward rates from the yield curve can be used to obtain the needed forward rates.

Data

The raw data for this study consisted of monthly rates of returns for all NYSE securities and monthly measures of the risk-free rate of interest.

Monthly data on security returns are obtained from the Center for Research in Security Prices (CRSP) at the University of Chicago. The same service also provides the return on a value weighted index of all the NYSE stocks.

Monthly returns on high grade commercial paper from 1926 to 1951 were used as a proxy for the return on a riskless asset. From 1952 to 1978, the return on a Treasury Bill with 30 days to maturity was used for this purpose.

IV. Estimating the Relationship between Risk Premiums and NYSE Betas

The structural econometric model that is estimated in a given cross section is,[3]

$$\tilde{r}_{it} = a + b\beta_{ist} + \tilde{e}_{it}.$$

Any linear estimator of this relationship is obviously a linear combination of the dependent variable. Since the dependent variable is a rate of return, any linear estimator is a rate of return on a portfolio. The unbiasedness condition for an estimator is a set of constraints on this portfolio that assures that the expected rate of return on the portfolio is the coefficient that we are estimating. Once a set

[3] Procedures specific to the implementation of the three moment CAPM, the multiperiod CAPM, and the unanticipated inflation CAPM are not discussed because of unresolved issues relating to the estimation or *ex-ante* systematic skewness, *ex-ante* consumption betas and real betas. The after-tax version of the CAPM and its refinements are considered in Litzenberger and Ramaswamy (1979, 1980).

of portfolio weights $\{h_{it}, i = 1, 2, \cdots, N_t\}$ is chosen, the resulting portfolio rate of return is,

$$\sum_{i=1}^{N_t} h_{it} r_{it} = a \sum_{i=1}^{N_t} h_{it} + b[\sum_{i=1}^{N_t} h_{it}\beta_{ist}] + \sum_{i=1}^{N_t} h_{it} e_{it}. \tag{8}$$

The unbiasedness condition for an estimator of $(a + b)$ requires the following portfolio constraints,

$$\sum_{i=1}^{N_t} h_{it} = 1, \quad \text{and} \quad \sum_{i=1}^{N_t} h_{it}\beta_{ist} = 1.$$

That is, for any normal portfolio (i.e. portfolio weights summing to unity) having a beta of unity, equation (8) reduces to,

$$\sum_{i=1}^{N_t} h_{it} r_{it} = a + b + \sum_{i=1}^{N_t} h_{it} e_{it}.$$

Since the $E(\tilde{e}_{it}) = 0$, $\forall\ i$, it follows that such a portfolio is an unbiased estimator. The best linear unbiased estimator of $a + b$ would be the rate of return on the minimum variance normal portfolio having a beta of unity.

Without loss of generality the variance of any portfolio having a NYSE beta of unity may be expressed as

$$\text{Var}[\sum_{i=1}^{N_t} h_{it}\tilde{r}_{it}] = \text{Var}(\tilde{r}_{st}) + \text{Var}[\sum_{i=1}^{N_t} h_{it}\tilde{e}_{it}],$$

where:

$\tilde{r}_{st}$ = the excess rate of return on the value weighted NYSE portfolio

Note that $\text{Var}(\sum_{i=1} h_{it}\tilde{e}_{it}) = 0$ if and only if the h_{it} for each security corresponds to its weight in the NYSE value weighted index. Thus, the *best* unbiased estimator of $a_t + b_t$ is the excess rate of return on the value weighted NYSE portfolio itself, r_{st}. Assuming that observations of r_{st} are i.i.d., the BLUE estimation of $a + b$ is the average over time of the excess rate of return on the NYSE portfolio.

The unbiasedness conditions for a linear estimator of 'a' are,

$$\sum_{i=1}^{N_t} h_{it} = 1 \quad \text{and} \quad \sum_{i}^{N_t} h_{it}\beta_{ist} = 0.$$

Thus, the rate of return on any normal portfolio that has a zero (true) NYSE beta is an unbiased estimator of 'a'. In any cross-sectional month the best linear unbiased estimator of 'a' would be the rate of return on the minimum variance zero NYSE beta portfolio, r_{zst}.

Without loss of generality the variance of any portfolio having a zero NYSE beta may be expressed as

$$\text{Var}(\sum_{i}^{N_t} h_{it}\tilde{r}_{it}) = \text{Var}(\sum_{i}^{N_t} h_{it} e_{it})$$

Assume momentarily that the true NYSE betas are known. Using the single index model, which assumes that $\text{Cov}(e_{it}, e_{jt}) = 0\ \forall\ i, j \neq i$, the variance of a normal portfolio having a zero NYSE beta is,

$$\text{Var}(\sum_{i=1}^{N_t} h_{it} r_{it}) = \sum_{i=1}^{N_t} h_{it}^2 S_{it}^2$$

where:

S_{it}^2 = the residual risk for security i.

