TAXES, MARKET VALUATION AND CORPORATE FINANCIAL POLICY

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IN A well-known series of papers Franco Modigliani and Merton Miller[1] have outlined a general framework for the analysis of the effects of capital structure and dividend policies on the valuation of the corporation under uncertainty. What disagreement remains about their conclusions stems mainly from different beliefs about the effects of various market imperfections on their analysis.[2] Modigliani and Miller themselves have dealt comprehensively with one such imperfection, namely the tax system as it affects corporations directly.[3] However, while they have directed attention to the effects of the tax system as it relates to the taxation of corporate income, their papers are characterized by an almost total neglect of the complementary aspect of the system, which is the taxation of individuals. It is the purpose of this paper to extend their analysis to incorporate the effects of those features of the personal tax structure which are relevant for the valuation of the corporation.

Two features of the personal tax structure stand out in importance for the theory of valuation. First is the provision of the existing tax code which permits individuals as well as corporations to deduct interest payments from the computation of their taxable income. Second is the asymmetric tax treatment of income received in the form of dividends and of capital gains. The difficulty of introducing these institutional imperfections into the analysis arises from the progressive nature of the personal tax structure, which causes the relevant marginal tax rates to vary between investors in different income classes.

An important step towards recognizing the effects of the personal tax structure on corporate financial policy was made in a 1967 article in this journal by Farrar and Selwyn.[4] However, their analysis is limited by its concentration on the net income received by an investor with given tax rates from a share in a corporation, as that corporation pursues alternative financial policies. Their use of this net income concept as a criterion of optimality suffers by its implicit neglect of the market exchange opportunities open to an investor who does not find a particular set of financial policies congenial. To take into account these market exchange opportunities requires the development of a market valuation principle, so that the impact of alternative financial policies on the value of the corporation may be calculated: the Farrar-Selwyn paper lacks such a valuation principle.

The outline of this paper is as follows: in Section I the Farrar-Selwyn analysis and its results are considered in more detail. In Section II a market equilibrium condition is developed which takes account of the diversity of investor marginal tax rates. From this equilibrium condition a market valuation equation is developed in Section III. This is then used to discuss the effects of alternative dividend policies on the valuation of the corporation. In Section IV the effects of alternative capital structures are discussed within the framework of the same

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[1] [4, 5, 6, 7]

[2] Major attention has been focused on the effects of bankruptcy costs by P. A. Tinsley [12]. There remains also some dispute about the effects of dividend policy under uncertainty.

[3] [7].

[4] [1].

valuation model. In Section V attention is directed to the interactions of capital structure and dividend policies when the firm is subject to a share re-purchase constraint.

Throughout the paper we abstract from the postponability feature of the capital gains tax, and assume that taxes on capital gains as on dividends must be paid each period.

I

We summarize here the most general part of the Farrar-Selwyn analysis of corporate financial policy, in which they take into account the full array of personal and corporate tax rates.[5] They consider three different sets of corporate and personal financial strategies, but we shall restrict our attention to the first two, since the third concentrates on the postponability feature of the capital gains tax which we are neglecting. The strategies we shall consider are:

(i) Corporate earnings are paid out entirely as dividends and are taxed as personal income.

(ii) Corporate earnings are translated into capital gains with all gains being realized immediately by investors and taxed at capital gains rates.[6]

Farrar and Selwyn consider a single investor with given marginal tax rates, owning a share in a corporation and having a desired total (corporate and personal) debt per share. Within this framework they evaluate alternative corporate financial policies in terms of the after-tax income received by the investor.

For this purpose define:[7]

$\tilde{Y}$—the net income stream (including capital gains) available to an investor from holding one share of stock after all interest and taxes, personal and corporate, have been paid.

$\tilde{X}$—the operating income per share of the company before interest and tax payments.

r —the market rate of interest faced by personal and corporate borrowers and lenders alike.

D_c—the amount of corporate debt outstanding per share of common stock.

D_p—the amount of personal debt outstanding per share of common stock.

T_c, T_p, T_g—the marginal corporate, personal income, and capital gains tax rates.

Strategy 1: Earnings Paid as Dividends and Taxed as Income.

