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WITNESS: ROGER A. MORIN

BEFORE THE
WASHINGTON UTILITIES AND TRANSPORTATION COMMISSION

WASHINGTON UTILITIES AND TRANSPORTATION COMMISSION, **Complainant,** **v.** **PUGET SOUND ENERGY, INC.,** **Respondent.**	**Docket No. UE-060266** **Docket No. UG-060267**

THIRD EXHIBIT (NONCONFIDENTIAL) TO THE
PREFILED REBUTTAL TESTIMONY OF
ROGER A. MORIN
ON BEHALF OF PUGET SOUND ENERGY, INC.

AUGUST 23, 2006

Arithmetic versus Geometric Means in Estimating the Cost of Capital

The use of the arithmetic mean appears counter-intuitive at first glance, because we commonly use the geometric mean return to measure the average annual achieved return over some time period. For example, the long-term performance of a portfolio is frequently assessed using the geometric mean return.

But performance appraisal is one thing, and cost of capital estimation is another matter entirely. In estimating the cost of capital, the goal is to obtain the rate of return that investors expect, that is, a target rate of return. On average, investors expect to achieve their target return. This target expected return is in effect an arithmetic average. The achieved or retrospective return is the geometric average. In statistical parlance, the arithmetic average is the unbiased measure of the expected value of repeated observations of a random variable, not the geometric mean.

The geometric mean answers the question of what constant return you would have had to achieve in each year to have your investment growth match the return achieved by the stock market. The arithmetic mean answers the question of what growth rate is the best estimate of the future amount of money that will be produced by continually reinvesting in the stock market. It is the rate of return which, compounded over multiple periods, gives the mean of the probability distribution of ending wealth.

While the geometric mean is the best estimate of performance over a long period of time, this does not contradict the statement that the arithmetic mean compounded over the number of years that an investment is held provides the best estimate of the ending wealth value of the investment. The reason is that an investment with uncertain returns will have a higher ending wealth value than an investment which simply earns (with certainty) its compound or geometric rate of return every year. In other words, more money, or terminal wealth, is gained by the occurrence of higher than expected returns than is lost by lower than expected returns.

In capital markets, where returns are a probability distribution, the answer that takes account of uncertainty, the arithmetic mean, is the correct one for estimating discount rates and the cost of capital.

While the geometric mean is appropriate when measuring performance over a long time period, it is incorrect when estimating a risk premium to compute the cost of capital.

Theory

The geometric mean measure the magnitude of the returns, as the investor starts with one portfolio and ends with another. It does not measure the variability of the journey, as does the arithmetic mean. The geometric mean is backward looking. There is no difference in the geometric mean of two stocks or portfolios, one of which is highly volatile and the other of which is absolutely stable. The arithmetic mean, on the other hand, is forward looking in that it does impound the volatility of the stocks.

To illustrate, Table 1 shows the historical returns of two stocks, the first one is highly volatile with a standard deviation of returns of 65% while the second one has a zero standard deviation. It makes no sense intuitively that the geometric mean is the correct measure of return, one that implies that both stocks are equally risky since they have the same geometric mean. No rational investor would consider the first stock equally as risky as the second stock. Every financial model to calculate the cost of capital recognizes that investors are risk averse and avoid risk unless they are adequately compensate for undertaking it. It is more consistent to use the mean that fully impounds risk (arithmetic mean) than the one from which risk has been removed (geometric mean). In short, the arithmetic mean recognizes the uncertainty in the stock market while the geometric mean removes the uncertainty by smoothing over annual differences.

Empirical Evidence

If both the geometric and arithmetic mean returns over the 1926-2004 data are regressed against the standard deviation of returns for the firms in the deciles, the arithmetic mean outperforms the geometric mean in this statistical regression. Moreover the constant of arithmetic mean regression matches the average Treasury bond rate and therefore makes economic sense while the constant for the geometric mean matches nothing in particular. This is simply because the geometric mean is stripped of volatility information and, as a result, does a poor job of forecasting returns based on volatility.