The BLUE estimator of 'a' for a given cross-section month 'a_t' is, therefore, the minimum variance rate of return zero NYSE beta portfolio. The rate of return on this portfolio in month t may be obtained by solving the above described portfolio problem for the h_{it}'s and then calculating $\sum_{i=1}^{N_t} h_{it} r_{it}$. The resulting r_{zst} is

$$r_{zst} = \left[m_{pp} - \frac{m_{p\beta}^2}{m_{\beta\beta}} \right]^{-1} \cdot \left[m_{pr} - \frac{m_{p\beta} m_{\beta r}}{m_{\beta\beta}} \right] \tag{10}$$

where:

$$m_{pp} = \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{1}{S_{it}^2} \qquad m_{p\beta} = \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{\beta_{it}}{S_{it}^2} \qquad m_{pr} = \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{r_{it}}{S_{it}^2}$$

$$m_{\beta\beta} = \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{\beta_{it}^2}{S_{it}^2} \qquad m_{\beta r} = \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{r_{it} \beta_{it}}{S_{it}^2}$$

In the absence of measurement errors in betas, if r_{zst}'s were i.i.d. then a simple average of this would yield the BLUE estimator of 'a', the risk premium on the minimum variance NYSE portfolio.

Errors in the Measurement of Betas

The true NYSE betas are unobservable. If the previously described procedures were used with estimated betas, the cross sectional variance in the estimated betas $m_{\hat{\beta}\hat{\beta}}$ would be an upward biased and inconsistent estimator of the cross sectional variance in the true betas. This would give h_{it}'s that results in portfolio that has positive true NYSE beta for large samples and hence an upward biased estimator of 'a' the risk premium on a portfolio having a zero NYSE beta. To obtain a consistent estimator of 'a', a classical errors in variables approach is undertaken. In this approach, the 'normal' equations for estimation are adjusted as follows: The cross sectional variation in the true NYSE betas, that are unobserved, is replaced by the cross sectional variation in observed NYSE betas less the (sum) of the variances of the measurement errors of the NYSE betas, which has been computed above as $\mathrm{Var}(\hat{\beta}_{it})$. When solved, the resulting estimator is,

$$r_{zt} = \left[m_{pp} - \frac{m_{p\hat{\beta}}^2}{m_{\hat{\beta}\hat{\beta}} - Q} \right]^{-1} \cdot \left[m_{pr} - \frac{m_{\hat{p}\hat{\beta}} m_{\hat{\beta}r}}{m_{\hat{\beta}\hat{\beta}} - Q} \right] \tag{11}$$

where

$$Q = \frac{1}{N_t} \sum_{i=1}^{N_t} \frac{\mathrm{Var}(\hat{\beta}_{it})}{\hat{S}_{it}^2}.$$

Comparing relation (10) with relation (11) indicates that they are identical except for the Q term which is the adjustment due to the variability in the estimator of beta. Under the assumption that the error term is normally distributed and that the true variances of the measurement errors are known, $m_{\hat{\beta}\hat{\beta}} - Q$ is the maximum

likelihood estimator of $m_{\beta\beta}$, the cross sectional variation in the unobservable true NYSE betas. It also follows that $m_{p\hat{\beta}}$ and $m_{\hat{\beta}r}$ are maximum likelihood estimators of $m_{p\beta}$ and $m_{\beta r}$. Since the above described estimator of 'a' is a function of a maximum likelihood estimator, it is also a maximum likelihood estimator (see Kendall and Stuart [1973]).

V. Estimates of CAPM Parameters

The consistent estimators (as described in the previous section) of the parameters of the relationship between *ex-ante* premiums and NYSE betas are given in Table 1. Results for individually Bayesian adjusted and raw betas are presented.

Since the raw betas are not adjusted towards unity, the a_t's calculated each month would be expected to have a positive beta. Regressing the a_t's that were calculated using raw NYSE betas on the r_{st}'s gives a slope coefficient of 0.109 and an R^2 of 0.039. This suggests that the true NYSE beta on this portfolio is positive.

The standard deviation of the r_{zt}'s is less than the standard deviation of the $(r_{st} - r_{zt})$'s as the mathematics of the efficient frontier would suggest. Since individually Bayesian adjusted betas are adjusted towards unity, the r_{zt}'s calculated using the Bayesian adjusted betas would be expected to have a zero NYSE beta. However, regressing the r_{zt}'s that were calculated using Bayesian adjusted NYSE betas (the r_{zt}'s) on the r_{st}'s gives a slope of -0.144 and an R^2 of 0.0327. This suggests that the NYSE beta of this portfolio is negative. Unfortunately, an econometric rationale for a negative beta is not readily apparent. Again the standard deviation of the r_{zt}'s is lower than the standard deviation of the $(r_{st} - r_{zt})$'s as would be expected from the mathematics of the efficient frontier. The $\bar{r}_z$ calculated using Bayesian adjusted betas is lower than the $\bar{r}_z$ calculated using raw betas as would be expected given the correlation of these portfolios with the NYSE index. Note that the consistent estimators of 'a' and a' reported in TABLE 1 are lower than the corresponding inconsistent estimators obtained using gen-