When all the earnings of the corporation are paid out as dividends and are taxed as the personal income of the investor, his net income per share is given by:

$$\tilde{Y}=[(\tilde{X}-rD_c)(1-T_c)-rD_p](1-T_p) \tag{1.1}$$

Then the after-tax costs to the investor of personal and corporate debt are found by differentiating (1.1) partially with respect to D_p and D_c.

$$\begin{aligned} \frac{\partial \tilde{Y}}{\partial D_p} &= -r(1-T_p) \\ \frac{\partial \tilde{Y}}{\partial D_c} &= -r(1-T_p)(1-T_c) \end{aligned} \tag{1.2}$$

It follows from (1.2) that an investor's net income per share is reduced less by additional corporate debt than it is by additional personal debt; this is on account of the additional tax shield for corporate interest payments offered by the corporate income tax. In this sense corporate debt is 'cheaper' than personal debt for all investors, whatever their marginal tax rates.

Strategy 2: Earnings Transformed into Capital Gains and Taxed Immediately.

[5] The three other cases considered by Farrar and Selwyn may be obtained from this one by setting one or more of the tax rates equal to zero.

[6] As Myers [9] has pointed out, Farrar and Selwyn make an error in their computation of the capital gains tax liability. However, their error corresponds to our assumption that capital gains are taxed when earned rather than when realized.

[7] Those symbols covered with a tilde denote random variables.

In this case the net income available to the investor may be written:

$$\tilde{Y}=(\tilde{X}-rD_c)(1-T_c)(1-T_g)-rD_P(1-T_P) \tag{1.3}$$

The costs of personal and corporate debt are again found by partial differentiation with respect to D_p and D_c.

$$\frac{\partial\tilde{Y}}{\partial D_p}=-r(1-T_p)$$
$$\frac{\partial\tilde{Y}}{\partial D_c}=-r(1-T_c)(1-T_g) \tag{1.4}$$

Now corporate debt is 'cheaper' for an investor only if:

$$(1-T_c)(1-T_g)<(1-T_p)$$

or

$$T_p<T_c+T_g+T_cT_g \tag{1.5}$$

(1.5) indicates that the relative effects of corporate and personal debt on the net income per share received by the investor will depend upon his marginal tax rates T_p and T_g: in general, low tax bracket investors will find the impact of corporate debt on their net income relatively more favourable than will high tax bracket investors.

Thus if the criterion of maximizing the net income per share received by investors is accepted, the following conclusions may be drawn. First, as Farrar and Selwyn also show, and as is readily apparent, it will always be optimal for a corporation to use any residual earnings for share re-purchase rather than for dividend payment, so long as the investor's marginal tax rate on dividends exceeds his marginal tax rate on capital gains. Secondly, corporate debt will be advantageous for all investors in dividend-paying corporations, although the value of corporate debt to different investors will depend directly upon their marginal tax rates T_p. Finally, for a non-dividend paying corporation, different financial policies may be optimal for different investor groups, according to their marginal tax rates: for example, high marginal tax rate investors for whom $T_p>T_c+T_g+T_cT_g$ would appear to prefer the corporation to pursue a zero debt strategy[8] to maximize the amount of debt the investors may issue on their personal account, consistent with their desired total debt per share. Low marginal tax rate investors on the other hand would appear to prefer the corporation to pursue a maximum debt strategy: if such a strategy results in excessive debt per share from the investor's point of view it can always be partially undone by personal lending.

This possibility of a conflict of aims between different investor groups within the same corporation raises serious problems, both for the financial theorist in search of clear decision rules for corporate financial policy, and for the financial manager who must somehow reconcile these divergent interests. Farrar and Selwyn suggest tentatively that investors within a single corporation may be relatively homogeneous with respect to marginal tax rates, if different tax clienteles of investors find different characteristics of operating income streams $\tilde{X}$ attractive; but they conclude that "a certain amount of creative artistry will continue to be needed in the design of optimal financial policies."[9]

Fortunately we need only accept the Farrar-Selwyn conclusion if we accept their criterion of optimality, namely the maximization of the after-tax income flow to the investor. As we have previously suggested, such a criterion is open to objection. Its validity relies on the implicit assumption that the investor is locked into his existing shareholding, and hence is interested only in the net income per share which will accrue to him from that shareholding. A more reasonable assumption is that the investor's opportunity set includes not only the possibility of borrowing and lending but also of trading securities in the capital market. If, following the usual assumptions of a perfect market for securities,[10] this oppor-

8Or even a negative debt strategy! i.e. they would prefer the corporation to become a net lender.

9[1, p. 454].

10The assumptions required are that all market participants are price-takers and that transactions costs may be neglected. Note that "transactunity set can be regarded as independent of the decisions of a single firm, then it follows that the welfare of all investors in the firm is maximized by the maximization of the market value of the firm. Thus once the investor's market exchange opportunities are recognized the potential conflict of aims suggested by Farrar and Selwyn is shown to be nugatory.