Table 1 Geometric vs. Arithmetic Returns

	Stock A	**Stock B**
1996	50.0%	11.61%
1997	-54.7%	11.61%
1998	98.5%	11.61%
1999	42.2%	11.61%
2000	-32.3%	11.61%
2001	-39.2%	11.61%
2002	153.2%	11.61%
2003	-10.0%	11.61%
2004	38.9%	11.61%
2005	20.0%	11.61%
Standard Deviation	64.9%	0.0%
Arithmetic Mean	26.7%	11.6%
Geometric Mean	11.6%	11.6%

The following illustration is frequently invoked in defense of the geometric mean. Suppose that a stock's performance over a two-year period is representative of the probability distribution, doubling in one year (r_1 = 100%) and halving in the next (r_2 = -50%). The stock's price ends up exactly where it started, and the geometric average annual return over the two-year period, r_g, is zero:

$$\begin{aligned} 1 + r_g &= [(1 + r_1)(1 + r_2)]^{1/2} \\ &= [(1 + 1)(1 - .50)]^{1/2} = 1 \\ r_g &= 0 \end{aligned}$$

confirming that a zero year-by-year return would have replicated the total return earned on the stock. The expected annual future rate of return on the stock is not zero, however. It is the arithmetic average of 100% and -50%, (100-50)/2 = 25%. There are two equally likely outcomes per dollar invested: either a gain of $1 when r = 100% or a loss of $0.50 when r = -50%. The expected profit is ($1-$.50)/2 = $.25 for a 25% expected rate of return. The profit in the good year more than offsets the loss in the bad year, despite the fact that the geometric return is zero. The arithmetic average return thus provides the best guide to expected future returns.

What Academics Have to Say

Bodie, Kane, and Marcus cite:

> Which is the superior measure of investment performance, the arithmetic average or the geometric average? The geometric average has considerable appeal because it represents the constant rate of return we would have needed to earn in each year to match actual performance over some past investment period. It is an excellent measure of **past** performance. However, if our focus is on future performance, then the arithmetic average is the statistic of interest because it is an unbiased estimate of the portfolio's expected future return (assuming, of course, that the expected return does not change over time). In contrast, because the geometric return over a sample period is always less than the arithmetic mean, it constitutes a downward-biased estimator of the stock's expected return in any future year.
>
> Again, the arithmetic average is the better guide to future performance.

Another way of stating the Bodie, Kane, Marcus argument in favor of the arithmetic mean is that the latter is the best estimate of the future value of the return distribution because it represents the expected value of the distribution. It is most useful for determining the central tendency of a distribution at a particular time, that is, for cross-sectional analysis. The geometric mean, on the other hand, is best suited for measuring an investment's compound rate of return over time, that is, for time-series analysis. This is the same argument made by Ibbotson Associates (2005) where it is shown, using probability theory, that future terminal wealth is given by compounding the arithmetic mean, and not the geometric mean. In other words, if we accept the past as prologue, the best estimate of a future year's return based on a random distribution of the prior years' returns is the arithmetic average. Statistically, it is our best guess for the holding-period return in a given year.

Brigham & Ehrhardt (2005) in their widely-used corporate finance text point out that the arithmetic average is more consistent with CAPM theory as one of its key underpinning assumptions is that investors are supposed to focus, in their portfolio decisions, upon returns in the next period and the standard deviation of this return. To the extent that this next period is one year, the preference for the arithmetic mean which derives from a set of single one year period returns follows. It is also noteworthy that one of the crucial assumptions inherent in the CAPM is that investors are single-period expected utility of terminal wealth maximizers who choose among alternative portfolios on the basis of each portfolio's expected return and standard deviation.