Table 1

CAPM Parameters

Bayesian Betas

$$r_{it} = r_{zst} + [r_{st} - r_{zst}]\beta_{is(\mathrm{ADJ})} + \epsilon_{it}$$

$$\hat{a} = \bar{r}_{rs} = 0.136 \qquad \hat{b} = \bar{r}_s - \bar{r}_{zs} = 0.519$$

$$\sigma(r_{zst}) = 4.73 \qquad \sigma(r_{st} - r_{zst}) = 8.14$$

Raw Betas

$$r_{it} = [r_{zst} + (r_{st} - r_{zst})(1 - \omega)] + [(r_{st} - r_{zst})\omega]\beta_{is(\mathrm{raw})} + \epsilon_{it}$$

$$\hat{a}' = 0.326, \qquad \hat{b}'_t = 0.330$$

$$\sigma(a_t) = 3.23 \qquad \sigma(b_t) = 6.14$$

where

$$a'_t = [r_{zst} + (r_{st} - r_{zst})(1 - \omega)], \qquad b'_t = [(r_{st} - r_{zst})\omega]$$

eralized least squares as would be expected from the econometric theory. GLS parameters are reported in TABLE 2.

VI. Examples and Conclusions

To illustrate the biases that arise by naively assuming a proportional relationship between NYSE betas and risk premiums, the parameters from Table 1 along with estimates of the risk free rate of interest and betas were used to estimate the cost of equity capital for two utilities: one with a beta substantially less than unity, Pacific Gas and Electric (PGE), and one with a beta close to unity, Consolidated Edison (Con Ed).

The relevant unadjusted and Bayesians betas are presented in Table 3 along with cost of equity capital estimates made by naively assuming a proportional relationship, and by using the estimated linear relationship in all of the calculations.

A risk free rate of interest of 9.29% per annum was used. This was obtained by averaging forward interest rates implied by Treasury Bill futures settlement prices on the International Monetary Market for October 1, 1979 (the assumed date of the rate case). Assuming a nine month lag between the rate case and its implementation, Treasury Bill futures contracts for delivery in June 1980 and thereafter were used in the average. For the main model the same estimates of the risk premium on the NYSE index was used (i.e., $a + b$). The monthly cost of equity capital estimates were compounded to obtain annual estimates.

The differences in the cost of equity capital estimates, which illustrate the so called "zero beta effect", are substantial for PG&E since its NYSE beta estimates are less than unity. The zero beta effect is negligible for Con Ed since its beta is close to unity.

Table 2

Bayesian Betas	
$\hat{a} = 0.321$	$\hat{b} = 0.335$
$\sigma(\hat{a}_t) = 3.26$	$\sigma(\hat{b}_t) = 6.23$
Raw Betas	
$\hat{a} = 0.420$	$\hat{b}' = 0.236$
$\sigma(\hat{a}'_t) = 3.04$	$\sigma(\hat{b}'_t) = 5.19$

Table 3

Maximum Likelihood Estimates of the Cost of Equal Capital

	Unadjusted/Global adjusted betas			Individually Adjusted Bayesian betas		
Company	Raw beta	Proportional	Linear	Beta	Proportional	Linear
PGE	0.48	13.49	15.78	0.53	13.87	14.74
Con Ed	1.06	18.68	18.42	1.05	18.61	18.50

These two companies, as well as utilities in general, have residual standard deviations that are smaller than those of most industrial firms. Hence the individual Bayesian adjustment procedure did not adjust the betas of the sample companies as much towards unity as a global procedure would have. The effect of the individual Bayesian adjustment procedure on the estimated parameters presented in Table 2 can be loosely viewed as reflecting the average adjustment towards unity. Therefore, for a utility such as PG&E having a NYSE beta less than unity and having a lower than average residual risk and the cost of capital estimates obtained using a linear relationship between risk premiums and betas estimated with individually adjusted Bayesian betas would be lower than that obtained using a linear relationship estimated with unadjusted or globally adjusted betas. The difference between the estimates obtained using the individually Bayesian adjusted estimates and the raw betas is negligible for Con Ed since its beta is close to unity. The difference between the estimates for PG&E are substantial and indicate the importance of future research on the revision of betas towards unity.

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DISCUSSION

RICHARD S. BOWER*: As a regulator I find the three papers stimulating and helpful. Each is reassuring because it supports some aspect of regulatory practice, rewarding because it suggests an opportunity to improve practice and less than totally satisfying because it does not provide all the answers.

Bruce Greenwald's paper on admissable rate bases may be too rich to digest at a single sitting. Greenwald starts conventionally by stating that the Hope decision criteria for fairness to investors and capital attraction are met by any rate base valuation formula which permits market value to equal rate base and which causes rate base to increase dollar for dollar with new investment. He then argues, less conventionally, that to be admissable a formula must allow regulators to establish cash revenue requirements and rate base appreciation through time and

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