A related criticism of the Farrar-Selwyn approach is that it is essentially comparative static, and takes no account of any possible dynamic impact of the issuance of corporate debt on the net income of the investor $\tilde{Y}$, in the period in which the debt is issued. There will be such a dynamic impact to the extent that the value of the corporation is changed by the issuance of debt.

Hence the fundamental limitation of the Farrar-Selwyn approach stems from its lack of a market valuation principle; this prevents it from taking into account the market exchange opportunities open to the investor, or dealing with the dynamic effects of debt issue. The basis for such a market valuation principle will be developed in the following section.

II

In this section we develop the basic condition for capital market equilibrium under uncertainty when investors have different marginal tax rates. The basic framework of analysis is the Capital Asset Pricing Model of Lintner,[11] Sharpe[12] and Mossin,[13] generalized to incorporate the effects of the taxes investors must pay on their income from dividends and capital gains.

Following the usual assumptions of this model, we take the market for securities to consist of m risk-averse investors who are concerned with selecting portfolios to hold over the same single-period horizon. We assume that the utility functions U_i ($i=1, \ldots, m$) of the investors may be defined on the mean V_i, and variance S^2_i, of the *after-tax* returns on the portfolios, so that

$$\left.\begin{aligned} &U_i = U_i(V_i, S^2_i) \\ \text{where}\quad &U'_i = \frac{\partial U_i}{\partial V_i} > 0 \\ &U''_i = \frac{\partial U_i}{\partial S^2_i} < 0 \end{aligned}\right\} \; i=1, \ldots, m \tag{2.1}$$

The investors trade in $(n+1)$ securities; security 0 is assumed to have an initial unit price of unity, and a known terminal unit price q, and the whole of the return from this security is assumed to be subject to tax as ordinary income. The remaining n securities have initial unit prices p_j ($j=1, \ldots, n$) and uncertain terminal unit prices $\tilde{\pi}_j$; in addition, each unit of security j ($j=1, \ldots, n$) pays a terminal dividend d_j which is known at the beginning of the period. Thus the return on each risky security has two components, a known dividend and an uncertain terminal price. It is assumed that all investors agree in their assessments of the mean values of terminal price $\tilde{\pi}_j$ ($j=1, \ldots, n$), and of the covariances between the terminal prices of the securities s_{jk} ($j=1, \ldots, n$; $k=1, \ldots, n$). Each investor i ($i=1, \ldots, m$) comes to the market with an initial endowment of x^o_{ji} units of security j ($j=0, 1, \ldots, n$), and by trading with other investors achieves an equilibrium asset position x_{ji} ($j=0, 1, \ldots, n$). We are concerned with the conditions for all the investors to be in personal portfolio equilibrium and for the security markets to clear. Finally, we assume for simplicity that each investor has marginal tax rates on dividend and capital gains income t_{di} and t_{gi} which are constant and independent of his portfolio choice.

Individual Portfolio Equilibrium

The expected after-tax return on investor i's portfolio is given by:

$$V_i = \sum_{j=1}^{n} [\bar{\pi}_j - (\bar{\pi}_j - p_j)t_{gi} + d_j(1 - t_{di})]x_{ji} + [q - (q - 1)t_{di}]x_{oi} \tag{2.2}$$

tions costs" in this context includes any unpaid capital gains tax liability: we have assumed capital gains taxes are payable when the gains are made.

11 [3]

12 [11]

13 [8]

and the variance of the after-tax return is:

$$S^2_i = \sum_{j=1}^{n} \sum_{k=1}^{n} s_{jk}\, x_{ji}\, x_{ki}\, (1 - t_{gi})^2 \tag{2.3}$$

Thus the investor may be represented as maximizing a utility function

$$U_i = U_i(V_i, S^2_i) \tag{2.4}$$

subject to his budget constraint

$$\sum_{j=1}^{n} p_j(x_{ji} - x^o_{ji}) + (x_{oi} - x^o_{oi}) = 0 \tag{2.5}$$

where V_i and S^2_i are given by (2.2) and (2.3).