Brealey, Myers, and Allen (2006) in their leading graduate textbook in corporate finance opt strongly for the arithmetic mean. The authors illustrate the distinction between arithmetic and geometric averages and conclude that arithmetic averages are appropriate when estimating the cost of capital:

> The proper uses of arithmetic and compound rates of return from past investments are often misunderstood. Therefore, we call a brief time-out for a clarifying example.
>
> Suppose that the price of Big Oil's common stock is \$100. There is an equal chance that at the end of the year the stock will be worth \$90, \$110, or \$130. Therefore, the return could be –10 percent, +10 percent or +30 percent (we assume that Big Oil does not pay a dividend). The expected return is 1/3(-10+10+30)= +10 percent.
>
> If we run the process in reverse and HQ Distributionunt the expected cash flow by the expected rate of return, we obtain the value of Big Oil's stock:
>
> $$PV = \frac{110}{1.10} = \$100$$
>
> The expected return of 10 percent is therefore the correct rate at which to discount the expected cash flow from Big Oil's stock. It is also the opportunity cost of capital for investments which have the same degree of risk as Big Oil.
>
> Now suppose that we observe the returns on Big Oil stock over a large number of years. If the odds are unchanged, the return will be –10 percent in a third of the years, +10

> percent in a further third, and +30 percent in the remaining years. The arithmetic average of these yearly returns is
>
> $$\frac{-10 + 10 + 30}{3} = +10\%$$
>
> Thus the arithmetic average of the returns correctly measures the opportunity cost of capital for investments of similar risk to Big Oil stock.
>
> The average compound annual return on Big Oil stock would be
>
> $$(.9 \times 1.1 \times 1.3)^{1/3} - 1 = .088, \text{ or } 8.8\%$$
>
> less than the opportunity cost of capital. Investors would not be willing to invest in a project that offered an 8.8 percent expected return if they could get an expected return of 10 percent in the capital markets. The net present value of such a project would be
>
> $$NPV = -100 + \frac{108.8}{1.1} = -1.1$$
>
> Moral: If the cost of capital is estimated from historical returns or risk premiums, use arithmetic averages, not compound annual rates of return (geometric averages)."

Richard A. Brealey, Stewart C. Myers, and Paul Allen, *Principles of Corporate Finance*, 8th Edition, Irwin McGraw-Hill, 2006, page 156-7.)

The widely-cited Ibbotson & Associates publication also contains a detailed and rigorous discussion of the impropriety of using geometric averages in estimating the cost of capital[1].

> The arithmetic average equity risk premium can be demonstrated to be most appropriate when discounting future cash flows. For use as the expected equity risk premium in either the CAPM or the building block approach, the arithmetic mean or the simple difference of the arithmetic means of stock market returns and riskless rates is the relevant number. This is because both the CAPM and the building block approach are additive models, in which the cost of capital is the sum of its parts. The geometric

[1] Ibbotson Associates, Stocks Bonds Bills and Inflation, Valuation Edition 2005 Yearbook, page 75.

> average is more appropriate for reporting past performance, since it represents the compound average return.
>
> The argument for using the arithmetic average is quite straightforward. In looking at projected cash flows, the equity risk premium that should be employed is the equity risk premium that is expected to actually be incurred over the future time periods.
>
> The best estimate of the expected value of a variable that has behaved randomly in the past is the average (or arithmetic mean) of its past values.

In their widely publicized research on the market risk premium, Dimson, Marsh and Staunton (2002) state

> The arithmetic mean of a sequence of different returns is always larger than the geometric mean. To see this, consider equally likely returns of +25 and –20 percent. Their arithmetic mean is 2½ percent, since (25 – 20)/2 = 2½. Their geometric mean is zero, since (1 + 25/100) x (1 – 20/100) – 1 = 0. But which mean is the right one for discounting risky expected future cash flows? For forward-looking decisions, the arithmetic mean is the appropriate measure.
>
> To verify that the arithmetic mean is the correct choice, we can use the 2½ percent required return to value the investment we just described. A $1 stake would offer equal probabilities of receiving back $1.25 or $0.80. To value this, we discount the cash flows at the arithmetic mean rate of 2½ percent. The present values are respectively $1.25/1.015 = $1.22 and $0.80/1.025 = $0.78, each with equal probability, so the value is $1.22 x ½ + $0.80 x ½ = $1.00. If there were a sequence of equally likely returns of +25 and –20 percent, the geometric mean return will eventually converge on zero. The 2½ percent forward-looking arithmetic mean is required to compensate for the year-to-year volatility of returns."