The first-order conditions for the constrained maximum are found by setting up the Lagrangean expression

$$L = U_i(V_i, S^2_i) - \lambda\Big[\sum_{j=1}^{n} p_j(x_{ji} - x^o_{ji}) + (x_{oi} - x^o_{oi})\Big] \tag{2.6}$$

and setting equal to zero its partial derivatives with respect to $x_{ji}(j=0, 1, \ldots, n)$ and λ.

$$\frac{\partial L}{\partial x_{ji}} = U'_i \frac{\partial V_i}{\partial x_{ji}} + U''_i \frac{\partial S^2_i}{\partial x_{ji}} - \lambda p_j = 0 \qquad j=0, 1, \ldots, n \tag{2.7}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{j=1}^{n} p_j(x_{ji} - x^o_{ji}) + (x_{oi} - x^o_{oi}) = 0 \tag{2.8}$$

But from (2.2) we see that:

$$\frac{\partial V_i}{\partial x_{oi}} = q - (q - 1)t_{di} \tag{2.9}$$

$$\frac{\partial V_i}{\partial x_{ji}} = \bar{\pi}_j - (\bar{\pi}_j - p_j)t_{gi} + d_j(1 - t_{di}) \qquad j=1, \ldots, n \tag{2.10}$$

and from (2.3)

$$\frac{\partial S^2_i}{\partial x_{oi}} = 0 \tag{2.11}$$

$$\frac{\partial S^2_i}{\partial x_{ji}} = 2 \sum_{k=1}^{n} s_{jk}\, x_{ji}\, x_{ki}\, (1 - t_{gi})^2 \qquad j=1, \ldots, n \tag{2.12}$$

Substituting for these expressions in (2.7) we obtain as conditions for the constrained maximum, in addition to the budget equation (2.8):

$$\frac{\partial L}{\partial x_{oi}} = U'_i[q - (q - 1)t_{di}] - \lambda = 0 \tag{2.13}$$

$$\frac{\partial L}{\partial x_{ji}} = U'_i[\bar{\pi}_j - (\bar{\pi}_i - p_j)t_{gi} + d_j(1 - t_{di})] + U''_i\Big[2 \sum_{k=1}^{n} s_{jk}(1 - t_{gi})^2 x_{ki}\Big] - \lambda p_j = 0 \qquad j=1, \ldots, n \tag{2.14}$$

Eliminating λ between (2.13) and (2.14) and re-arranging, we obtain:

$$\sum_{k=1}^{n} s_{jk}\, x_{ki} = \frac{w_i}{(1-t_{gi})^2}[\bar{\pi}_j(1 - t_{gi}) + p_j t_{gi} + d_j(1 - t_{di}) - p_j(q - (q - 1)t_{di})] \qquad j=1, \ldots, n \tag{2.15}$$

Where $w_i = -\frac{1}{2}\frac{U'_i}{U''_i}$, is proportional to the investor's marginal rate of substitution between expected return and variance.

Assuming that the second order conditions for the constrained maximum are satisfied, equations (2.15) give the equilibrium relationship between the per unit covariance of return on security j $(j=1, \ldots, n)$ and the after-tax risk premium expected per unit of security j. The n equations (2.15) in conjunction with the budget equation (2.8) suffice to determine uniquely the investor's equilibrium holding of each of the $(n+1)$ securities. It may be observed that since w_i enters only as a scaling constant in (2.15), the investor's relative holdings of the n risky securities are independent of the exact shape of his utility function, but not of his marginal tax rates t_{di} and t_{gi}.

Market Equilibrium

Equilibrium in the securities market requires first that each individual investor be in portfolio equilibrium, so that (2.8) and (2.15) must hold for all investors (i = 1, . . . , m); and secondly that the market for all securities clears so that:

$$\sum_{i=1}^{m} x_{ji} = \sum_{i=1}^{m} x^o_{ji} = x^o_j \qquad (2.16)$$
$$j=0, 1, \ldots, n$$

where x^o_j is the outstanding supply of security j.[14]

Then summing (2.15) over all investors (i = 1, …, m) we obtain

$$h \sum_{k=1}^{n} s_{jk}\, x^o_k = [\overline{\pi}_j + d_j - qp_j]$$
$$- T_g[\overline{\pi}_j - p_j] \quad j=1, \ldots, n$$
$$- T_d[d_j - p_j(q-1)] \qquad (2.17)$$

where

$$h = \left[\sum_{i=1}^{m} \frac{w_i}{(1-t_{gi})^2}\right]^{-1}$$

$$T_g = \left[\sum_{i=1}^{m} \frac{w_i\, t_{gi}}{(1-t_{gi})^2}\right]\left[\sum_{i=1}^{m} \frac{w_i}{(1-t_{gi})^2}\right]^{-1}$$

$$T_d = \left[\sum_{i=1}^{m} \frac{w_i\, t_{di}}{(1-t_{gi})^2}\right]\left[\sum_{i=1}^{m} \frac{w_i}{(1-t_{gi})^2}\right]^{-1}$$

Note that T_d and T_g are weighted averages of investors' marginal tax rates on dividends and capital gains, where the weights depend upon investors' marginal rates of substitution between expected return and variance of return.