Lastly, on the practical side, Bruner, Eades, Harris, and Higgins (1998) found that 71% of the texts and tradebooks in their extensive survey of practice supported use of an arithmetic mean for estimation of the cost of equity.

Mean Reversion Argument

Some academics have argued that if stock returns were expected to revert to a trend, this would suggest the use of a geometric mean since the geometric mean is, by definition, an estimate of a smoothed long run trend increment. These same academics have argued that the historical estimate of the market risk premium ("MRP") is upward-biased by the buoyant performance of the stock market prior to 2002, and because of the extraordinary and unusually high realized MRPs in those years, investors expect a return to lower MRPs in the future, bringing the average MPR to a more "normal" level.

The presence or absence of mean reversion is an empirical issue. The empirical findings are weak and highly contradictory; the empirical evidence is inconclusive and unconvincing, certainly not enough to support the "mean reversion" hypothesis. The weight of the empirical evidence on this issue is that the more sophisticated tests of mean reversion in the MRP demonstrate that the realized MRP over the last 75 years or so was almost perfectly free of mean reversion, and had no statistically identifiable time trend. It is also noteworthy that most of these studies were performed prior to the stock market's debacle in 2000-2002, years of extraordinary and unusually low realized MRPs. The stock's market dismal performance of 2000-2002 has certainly taken the wind out of the mean reversion school's sails.

An examination of historical MRPs reveals that the MRP is random with no observable pattern and. To the extent that the estimated historical equity risk premium follows what is known in statistics as a random walk, one should expect the equity risk premium to remain at its historical mean. Therefore, the best estimate of the future risk premium is the historical mean.

Ibbotson Associates (2005) find no evidence that the market price of risk or the amount of risk in common stocks has changed over time:

> Our own empirical evidence suggests that the yearly difference between the stock market total return and the U.S. Treasury bond income return in any particular year is random……..there is no discernable pattern in the realized equity risk premium.

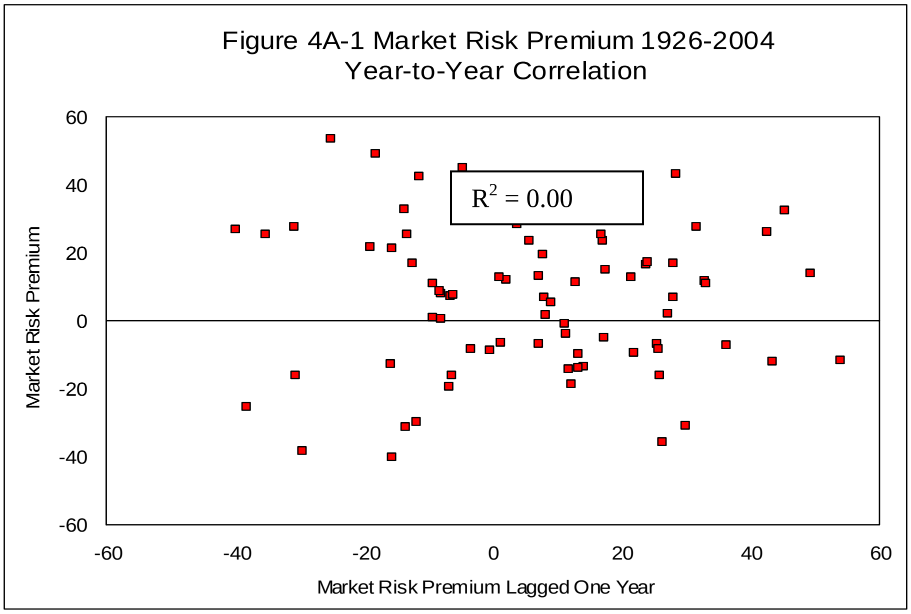
Ibbotson Associates, *Stocks Bonds Bills and Inflation, Valuation Edition 2005 Yearbook* 74-75.