Define the following new variables:

$r = q - 1$ the riskless rate of interest

$\delta_j = \frac{d_j}{p_j}$ the prospective dividend yield on security j (j=1, . . . , n)

$\widetilde{R}_j = \frac{\widetilde{\pi}_j + d_j - p_j}{p_j}$ the rate of return on security j (j=1, . . . , n)

Note that

$$\sum_{k=1}^{n} \frac{s_{jk}}{p_j} x^o_k = M \sum_{k=1}^{n} Q_k \, COV(\widetilde{R}_j \, \widetilde{R}_k)$$

where M is the total market value of all securities and Q_k (k=1,, . . . , n) is the share of security k in that total market value.[15]

Then dividing (2.17) by p_j, transposing terms and making the above substitutions yields:

$$\overline{R}_j - r = hM \sum_{k=1}^{n} Q_k \, COV(\widetilde{R}_j \widetilde{R}_k)$$
$$+ T_d(\delta_j - r) + T_g(\overline{R}_j - r) \qquad (2.18)$$
$$j=1, \ldots, n$$

or
$$\overline{R}_j - r = HM \sum_{k=1}^{n} Q_k \, COV(\widetilde{R}_j \widetilde{R}_k)$$
$$+ T(\delta_j - r) \qquad (2.19)$$
$$j=1, \ldots, n$$

where $H = h/(1-T_g)$[16]

$T = (T_d - T_g)/(1-T_g)$[17]

Finally (2.19) may be simplified by noting that $\sum_{k=1}^{n} Q_k M \, \widetilde{R}_k = \widetilde{R}_m$ where $\widetilde{R}_m$ is the rate

[14]As usual in a general equilibrium system one of these equations is redundant since it can be obtained from the other equations and the summation of investors' budget equations.

[15]This is because

$$\sum_{k=1}^{n} \frac{s_{jk}}{p_j} x^o_k = \sum_{k=1}^{n} \frac{COV(\widetilde{\pi}_j \, \widetilde{\pi}_k)}{p_j p_k} p_k \, x^o_k$$
$$= \sum_{k=1}^{n} M \, Q_k \, COV(\widetilde{R}_j \, \widetilde{R}_k)$$

[16]The value of H may be found by multiplying equation (2.19) by $p_j \, x^o_j$ and summing over j (j=1, . . . , n). This yields

$$H = \overline{R}_m - r - T(\delta_m - r)$$

where $\overline{R}_m$ and δ_m are the expected return and dividend yield on a value-weighted market portfolio.

[17]If $t_{di} \geq t_{gi} \geq 0$ for all investors, then $0 \leq T \leq 1$.

of return on the whole market portfolio, so that

$$M \sum_{k=1}^{n} Q_k \, COV(\tilde{R}_j \tilde{R}_k) = COV(\tilde{R}_j \tilde{R}_m) \qquad (2.20)$$

and (2.19) becomes:

$$\overline{R_j} - r = H \, COV(\tilde{R}_j \tilde{R}_m) + T(\delta_j - r) \qquad (2.21)$$

$$j = 1, \ldots, n$$

Equation (2.21) then expresses the basic principle of market valuation under uncertainty when different investors have different marginal tax rates. It asserts that the expected or required risk premium on security j $(j=1, \ldots, n)$, $(\overline{R_j} - r)$, is a function of that security's risk characteristics $COV(\tilde{R}_j \tilde{R}_m)$, and of its expected dividend yield δ_j. The intuitive interpretation of this result is that for a given level of risk, investors require a higher total return on a security the higher is its prospective dividend yield, because of the higher rate of tax levied on dividends than on capital gains.

III

In the previous section we have derived a market equilibrium condition relating the required rate of return on a security to its risk characteristics and its expected dividend yield. For the discussion of dividend and capital structure policies it is helpful to transform this equation into a relationship between the value of a firm, the characteristics of its operating income stream, and its financial policies. For this purpose we restrict our attention to the convenient no-growth case and assume that the future may be regarded as a series of identical periods in each of which the market equilibrium condition (2.21) is expected to hold.

Each period the corporation[18] has an uncertain operating income stream $\tilde{X}$, and the joint probability distribution of $\tilde{X}$ and the returns on all other assets in the economy is assumed to be stable through time; corporate tax at the rate τ is levied on this income stream each period. The corporation is assumed to pay a constant dividend, D, each period, and to repurchase or issue stock at the end of each period so that its market value at the beginning of the following period is a constant, V. If we assume also that investors' risk preferences and tax rates remain constant through time, then all periods are identical and the expected future earnings of the corporation will be capitalized at a constant rate we shall denote by ρ. We have excluded the possibility of a change in the value of the firm due to a recapitalization of future earnings prospects; therefore, the uncertain end-of-period value of the firm, $\tilde{V}_t$, after dividends have been paid but before any shares have been issued or repurchased, will be equal to the beginning-of-period value of the firm, V, plus the net operating income stream, $\tilde{X}(1-\tau)$, less the amount of dividends paid, D.

$$\text{i.e.} \quad \tilde{V}_t = V + \tilde{X}(1-\tau) - D \qquad (3.1)$$

It follows then that if $\tilde{R}$ is the rate of return on the corporation's securities

$$1 + \tilde{R} = (\tilde{V}_t + D)/V \qquad (3.2)$$

$$\text{and} \quad COV(\tilde{R}, \tilde{R}_m) = COV[(V + \tilde{X}(1-\tau))/V, \tilde{R}_m] = COV[\tilde{X}(1-\tau)/V, \tilde{R}_m] \qquad (3.3)$$

Hence, the value of the corporation, V, may be written as the capitalized value of the expected earnings after tax

$$V = \frac{\overline{X}(1-\tau)}{\rho} \qquad (3.4)$$

where the capitalization rate ρ is given by the market equilibrium condition (2.21)

$$\rho = r + H \, COV(\tilde{R}, \tilde{R}_m) + T[\frac{D}{V} - r]$$

$$= r + H \, COV[\tilde{X}(1-\tau)/V, \tilde{R}_m] + T[\frac{D}{V} - r], \qquad (3.5)$$

for $\frac{D}{V}$ will be the prospective dividend yield

[18] In this and the following section we drop the firm subscript since we are always referring to the same firm.

on the corporation's securities. Then substituting for ρ in (3.4) and rearranging, we obtain:

$$V = \frac{\overline{X}(1-\tau) - H\,COV[\widetilde{X}(1-\tau), \widetilde{R}_m] - TD}{r(1-T)} \quad (3.6)$$

(3.6) is a general valuation equation for the corporation, expressing its value as function of the net operating income stream, $\widetilde{X}(1-\tau)$, and the amount of dividends paid each period, D.

To calculate the effect of alternative dividend policies on the value of the corporation it is only necessary to differentiate (3.6) partially with respect to D.

$$\frac{\partial V}{\partial D} = \frac{-T}{r(1-T)} \quad (3.7)$$

(3.7) shows that if the criterion of market value maximization is accepted and if $T > 0$, it is non-optimal for all investors in a corporation for the corporation to pay dividends: share re-purchase is a preferred alternative for all investors, whatever their marginal tax rates. Since we observe that most corporations do in fact pay regular dividends, such behavior must be rationalized by the assumption that such corporations are behaving under an actual or perceived constraint on systematic share re-purchase as an alternative to dividend payment. In our discussion of capital structure policy we shall assume that the corporation is subject to such a constraint.

IV

Suppose that a corporation has an amount of bonds B outstanding at an interest cost r. Then the market value of the corporation's equity E, is given by:

$$E = V - B \quad (4.1)$$

But E may also be regarded as the expected net earnings of equity holders $(\widetilde{X} - rB)(1-\tau)$, capitalized at a rate ρ_E appropriate to the risk and composition of the equity income stream.

$$E = \frac{(\overline{X} - rB)(1-\tau)}{\rho_E} \quad (4.2)$$

An argument analogous to that used in the previous section to derive ρ, shows that ρ_E is given by:

$$\rho_E = r + H\,COV[\widetilde{X}(1-\tau)/E, \widetilde{R}_m] + T[\frac{D}{E} - r] \quad (4.3)$$

Substituting for ρ_E in (4.2) and rearranging yields:

$$E = \frac{(\overline{X} - rB)(1-\tau) - H\,COV[\widetilde{X}(1-\tau), \widetilde{R}_m] - TD}{r(1-T)} \quad (4.4)$$

The constraint on systematic share repurchase may be written:

$$D + (1-\tau)\,rB \geqq \overline{X}(1-\tau) \quad (4.5)$$

(4.5) requires that the amount paid out in dividends and net interest payments be at least as great as the mean net income stream.