In statistical parlance, there is no significant serial correlation in successive annual market risk premiums, that is, no trend. Ibbotson Associates go on to state that it is reasonable to assume that these quantities will remain stable in the future:

> The best estimate of the expected value of a variable that has behaved randomly in the past is the average (or arithmetic mean) of its past values.

Id. at 75. Nowhere is it suggested by Ibbotson Associates that the market risk premium has declined over time.

Because there is little evidence that the MRP has changed over time, it is reasonable to assume that these quantities will remain stable in the future. Figure 4A-1 below shows the relationship, or the lack of relationship, between year-to-year MRP's reported in the Ibbotson Associates Valuation yearbook, 2005 edition for the 1926-2004 period. The relationship is virtually absent, as indicated by the low R^2 of zero between successive MRPs. In other words, there is no history in successive MRPs as indicated by the zero serial correlation coefficient.



In short, the determination of the cost of capital with the CAPM requires an unbiased estimate of the expected annual return. The expected arithmetic return provides the appropriate measure for this purpose.

Formal Demonstration

This section shows why arithmetic rather than geometric means should be used for forecasting, discounting, and estimating the cost of capital[2]. By definition, the cost of equity capital is the annual discount rate that equates the discounted value of expected future cash flows (from dividends and the sale of the stock at the end of the investor's investment horizon) to the current market price of a share in the firm. The discount rate that equates the discounted value of future expected dividends and the end of period expected stock price to the current stock price is a prospective arithmetic, rather than a prospective geometric mean rate of return. Since future dividends and stock prices cannot be predicted with certainty, the "expected" annual rate of return that investors require is an average "target" percentage rate around which the actual, year-by-year returns will vary. This target rate is, in effect, an arithmetic average.

A numerical illustration will clarify this important point. Consider a non-dividend paying stock trading for $100 which has, in every year, an equal chance of appreciating by 20% or declining by 10%. Thus, after one year, there is an equal chance that the stock's price will be $120 and an equal chance the price will be $90. Figure 4A-2 presents all

[2] This section is adapted from a similar treatments and demonstration in Brealey, Myers, and Allen (2006) and Ibbotson Associates (2005).

possible eventualities after two periods have elapsed (the rates of return are presented at the end of the lines in the diagram).

The possible stock prices are shown in the Table 2.