Note that since

$$V = B + E$$

$$\frac{dV}{dB} = 1 + \frac{dE}{dB} \quad (4.6)$$

We shall consider the effects of alternative debt levels on the value of the corporation under two assumptions:

(i) Constraint on share re-purchase not binding so that

$$D + (1-\tau)\,rB > \overline{X}(1-\tau)$$

Then from (4.4)

$$\frac{dE}{dB} = -\frac{(1-\tau)}{(1-T)} \quad (4.7)$$

and

$$\frac{dV}{dB} = \frac{\tau - T}{1 - T} \text{ from (4.6)} \quad (4.8)$$

(4.8) shows that a high leverage strategy will maximize the value of the corporation, and hence will be advantageous for all investors so long as the corporate tax rate

τ exceeds the market's "effective tax rate" T. However, the relative advantage of corporate debt is reduced by the existence of investor taxes $(T>0)$.

(ii) Constraint on share re-purchase binding so that

$$D+(1-\tau)\,rB=\overline{X}(1-\tau)$$

Note that this constraint now implies that

$$\frac{dD}{dB}=-(1-\tau)r \qquad (4.9)$$

so that the issuance of debt reduces the amount of dividends that must be paid by the net interest cost of the debt.

Then taking into account this relationship,

$$\frac{dV}{dB}=\frac{\partial V}{\partial B}+\frac{\partial V}{\partial D}\cdot\frac{dD}{dB} \qquad (4.10)$$

$$=-\frac{\tau-T}{1-T}+\frac{T(1-\tau)}{(1-T)}$$

$$\frac{dV}{dB}=\tau \qquad (4.11)$$

But this is precisely the result obtained by Modigliani and Miller neglecting investor taxes:[19] that is, if an amount of bonds B is issued, the value of the corporation is increased by τB. We conclude then that if the corporation is subject to a binding constraint on share re-purchase, the original Modigliani-Miller cost-of-capital propositions are unaffected by the existence of investor taxes.

However, the whole of the above argument has been conducted on the assumption that there is no relationship between the amount of debt issued in a period and the amount of dividends that must be paid in that period. In fact, if there is a constraint on share repurchase which precludes use of the proceeds of a bond issue to repurchase shares and if the share repurchase constraint is binding, there will be a necessary connection between the amount of debt issued in a period and the amount of dividends paid in that period. Since this consequent change in dividends paid will affect valuation, a full analysis of the effects of debt issuance must take this effect into account. To this problem we turn in the next section.

V

We now consider the effect on the current value of the corporation V, of an expected issue of debt $\triangle B$, at the end of the first period, assuming that the corporation pays a constant dividend and issues no further debt in subsequent periods. We assume also that the corporation is subject to a binding constraint on share re-purchase.

Denote by $\widetilde{V}'_t$ the value of the corporation at the end of one period after the debt has been issued and dividends have been paid. Then the argument of the previous section implies that with a binding constraint on share re-purchase

$$\frac{d\widetilde{V}'_t}{d\triangle B}=\tau \qquad (5.1)$$

and hence

$$\frac{d\overline{V}'_t}{d\triangle B}=\tau \qquad (5.2)$$

so that the end-of-period expected value of the corporation is increased by τ times the amount of debt issued.

The total return to investors from owning the corporation over this peirod is

$$\frac{\widetilde{V}'_t+\widetilde{X}_1(1-\tau)-V}{V} \qquad (5.3)$$

where $\widetilde{X}_1(1-\tau)$ is the realized net operating income of the corporation during the period. Market equilibrium requires that the expected value of (5.3) be equal to:

$$r(1-T)+H\,COV[(\widetilde{V}'_t+\widetilde{X}_1(1-\tau))/V,\widetilde{R}_m]-T\frac{D_1}{V} \qquad (5.4)$$

where D_1 is the amount of dividends paid in the first period.

Then equating these two expressions and solving for V, we obtain:

[19] [7]

$$V = \frac{\overline{V}'_t + \overline{X}_1(1-\tau)}{1+r(1-T)} \tag{5.5}$$

$$\frac{-H\,COV[\widetilde{V}'_t + \widetilde{X}_1(1-\tau, \widetilde{R}_m] - TD_1}{1+r(1-T)}$$

The constraint on share re-purchase for the first period may be written:

$$D_1 + (1-\tau)\,rB \geqq \overline{X_1}(1-\tau) + \triangle B \quad (5.6)$$

Note that (5.6) explicitly excludes use of the proceeds of the bond issue for share repurchase.