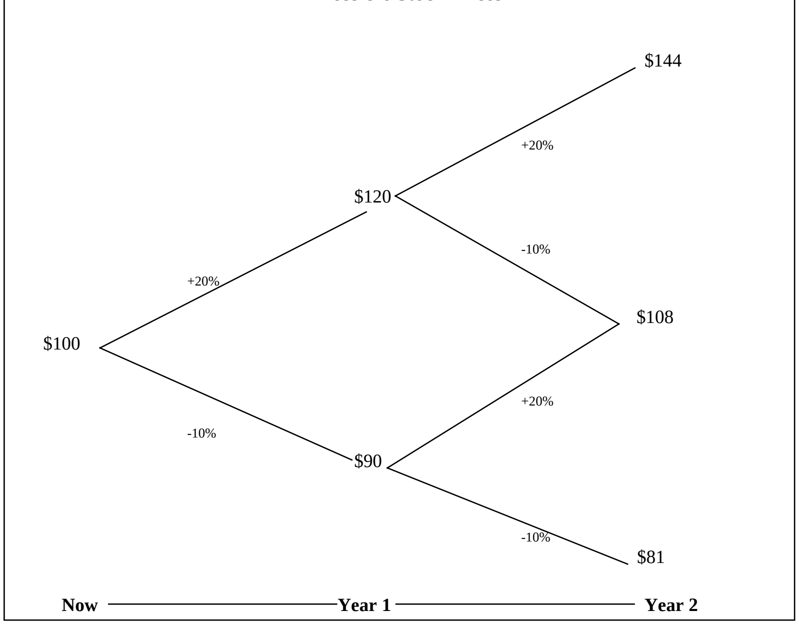
TABLE 2
STOCK PRICES AFTER TWO PERIODS

Price	Chance
$144	1 chance in 4
$108	2 chances in 4
$81	1 chance in 4

The expected future stock price after two periods is then:

$$1/4\ (\$144) + 2/4\ (\$108) + 1/4\ (\$81) = \$110.25$$

Figure 4A-2
Possible Stock Prices



The cost of equity capital is calculated as the discount rate that equates the present value of the future expected cash flows to the current stock price. In the present simple example, the only cash flow is the gain from selling the stock after two periods have elapsed. Thus, using the expected stock price of $110.25 calculated above, the expected rate of return is that r, which solves the following equation:

$$\text{Current Stock Price} = \frac{\text{Expected Stock price}}{(1 + r)^2}$$

The factor $(1 + r)^2$ discounts the expected stock price to the present. Substituting the numerical values, we have:

$$\$100 = \frac{\$100.25}{(1+r)^2}$$

$$r = 5\%$$

Thus, the cost of equity capital is 5%. This 5% cost of equity capital is equal to the prospective arithmetic mean rate of return, which is the probability-weighted average single period rate of return on equity. Since in every period there is an equal chance that the stock's return will be 20% or -10%, the probability-weighted average is:

$$1/2\ (20\%) \quad + \quad 1/2\ (-10\%) \quad = \quad 5\%$$

However, the 5% cost of equity capital is not equal to the prospective geometric mean rate of return, which is a probability-weighted average of the possible compounded rates of return over the two periods. Now consider the prospective geometric mean rate of return. Table 3 shows the possible compounded rates of return over two periods, and the probability of each.

TABLE 3
STOCK PRICES AND RETURNS AFTER TWO PERIODS

Price	**Chance**	**Compounded Return**
$144	1 chance in 4	20.00%
$108	2 chances in 4	3.92%
$81	1 chance in 4	-10.00%

Thus, the prospective geometric mean rate of return is:

$$1/4\ (20\%) \;=\; 2/4\ (3.92\%) + 1/4\ (-10\%) \;=\; 4.46\%$$

This return is not equal to the 5% cost of equity capital.

The example can easily be extended to include the case of a dividend-paying company and reached the same conclusion: the implied discount rate calculated in the DCF model is an expected arithmetic rather than an expected geometric mean rate of return.

The foregoing analysis shows that it is erroneous to use a prospective multi-year geometric mean rate of return as a "target" rate of return for each year of the period. If, for example, investors currently require an expected future rate of return on an investment of 13% each year, then 13% is the appropriate annual rate of return on equity for ratemaking purposes. Consequently, in using a risk premium approach for the purposes of rate of return regulation, the single-year annual required rate of return should be estimated using arithmetic mean risk premiums.

It should be pointed out that the use of the arithmetic mean does not imply an investment holding period of one year. Rather, it is premised on the uncertainty with respect to each year's return during the holding period, however how many years that may be. When computing the arithmetic average of historic annual returns in order to calculate the average return (expected value of the return), every achieved return outcome is one possible future outcome for each year the security will be held. Each historic return has an equal probability of occurring during each year of the holding period. The resulting expected value of the risk premium is the arithmetic average of all of the past premiums considered, regardless of the length of the expected holding period.

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