Now $$\frac{dV}{d\triangle B} = \frac{\partial V}{\partial \triangle B} + \frac{\partial V}{\partial D_1} \cdot \frac{dD_1}{d\triangle B} \tag{5.7}$$

(5.7) shows that the total impact of the expected debt issue on the value of the corporation has two components: the direct impact, $\frac{\partial V}{\partial \triangle B}$, and an indirect impact, $\frac{\partial V}{\partial D_1} \cdot \frac{\partial D_1}{\partial \triangle B}$, due to the consequent change in first period dividends. If the repurchase constraint (5.6) is binding, $\frac{\partial D_1}{\partial \triangle B} = 1$, so that:

$$\frac{dV}{d\triangle B} = \frac{\tau - T}{1+r(1-T)} \tag{5.8}$$

Thus when the repurchase constraint is binding, (5.8) shows that corporate debt issuance will be advantageous so long as the corporate tax rate τ exceeds T. However, it is clear that when a binding repurchase constraint requires the proceeds of a bond issue to be paid out in dividends, the prima facie advantage of adding debt to the capital structure may be substantially reduced. The reason for this is that, while the value of the corporation will tend to be raised by the expected corporate tax savings due to the bond issue, it will tend to be reduced by the higher personal taxes which investors must pay on the increased first period dividend entailed by the bond issue.

VI

In this paper we have discussed the impact of the personal tax structure on optimal corporate financial policy. In Section 1, we argued that the Farrar-Selwyn analysis was misleading on account of its neglect of the market trading opportunities open to investors. It was argued that once these were acknowledged, the welfare of all investors in the corporation would be maximized by the maximization of the market value of the firm. This was therefore accepted as the appropriate criterion of financial policy. Section III was concerned with the effect of dividend policy on the value of the corporation within the framework of the market equilibrium condition developed in Section II. It was shown that so long as the market's "effective tax rate" T, exceeds zero, the payment of dividends will be detrimental to the interests of all investors. A constraint on systematic share repurchase was then invoked to explain the observed behavior of corporations.

Section IV analyzed the effect of alternative capital structure policies on the value of the corporation allowing for this repurchase constraint. It was shown that if the constraint is binding the effects of alternative capital structures are the same as found by Modigliani and Miller neglecting investor taxes. However, if the constraint is not binding then the advantages of a high debt capital structure are reduced by the existence of investor taxes.

Finally, Section V extended the analysis of the previous section to the case in which a binding share repurchase constraint precludes use of the proceeds of a bond issue to repurchase outstanding shares. It was shown that in this case the advantages of issuing corporate debt may be substantially reduced by the consequent need to pay out the proceeds in dividends.

The author hopes to present in a later paper the results of some attempts to derive empirical estimates of the market's "effective tax rate" T. If T is approximately zero, then the Modigliani-Miller propositions concerning capital structure and dividend policies remain substantially unaltered by the existence of investor taxes. But if T is non-zero, then their results must be altered along the lines suggested in this paper.

REFERENCES

1. Farrar, Donald E. and Lee L. Selwyn, "Taxes, Corporate Financial Policies and Returns to Investors", *National Tax Journal,* Vol. 20, No. 4 (December 1967), pp. 444-454.

2. Gordon, Myron J., "The Savings Investment and Valuation of a Corporation", *Review of Economics and Statistics,* Vol. 45, No. 1, (February 1962), pp. 37-51.

3. Lintner, John, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets", *Review of Economics and Statistics,* Vol. 47, No. 1 (February 1965), pp. 13-37.

4. Miller, Merton H. and Franco Modigliani, "Dividend Policy, Growth and the Valuation of Shares", *Journal of Business,* Vol. 34, No. 4 (October 1961), pp. 411-433.

5. Modigliani, Franco and Merton H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment", *American Economic Review,* Vol. 48, No. 3, (June 1958), pp. 261-297.

6. ————, "The Cost of Capital, Corporation Finance, and the Theory of Investment: Reply", *American Economic Review,* Vol. 49, No. 4, (September 1959), pp. 655-669.

7. ————, "Corporate Income Taxes and the Cost of Capital: A Correction", *American Economic Review,* Vol. 53, No. 3, (June 1963), pp. 433-443.

8. Mossin, Jan. "Equilibrium in a Capital Asset Market", *Econometrica,* Vol. 34, No. 4 (October 1966), pp. 768-783.

9. Myers, S. C., "Taxes, Corporate Financial Policy and the Return to Investors: Comment", *National Tax Journal,* Vol. 20, No. 4 (December 1967), pp. 455-462.

10. Robichek, Alexander A. and Stewart C. Myers, *Optimal Financing Decisions,* Englewood Cliffs, Prentice-Hall, 1965.

11. Sharpe, W. F., "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk", *Journal of Finance,* Vol. 19, No. 3 (September, 1964), pp. 425-442.

12. Tinsley, P.A., "Capital Structure, Precautionary Balances, and Valuation of the Firm: The Problem of Financial Risk", *Journal of Financial and Quantitative Analysis,* Vol. 5, No. 1 (March 1970), pp. 33-62.